Effective time reversal and echo dynamics in the transverse field Ising model

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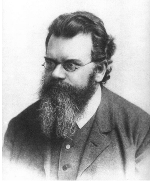
Outline

1. Irreversibility in classical systems
2. Quantifying Irreversibility in Quantum Many-Body Systems
3. Echos in the transverse field Ising model
4. Turning on interactions

Irreversibility in classical systems

H-Theorem

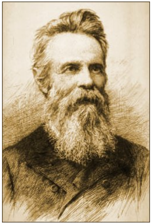


$$\frac{dS}{dt} \geq 0$$

This provides an analytical proof of the Second Law [...]

[Boltzmann, 1872]

Loschmidt's paradox



Obviously, in every arbitrary system the course of events must become retrograde when the velocities of all its elements are reversed.

[Loschmidt, 1876]

[Image source: Wikipedia]

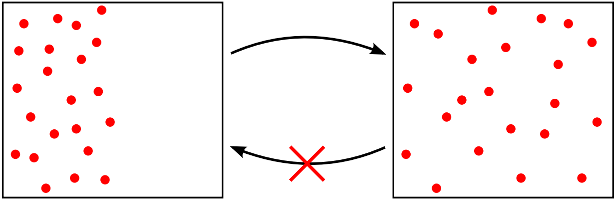
Irreversibility in classical systems

H-Theorem

dS

Irreversibility - “Then try and do it!”

The dynamics of a system are irreversible if it is practically impossible to induce the evolution from a uniform (disordered) state to an ordered state, although the other direction is well possible.



the velocities of all its elements are reversed.

[Loschmidt, 1876]

[Image source: Wikipedia]

Quantifying irreversibility in quantum systems

The protocol

[Peres, PRA (1984)]

Consider imperfect effective time reversal

$$H \to -H + \epsilon V$$

in an echo experiment

$$|\psi_0\rangle \overset{e^{-\mathrm{i}H\tau}}{\mapsto} |\psi(\tau)\rangle \overset{e^{\mathrm{i}(H-\epsilon V)\tau}}{\mapsto} |\psi(2\tau)\rangle \overset{?}{\approx} |\psi_0\rangle$$

Loschmidt echo

Natural measure for resemblance the of q.m. states

$$\mathcal{L}(\tau) = \left|\langle\psi_0|\psi(2\tau)\rangle\right|^2$$

[Gorin et al., Phys. Rep. (2006); Jacquod and Petitjean, Adv. Phys. (2009)]

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Eigenstate Thermalisation Hypothesis

[Deutsch, PRA (1991); Srednicki, PRE (1994); Rigol et al., Nature (2008)]

Close-by energy eigenstates, although orthogonal,

$$|\langle E_i|E_{i+1}\rangle| = 0$$

are indistinguishable by local observables,

$$|\langle E_i|O|E_i\rangle - \langle E_{i+1}|O|E_{i+1}\rangle| \sim \text{dim}(\mathcal{H})^{-1/2} .$$

Quantifying irreversibility in quantum systems

The protocol [Peres, PRA (1984)]

Consider imperfect effective time reversal

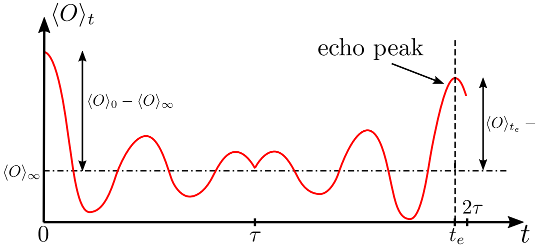
Loschmidt echo

Natural measure for

Measuring (ir-)reversibility in quantum many-body systems

Only observables can be used to quantify (ir-)reversibility in experiment.

$$E^*[X] = \frac{|X_{t_e} - X_\infty|}{|X_0 - X_\infty|}$$



$$|\langle E_i|O|E_i\rangle - \langle E_{i+1}|O|E_{i+1}\rangle| \sim \mathrm{dim}(\mathcal{H})^{-1/2} \ .$$

Experimental realisation

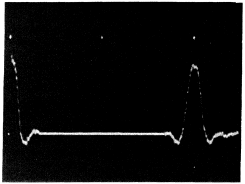
NMR and Magic Echo techniques

[RHIM ET AL., PRB (1971); SCHNEIDER AND SCHMIEDEL, PLA (1969)]

Experiments on dipolar-coupled spins

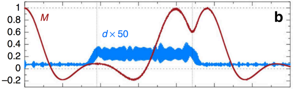
$$H = \sum_{i<j} b_{ij} \left(\vec{S}_i \cdot \vec{S}_j - 3 S_i^z S_j^z \right)$$

Magic echo technique: $H \to -\frac{1}{2} H + \epsilon V$



Other options for experimental realisation

- Highly controllable quantum simulators
  [BLOCH ET AL., NATURE PHYSICS (2012); GEORGESCU ET AL., REV. MOD. PHYS. (2014)]
- Recent theoretical result: time reversal by periodic driving
  [MENTINK ET AL., NATURE COMM. (2015)]



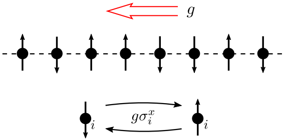
Echos in the transverse field Ising model

The transverse field Ising model

The Ising spin chain in a transverse magnetic field

$$H(g) = -J\sum_i S_i^z S_{i+1}^z + gJ\sum_i S_i^x$$

is a free fermion model.



- All quantities are computed analytically.

Echos in the transverse field Ising model

Mapping to free fermions

[Pfeuty, Ann. Phys. (1970)]

$$H(J,g) = J\left(-\sum_i S_i^z S_{i+1}^z + g\sum_i S_i^x\right)$$

$\updownarrow$ Jordan-Wigner + Fourier transformation

$$= \sum_{k>0} \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \begin{pmatrix} d_k^z(g) & -id_k^y \\ id_k^y & -d_k^z(g) \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}\ , \quad \{c_k, c_{k'}^\dagger\} = \delta_{k,k'}$$

$\updownarrow$ Bogoliubov rotation

$$= \sum_k \epsilon_k(g)\lambda_k^\dagger\lambda_k\ , \quad \epsilon_k(g) = \sqrt{g^2 - 2g\cos k + 1}$$

Computation of magnetisation and correlation functions in the ground state

[Lieb et al., Ann. Phys. (1961)]

$$\langle O\rangle = f_O\left[d_k^y, d_k^z(g)\right]$$

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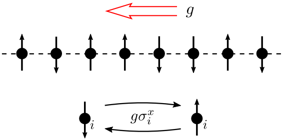

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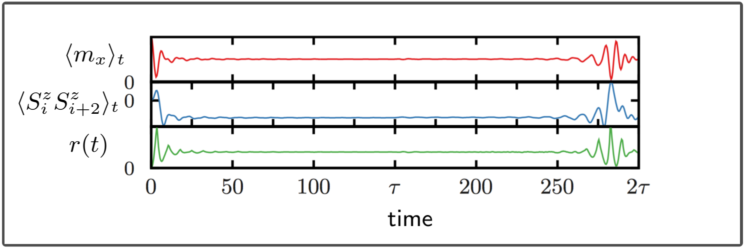
Imperfect effective time reversal

The echo operator for our protocol is

$$U_E^{g,\delta g}(t,\tau) = \begin{cases} e^{-\mathrm{i}H(g)\tau} & ,t<\tau \\ e^{\mathrm{i}H(g+\delta g)(t-\tau)}e^{-\mathrm{i}H(g)\tau} & ,t>\tau \end{cases}$$

Echos in the transverse field Ising model



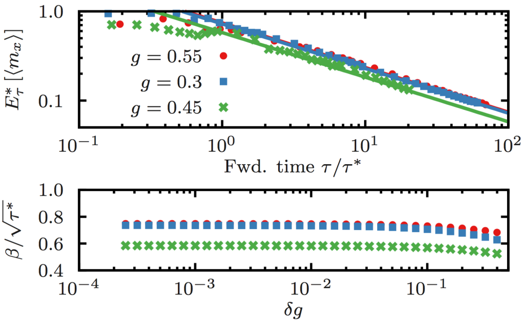
- Note the shift of the echo peak

$$t_e = (1 + \nu)\tau \neq 2\tau$$

- Reason: Quasiparticle velocities of $H(g)$ and $H(g + \delta g)$ differ.

Echos in the transverse field Ising model

Transverse magnetisation



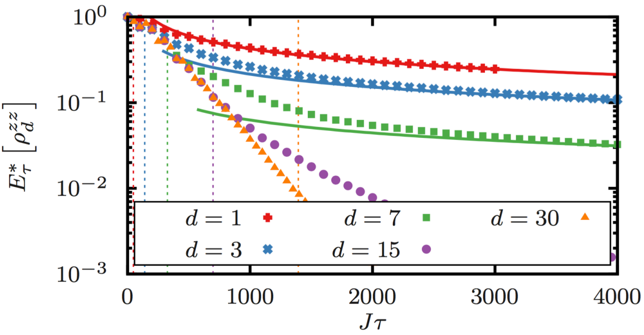
- Algebraic decay for $\tau \gg \tau^*$, $(\tau^*)^{-1} \propto \left(\epsilon_k^g - \nu\epsilon_k^{g+\delta g}\right)''_{k=k^*}$

$$E^*_\tau[\langle m_x \rangle] \approx \beta^{g,\delta g}_{k^*} \tau^{-1/2}$$

- Echo peak height at τ^* independent of δg.

Echos in the transverse field Ising model

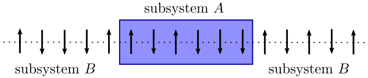
Longitudinal spin-spin correlation $\rho_n^{zz} = \langle S_i^z S_{i+n}^z \rangle$



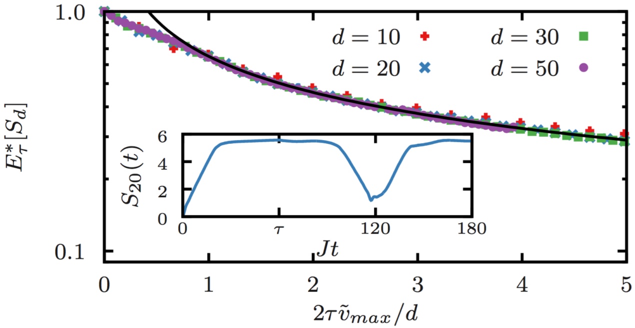
- Exponential-looking decay for $\tilde{v}_{\max}\tau \ll d$, $\tilde{v}_{\max} = \max_k \frac{d}{dk}(\epsilon_k^g - \nu_{k^*}^{g,g\delta}\epsilon_k^{g\delta})$
  - also found in simple quench dynamics [Calabrese et al., J.Stat.Mech. (2012)]
- Algebraic decay $\propto \tau^{-1/2}$ for long forward times

Echos in the transverse field Ising model

Entanglement entropy



$$S_A = \mathrm{tr}\left[\rho_A \log \rho_A\right], \quad \rho_A = \mathrm{tr}_B[\rho]$$



An alternative time reversal protocol

Consider the local Hamiltonian

$$H_P = -\alpha \sum_j (S_j^x S_{j+1}^y + h.c.) = 2\alpha \sum_k \sin k \ \left[\lambda_k^\dagger \lambda_{-k}^\dagger + \lambda_{-k}\lambda_k\right]$$

A pulse

$$e^{-\mathrm{i}H_P t_P} = \prod_k \left[\cos(2\alpha t_P \sin k) - \mathrm{i}\sin(2\alpha t_P \sin k)(\lambda_k^\dagger \lambda_{-k}^\dagger + \lambda_{-k}\lambda_k)\right]$$

leads to an imperfect inversion of the mode occupation $n_k = \langle \lambda_k^\dagger \lambda_k \rangle$. Hence,

$$e^{\mathrm{i}H_P t_P} e^{-\mathrm{i}H\tau} e^{-\mathrm{i}H_P t_P} = e^{\mathrm{i}(H+\delta H)\tau}$$

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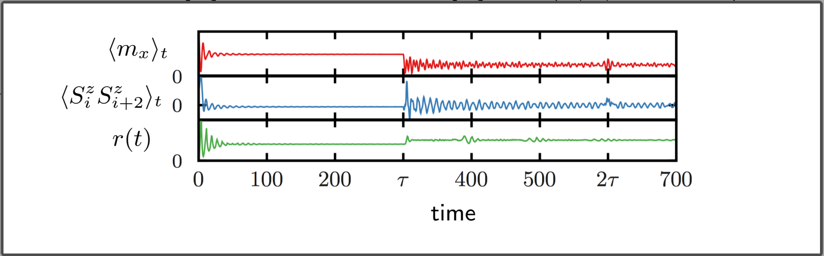

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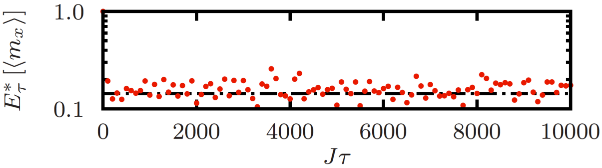
An alternative time reversal protocol

Echos in transverse magnetisation



- The transverse magnetisation never decays:

$$\langle m_x\rangle_{t_e=2\tau} = \langle m_x\rangle_\infty + \langle m_x\rangle_E + \langle m_x\rangle_\tau$$

$$\langle m_x\rangle_E = \frac{1}{2}\int\frac{dk}{2\pi}\sin\phi_k^{g,g_0}\left(1-\cos(4\alpha t_P\sin k)\right)$$

Turning on interactions

A longitudinal field component adds interactions

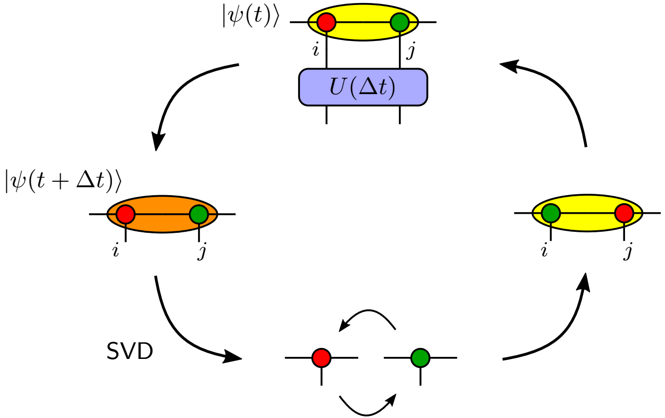
$$H = -J \sum_i S_i^z S_{i+1}^z - h_x \sum_i S_i^x - h_z \sum_i S_i^z$$

Dynamics computed by iTEBD.

Turning on interactions

The iTEBD cycle

[Kjäll et al., PRB (2013)]



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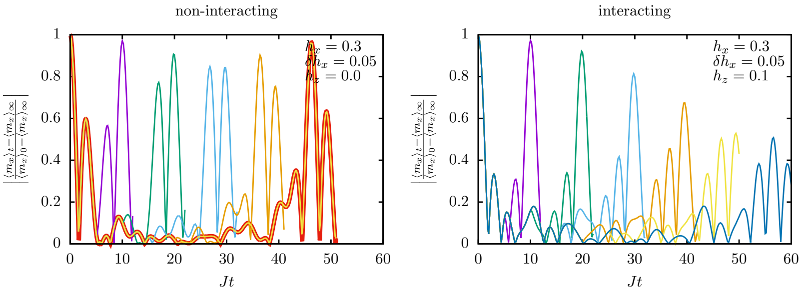


- Interactions change the decay characteristics drastically.

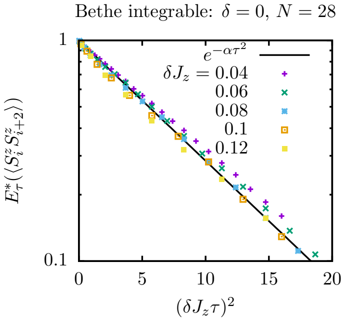
Turning on interactions

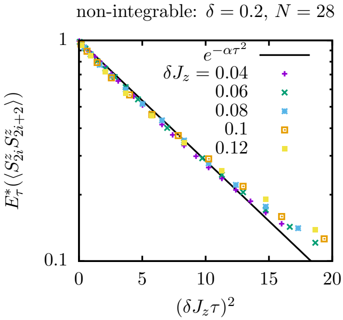
The role of integrability

The (dimerised) anisotropic spin-1/2 Heisenberg chain

$$H = J \sum_n (1 + (-1)^n \delta)(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + J_z \sum_n S_n^z S_{n+1}^z$$

Dynamics computed by Lanczos propagation ("ED").

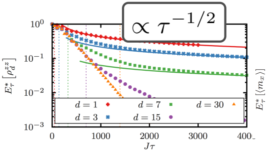


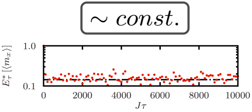


Conclusions

Echo peak decay of observables under imperfect effective time reversal:

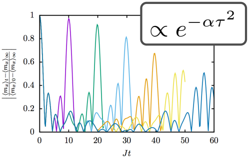
- non-interacting





[Schmitt and Kehrein, EPL **115** (2016)]

- interacting



Open questions

- Analytical understanding for interacting systems
- Connection to OTOCs: $\langle V(t)W(0)V(t)W(0)\rangle$ [Maldacena et al. (2015)]