The single-impurity Anderson model

The fundamental model to theoretically describe local moments in metals is the single-impurity Anderson model[1] Its Hamiltonian reads

$$\hat{H}_{\rm SIAM} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} \hat{c}^\dagger_{\vec{k}\sigma} \hat{c}_{\vec{k}\sigma} + \sum_{\sigma} \left(\epsilon_f + \frac{U}{2} \hat{f}^\dagger_{\bar{\sigma}} \hat{f}_{\bar{\sigma}} \right) \hat{f}^\dagger_\sigma \hat{f}_\sigma + \frac{1}{\sqrt{N}} \sum_{\vec{k}\sigma} \left[V_{\vec{k}} \hat{c}^\dagger_{\vec{k}\sigma} \hat{f}_\sigma + \text{h.c.} \right]$$

The operators $\hat{c}^{(\dagger)}_{\vec{k}\sigma}$ annihilate (create) states in a conduction band with dispersion $\epsilon_{\vec{k}}$, and $\hat{f}^{(\dagger)}_\sigma$ annihilates (creates) a fermionic state localized at lattice site $\vec{R} = 0$. The local level has an on-site energy ϵ_f and if two particles with opposite spin are present one has to pay the Coulomb energy U. Both subsystem talk to each other through a hybridization with strength $V_{\vec{k}}$. Usually one assumes $V_{\vec{k}} \equiv V$ independent of $\vec{k}$.

(a) Solve the SIAM in Hartree approximation for $\langle \hat{n}_f \rangle = \langle \hat{n}_{f,\uparrow} + \hat{n}_{f,\downarrow} \rangle = 1$, i.e. factorize

$$\hat{n}_{f,\uparrow}\hat{n}_{f,\downarrow} \rightarrow \hat{n}_{f,\uparrow}\langle \hat{n}_{f,\downarrow} \rangle + \langle \hat{n}_{f,\uparrow} \rangle \hat{n}_{f,\downarrow}$$

Allow for solutions $\langle \hat{n}_{f,\uparrow} \rangle \neq \langle \hat{n}_{f,\downarrow} \rangle$. When do these occur as function of the model parameters? Does the precise value and phase of the hybridization V play a role?
Hint: The solution is very easy when one uses Green's function methods.

(b) Discuss the physical relevance of the polarized Hartree solution. Do you expect it to be realized in impurity systems? What do you believe the system will do?

c) A very simplified version of the SIAM is the two-site model, i.e. we replace the band states by one level, too. Solve the resulting Hamiltonian in the limit $U = \infty$. What is the ground state and how does it depend on the model parameters?

[1] P.W. Anderson, Phys. Rev. **124**, 41 (1961).