Center for
Electronic Correlations and Magnetism
University of Augsburg

Theory of correlated fermionic condensed matter

4. Correlation-induced phenomena in fermionic matter

Superfluid Helium-3: From very low Temperatures to the Big Bang

XIV. Training Course in the Physics of Strongly Correlated Systems
Salerno, October 8, 2009

Dieter Vollhardt

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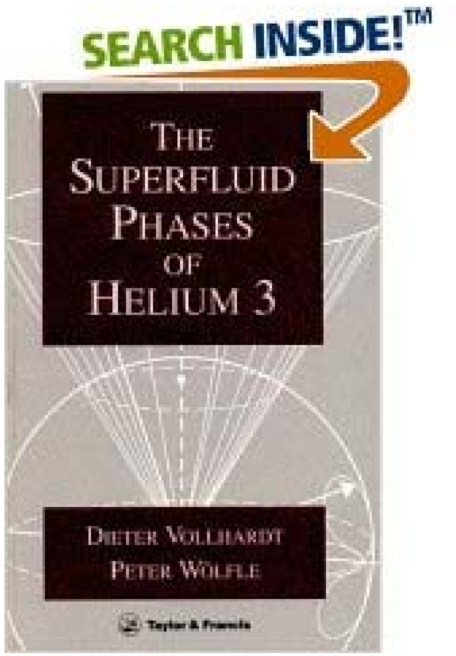
Contents:

- The quantum liquids ^{3}He and ^{4}He
- Superfluid phases of ^{3}He
- Broken symmetries and long-range order
- Topologically stable defects
- Big Bang simulation in the low temperature lab



The Superfluid Phases of Helium 3

D. Vollhardt and P. Wölfle

(Taylor & Francis, 1990)

Helium

Two stable Helium isotopes:

^{4}He: air, oil wells, ... Janssen/Lockyer (1868), Ramsay (1895)

^{3}He: $^6_3Li + ^1_0n \rightarrow ^3_1H + \alpha$ (1939)

$$\downarrow$$

$$^3_2He + e^-$$

$$\frac{\text{He}}{\text{air}} \approx 5\times10^{-6}, \quad \left.\frac{^3\text{He}}{^4\text{He}}\right|_{\text{air}} \approx 1\times10^{-6}$$

Research on macroscopic samples of ^{3}He since 1947

Helium

Atoms: spherical, hard core diameter ~ 2.5 Å

Interaction:
- hard sphere repulsion
- van der Waals dipole/multipole attraction

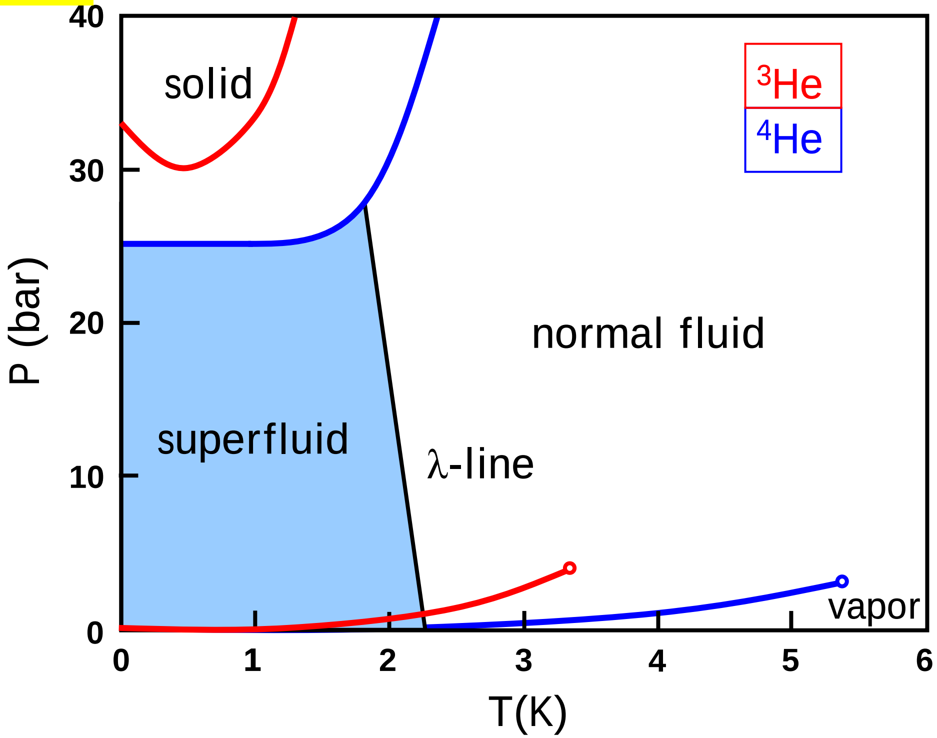
Boiling point: 4.2 K, ^4He Kamerlingh Onnes (1908)

Nobel Prize 1913

3.2 K, ^3He Sydoriak, *et al.* (1949)

Dense, simple liquids { isotropic
short-range interactions
extremely pure }

Helium

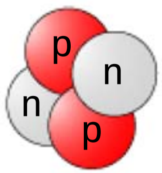
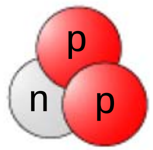


Atoms:
- spherical shape → weak attraction
- light mass → strong zero-point motion

T→0, P $\lesssim$ 30 bar: Helium remains liquid

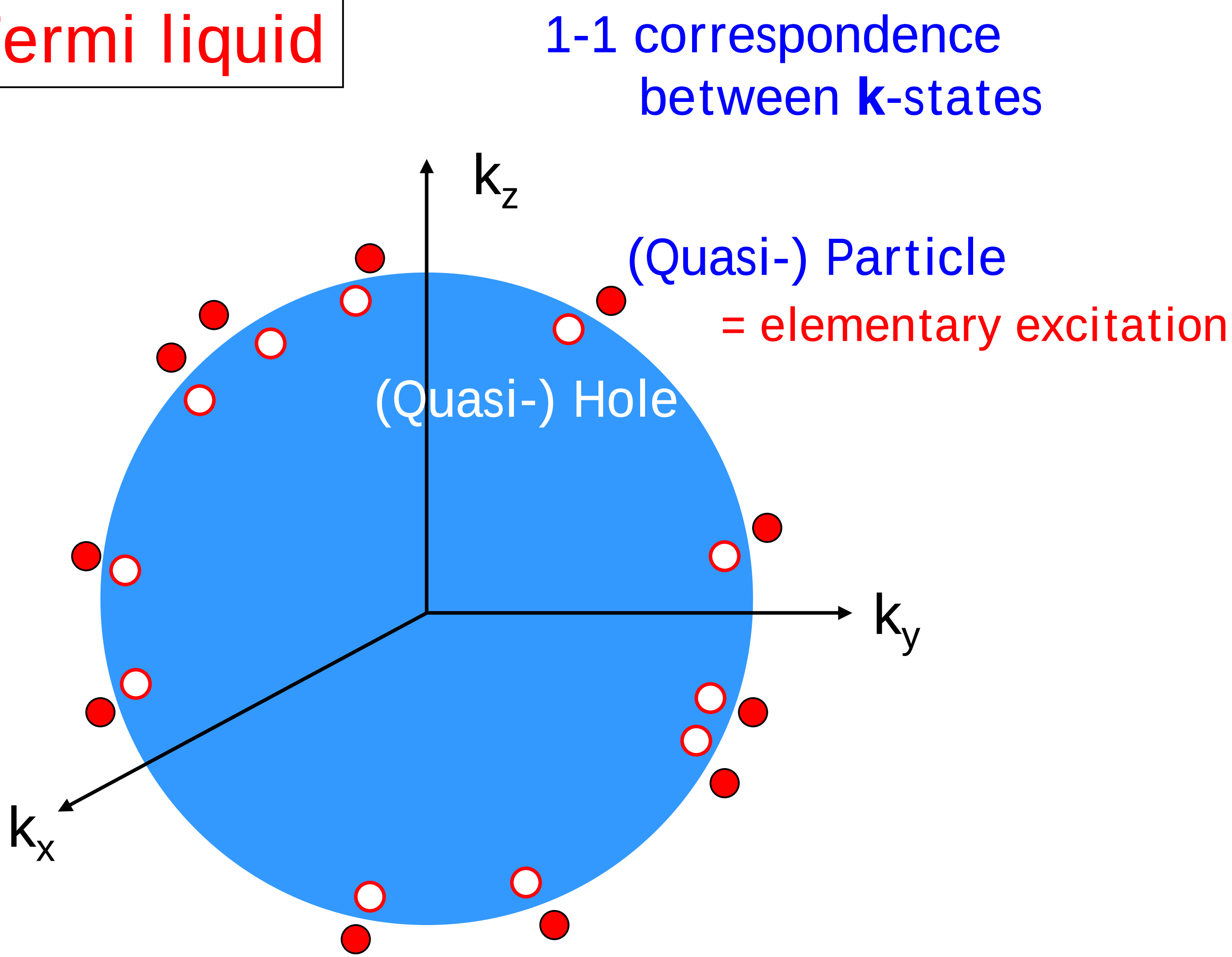
$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \to 0}$$ Macroscopic quantum phenomena

Helium

	^4He	^3He
Electron shell:	$2\ e^-$, $S = 0$	
Nucleus:		
	$S = 0$	$S = \frac{1}{2}\hbar$
Atom(!) is a	Boson	Fermion ➔ Quantum liquids
Phase transition	$T_\lambda = 2.2$ K ("BEC")	$T_c = ???$

Fermi liquid theory

Landau Fermi liquid

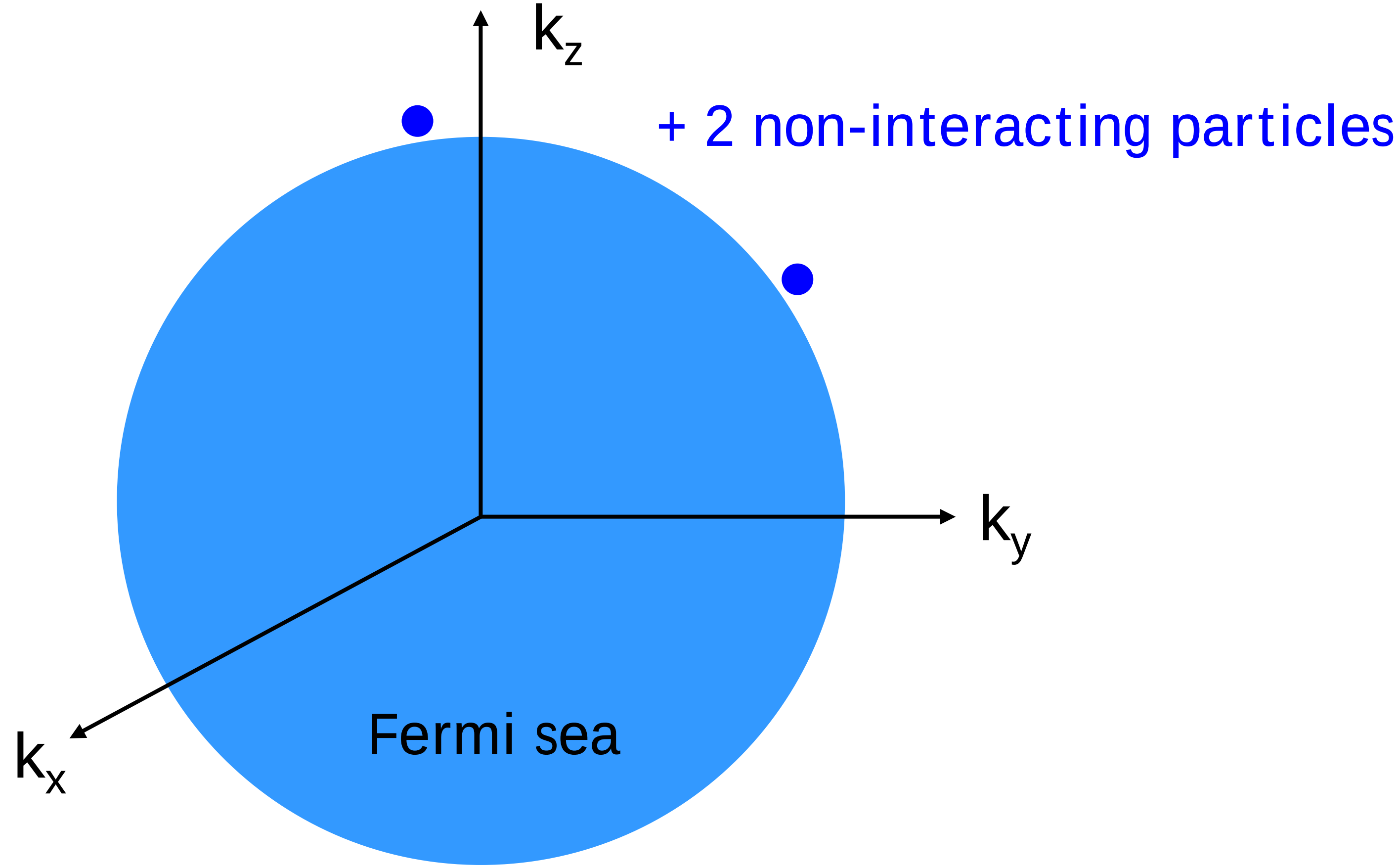


Prototype: Helium-3

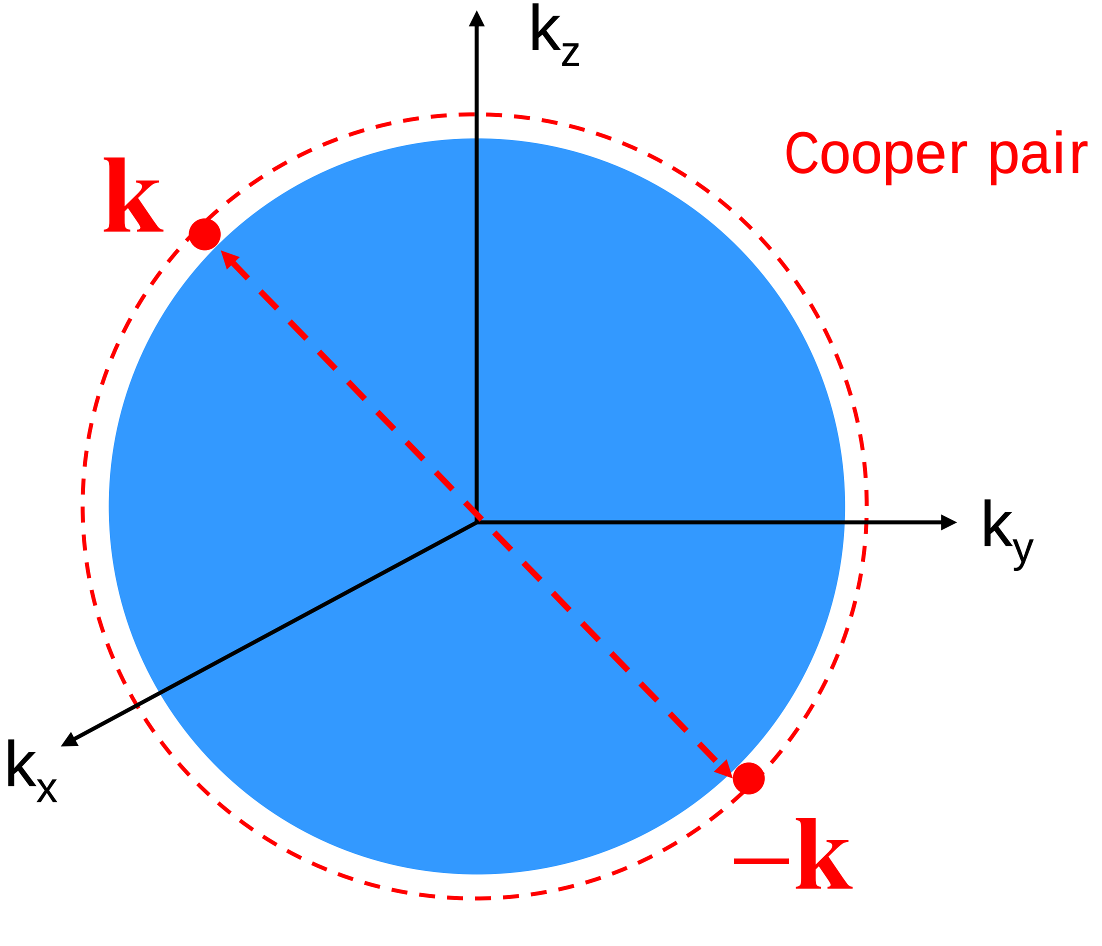
- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility

Instability ?

Instability of Landau Fermi liquid

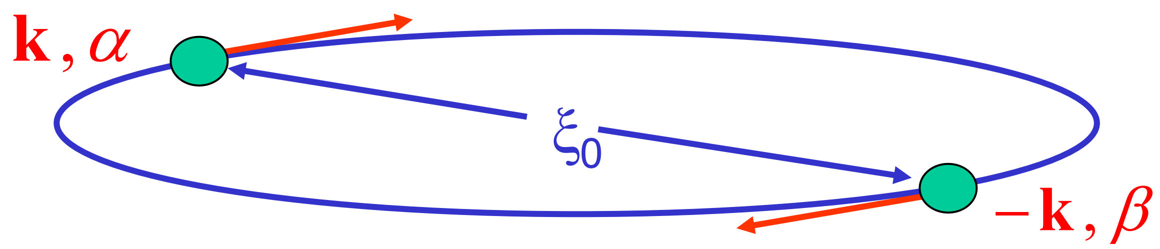


Arbitrarily weak attraction $\Rightarrow$ Cooper instability



Universal fermionic property

Arbitrarily weak attraction $\Rightarrow$ Cooper pair $(\mathbf{k},\alpha;-\mathbf{k},\beta)$



$$\Psi_{L=0,2,4,\ldots} = \psi(\mathbf{r})\left|\uparrow\downarrow - \downarrow\uparrow\right\rangle$$

S=0 (singlet)

$$\begin{aligned}\Psi_{L=1,3,5,\ldots} &= \psi_+(\mathbf{r})\left|\uparrow\uparrow\right\rangle \\ &+\psi_0(\mathbf{r})\left|\uparrow\downarrow + \downarrow\uparrow\right\rangle \\ &+\psi_-(\mathbf{r})\left|\downarrow\downarrow\right\rangle\end{aligned}$$

S=1 (triplet)

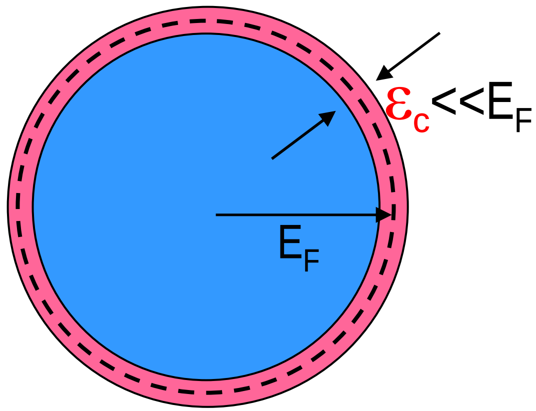
L = 0: isotropic wave function
L > 0: anisotropic wave function

Helium-3: Strongly repulsive interaction → L > 0 expected

BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs

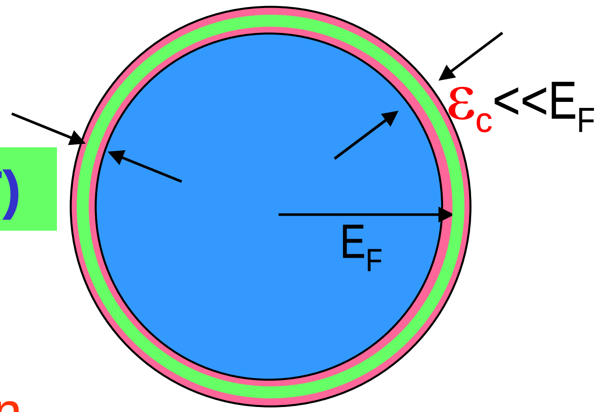


BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs

Energy gap $\Delta(T)$



→ "Pair condensate"
with macroscopically coherent wave function

Transition temperature $$T_c = 1.13\, \varepsilon_c \exp(-1/N(0)|V_L|)$$

"weak coupling theory"

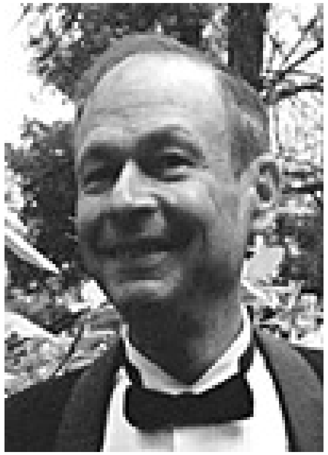
ε_c, V_L: Magnitude ? Origin ? ➔ T_c ?

Thanksgiving 1971: Transition in ^3He at $T_c = 0.0026$ K

Osheroff, Richardson, Lee (1972)

The Nobel Prize in Physics 1996

"for their discovery of superfluidity in helium-3"



David M. Lee
Cornell (USA)

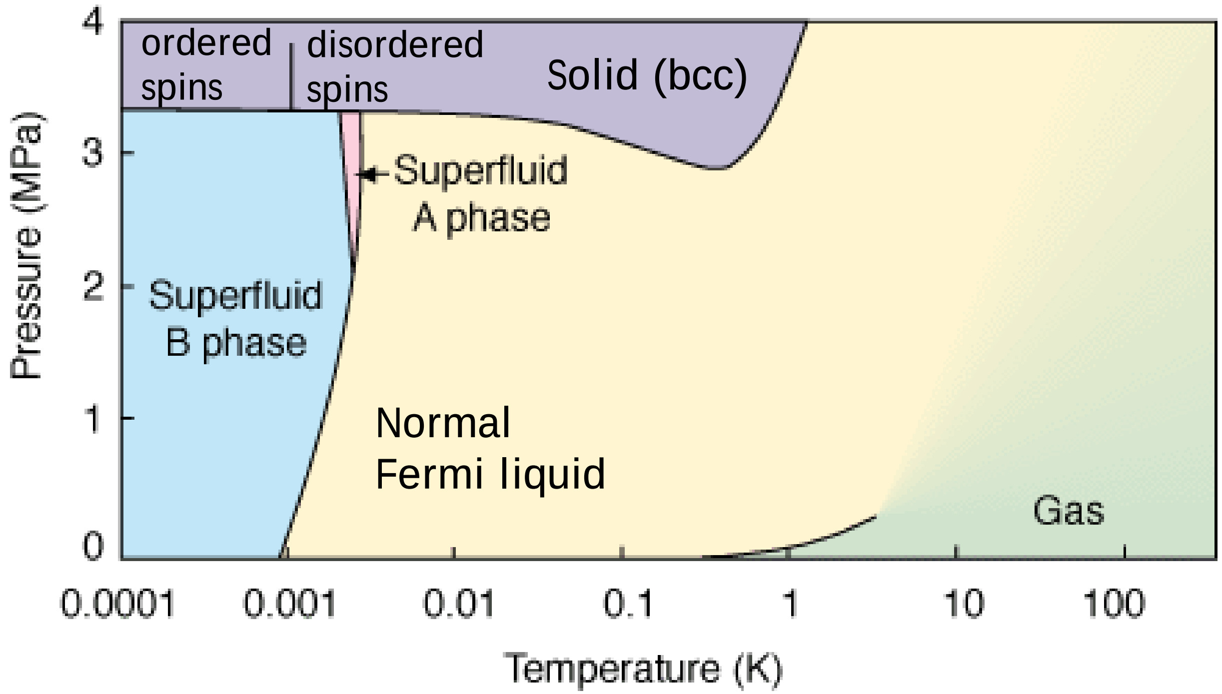
Douglas D. Osheroff
Stanford (USA)

Robert C. Richardson
Cornell (USA)

Phase diagram of Helium-3

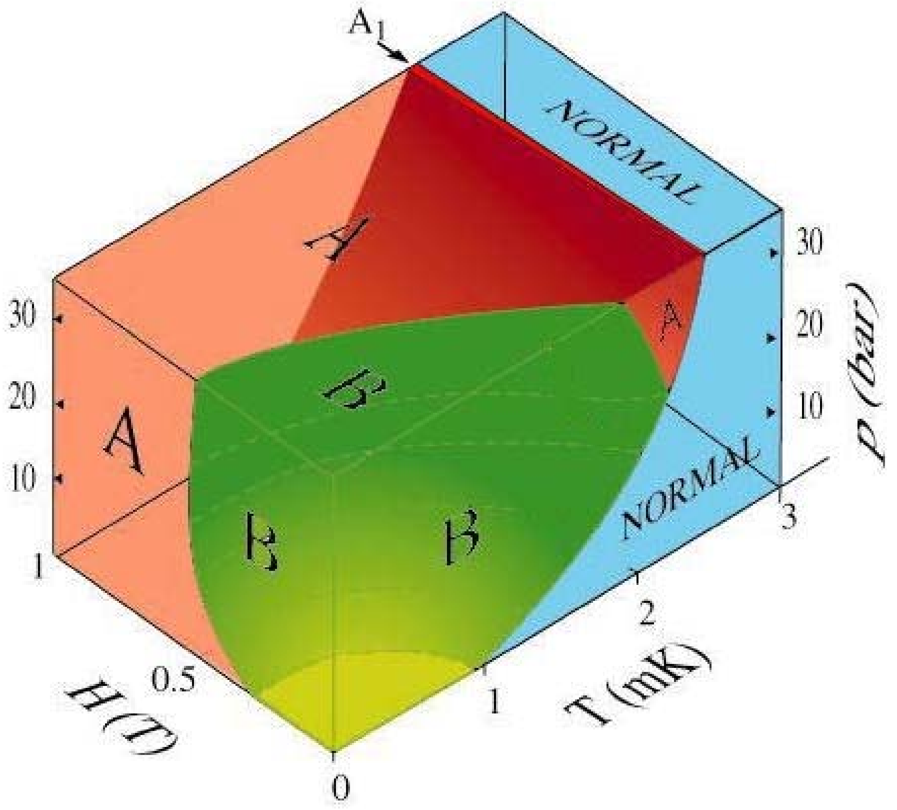
P-T phase diagram

Dense, simple liquid
- isotropic
- short-range interactions
- extremely pure
- nuclear spin S=1/2



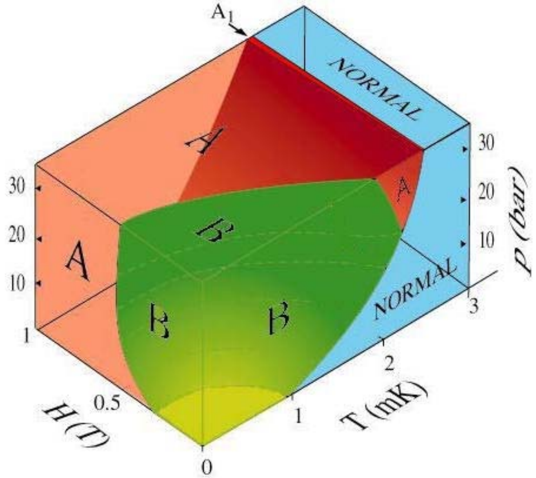
Phase diagram of Helium-3

P-T-H phase diagram



“Very low temperatures”: $T \ll T_{boiling} \sim$ 3-4 K
$\ll T_{backgr.\ rad.} \sim 3$ K

Superfluid phases of ^{3}He



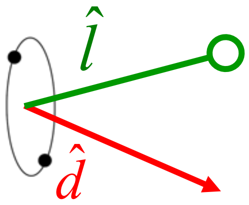
Theory + experiment: L=1, S=1 in all phases

Leggett
Wölfle
Mermin, ...

Attraction due to spin fluctuations

Anderson, Brinkman (1973)

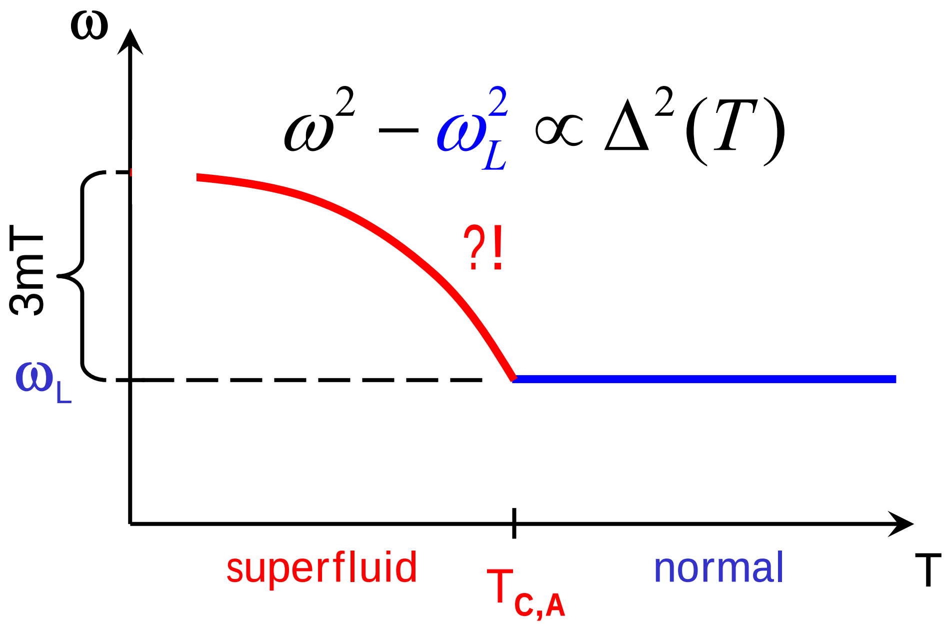
→ anisotropy directions in a ^{3}He Cooper pair



... and a mystery!

NMR experiment on nuclear spins $I = \frac{1}{2}\hbar$

Osheroff *et al.* (1972)



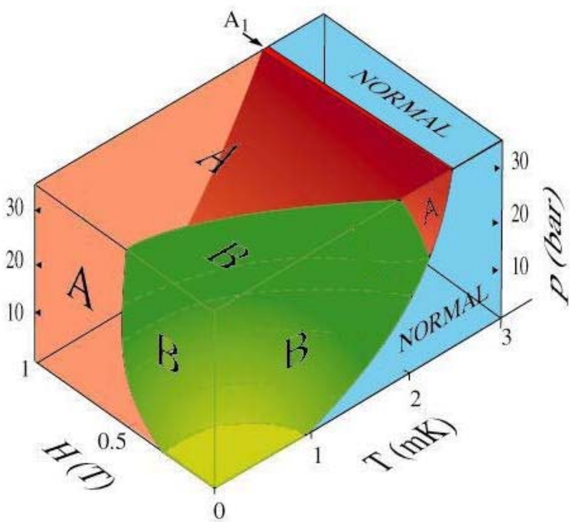
← Larmor frequency: $\omega_L = \gamma H$

Shift of ω_L	⇔	spin-nonconserving interactions
		→ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!

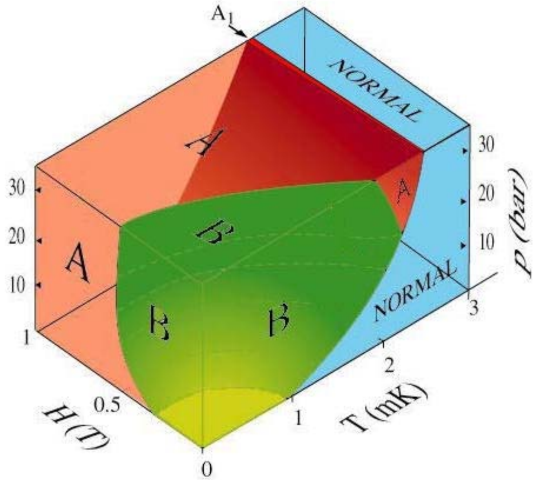
Leggett (1973)

The superfluid phases of ^3He



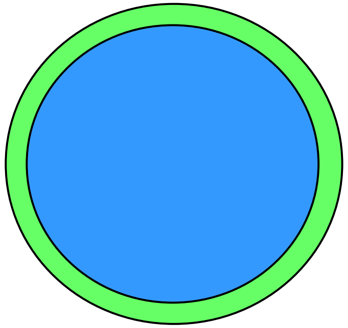
B-phase

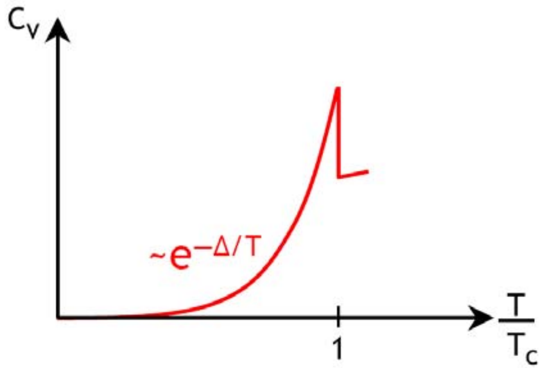
$$\Psi = |\uparrow\uparrow\rangle + |\uparrow\downarrow + \downarrow\uparrow\rangle + |\downarrow\downarrow\rangle$$



$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)
Vdovin (1963)



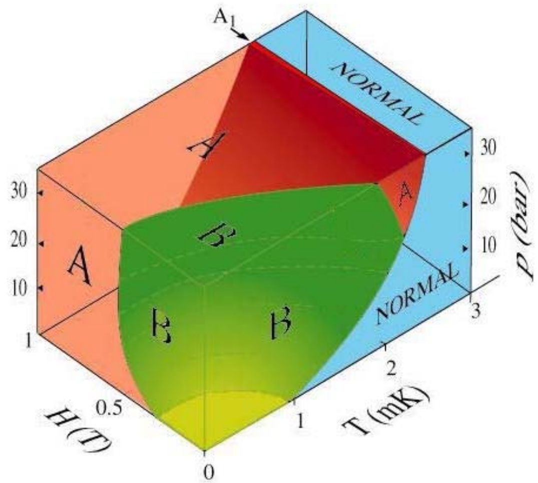


(pseudo-) isotropic state $\leftrightarrow$ s-wave superconductor

Weak-coupling theory: stable for all $T<T_c$

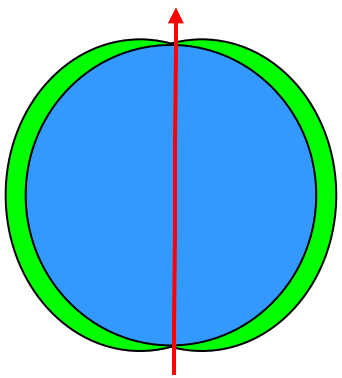
A-phase

$$\Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$$ → strong anisotropy



$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l})$$

Anderson, Morel (1961)



$\hat{l}$ Cooper pair orbital angular momentum

"Axial state" has point nodes

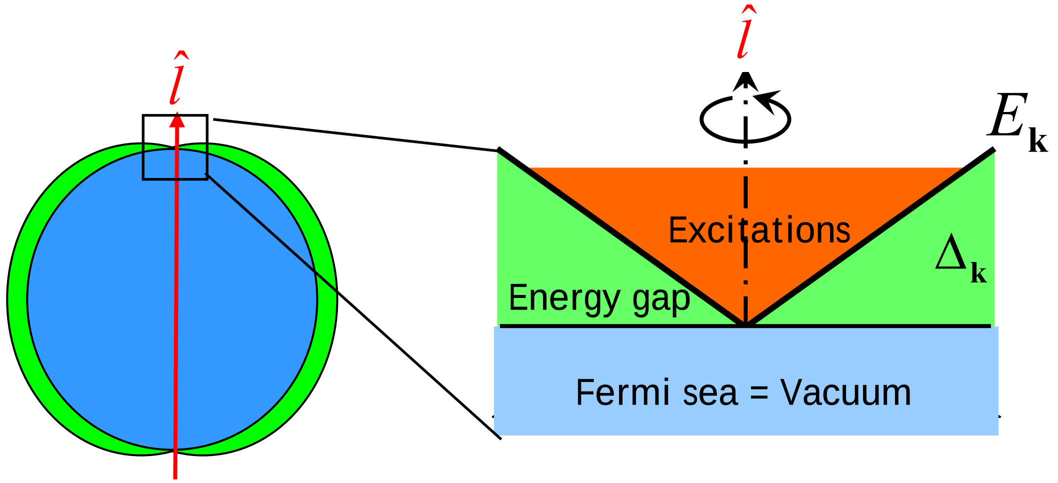
→ "unconventional" pairing in

- heavy fermion/high-T_c superconductors
- Sr_2RuO_4

Strong-coupling effect

^{3}He-A: Spectrum near poles

Volovik (1987)



$$E_{\mathbf{k}}^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{k}, \hat{l}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{k} \parallel +\hat{l} \\ -1 & \hat{k} \parallel -\hat{l} \end{cases}$$

2 chiralities

$$g^{ij} = v_F^2 l_i l_j + \left(\frac{\Delta}{k_F}\right)^2 (\delta_{ij} - l_i l_j)$$

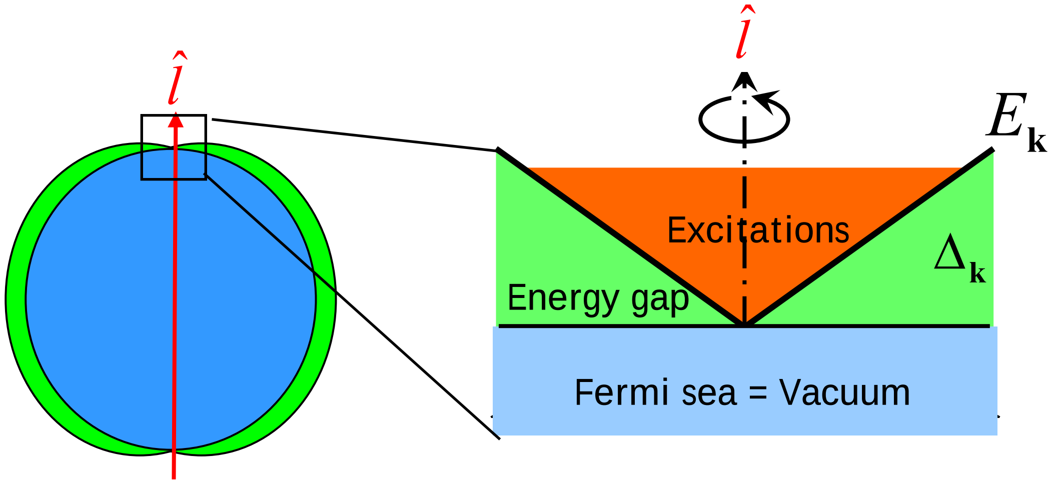
$$\mathbf{A} = k_F \hat{l}$$

$$\mathbf{p} = \mathbf{k} - e\mathbf{A}$$

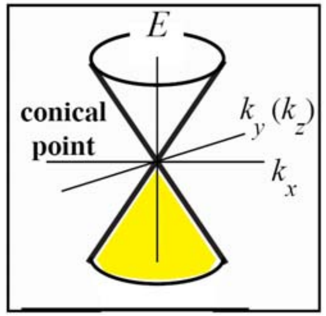
Lorentz invariance:
Symmetry enhancement
at low energies

^{3}He-A: Spectrum near poles

Volovik (1987)



⇔



Fermi point:
spectral flow

$$E_{\mathbf{k}}^2 = \mathrm{v}_{\mathrm{F}}^2 \left(k - k_F\right)^2 + \Delta_0^2 \sin^2(\hat{k}, \hat{l}) = \mathrm{g}^{\mathrm{ij}} p_i p_j$$

$$e = \begin{cases} +1 & \hat{k} \parallel +\hat{l} \\ -1 & \hat{k} \parallel -\hat{l} \end{cases}$$

2 chiralities

$$\mathrm{g}^{\mathrm{ij}} = \mathrm{v}_{\mathrm{F}}^2 l_i l_j + \left(\frac{\Delta}{k_F}\right)^2 (\delta_{ij} - l_i l_j)$$

⇔ Massless, chiral leptons, e.g., neutrino $E(\mathbf{p}) = cp$

→ Chiral anomaly of standard model

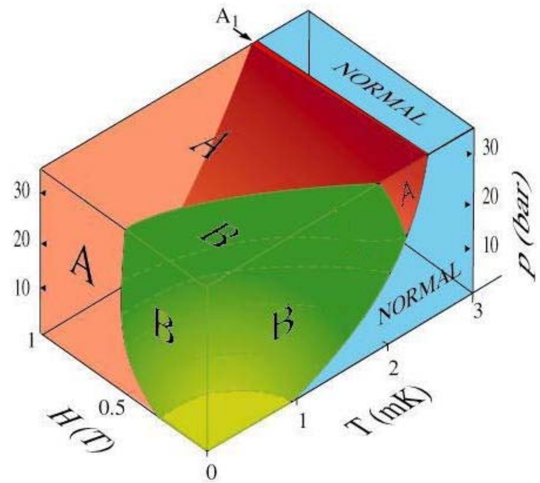
The Universe in a Helium Droplet, Volovik (2003)

A$_1$-phase

finite magnetic field

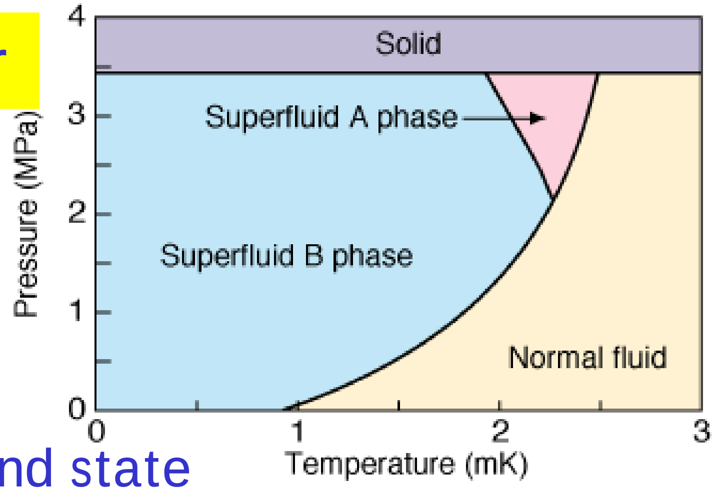
$$\Psi = |\uparrow\uparrow\rangle$$

Long-range ordered magnetic liquid



Broken Symmetries,
Long Range Order

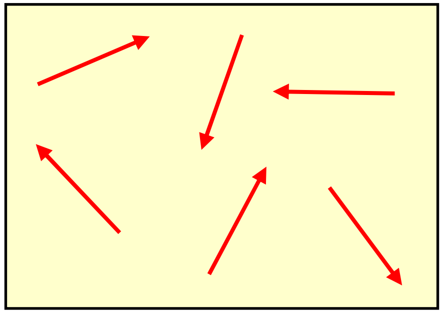
Broken Symmetries, Long Range Order



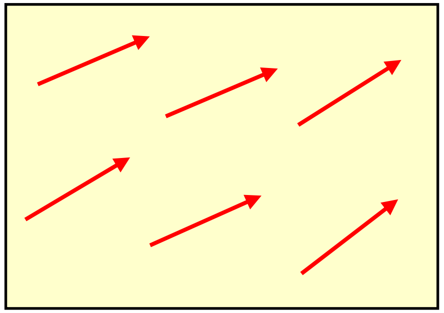
Normal ^{3}He ↔ ^{3}He-A, ^{3}He-B:
2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

I. Ferromagnet



$T>T_c$



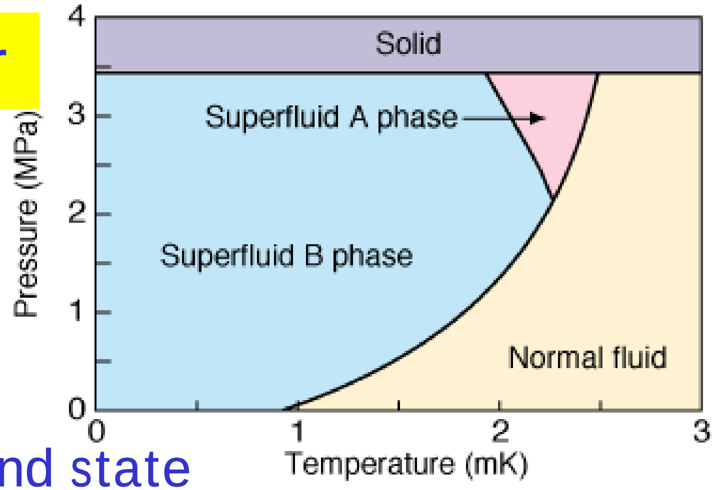
$T<T_c$

Average magnetization: $\langle \mathbf{M} \rangle = 0$ — $\langle \mathbf{M} \rangle \neq 0$ Order parameter

Symmetry group: SO(3) — U(1)⊂ SO(3)

$T<T_C$: SO(3) rotation symmetry in spin space spontaneously broken

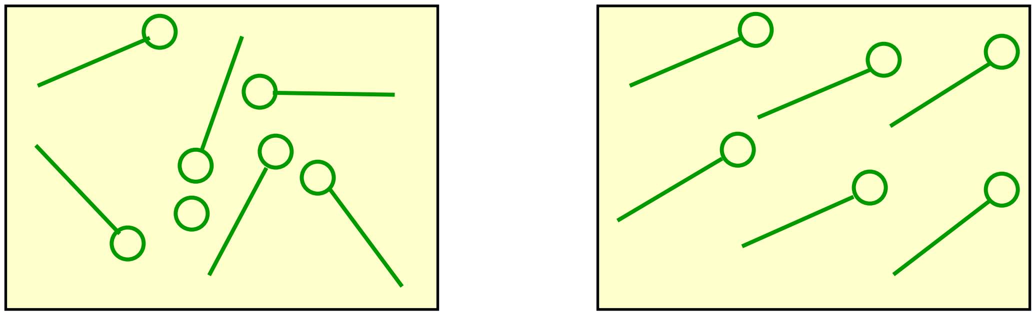
Broken Symmetries, Long Range Order



2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

II. Liquid crystal

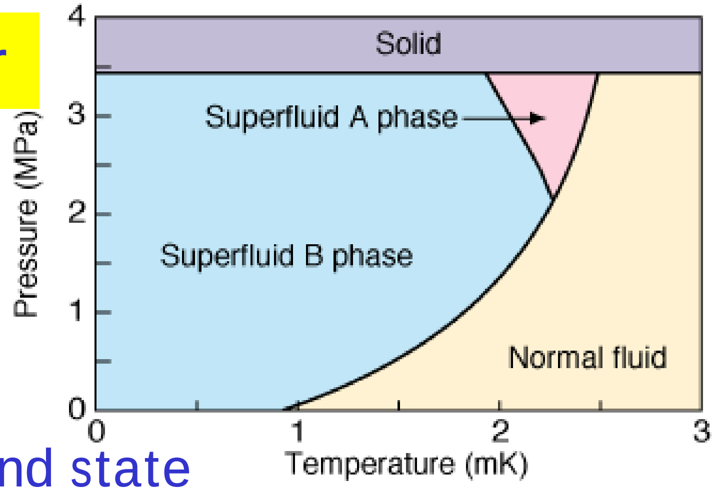


$T>T_c$ $T<T_c$

Symmetry group: SO(3) $U(1) \subset SO(3)$

$T<T_C$: SO(3) rotation symmetry in real space spontaneously broken

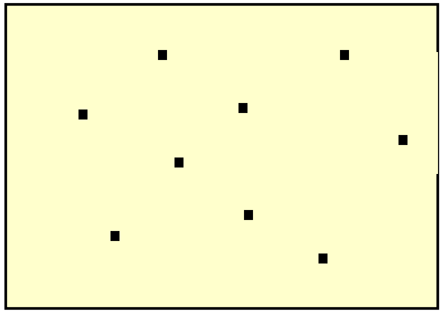
Broken Symmetries, Long Range Order



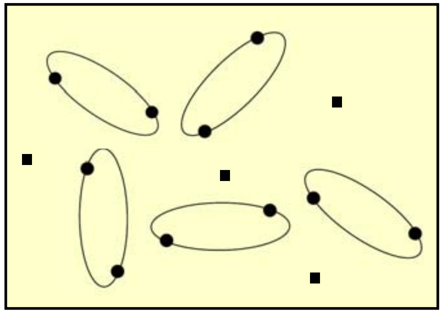
2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



$T>T_c$



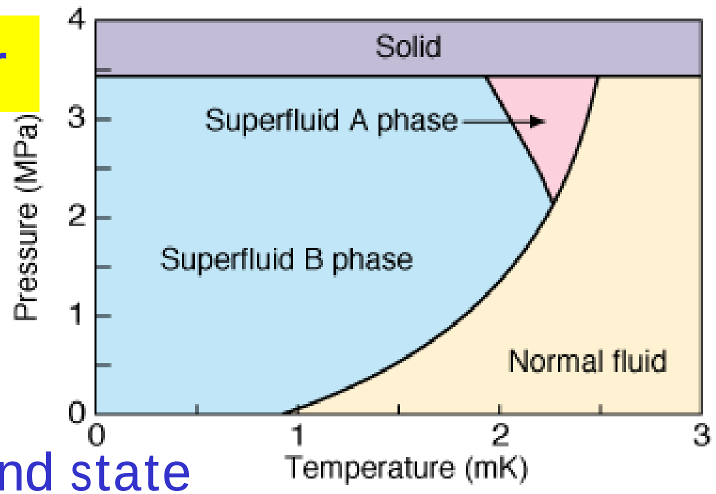
$T<T_c$

Pair amplitude $\langle c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \rangle$ 0 $\Delta e^{i\phi}$ "Order parameter"

Gauge transf. $c^\dagger_{\mathbf{k}\sigma} \rightarrow c^\dagger_{\mathbf{k}\sigma} e^{i\varphi}$: gauge invariant — not gauge invariant

Symmetry group U(1) —

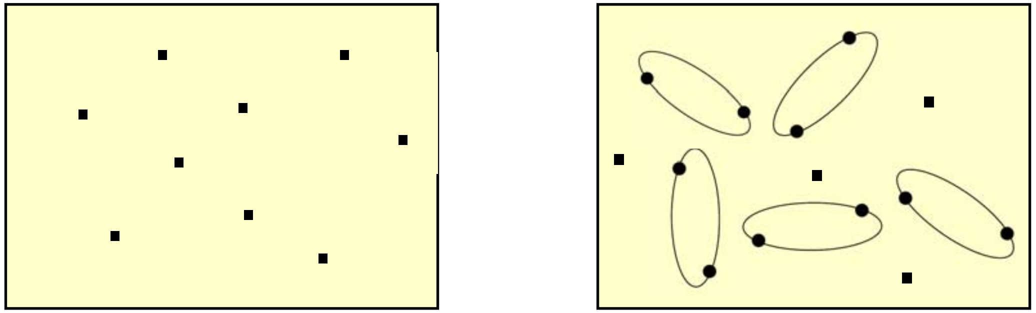
Broken Symmetries, Long Range Order



2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor

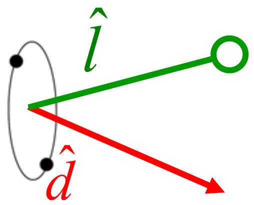


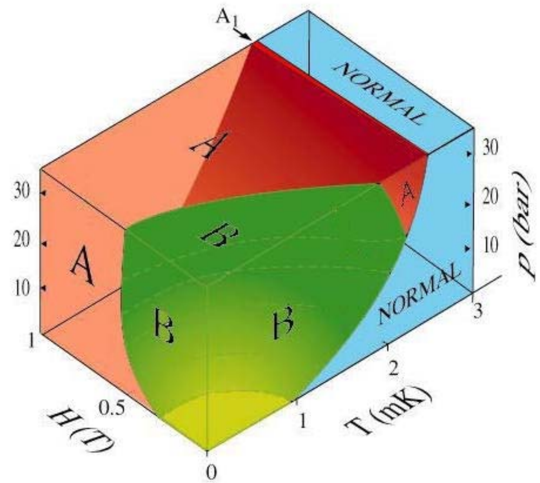
$T>T_c$ $T<T_c$

$T<T_C$: U(1) “gauge symmetry“ spontaneously broken

Broken symmetries in superfluid ^{3}He

L=1, S=1 in all phases

Cooper pair: 



Quantum coherence in	phase anisotropy direction for spin anisotropy direction in real space	Superfluid, magnetic liquid crystal

Characterized by $2\times(2L+1)\times(2S+1)$ = 18 real numbers

3x3 order parameter matrix $A_{i\mu}$

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry spontaneously broken Leggett (1975)

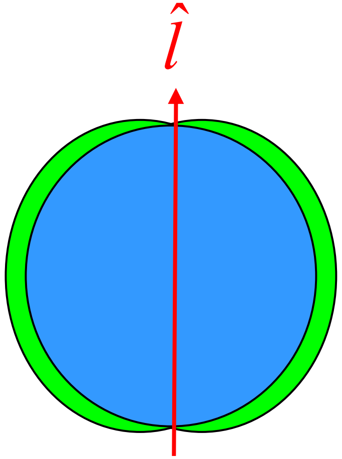
Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, DV (1986)

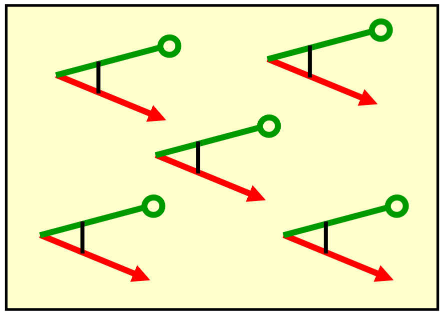
3He-A $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

$U(1)_{S_z} \times U(1)_{L_z-\varphi}$

„Unconventional" pairing

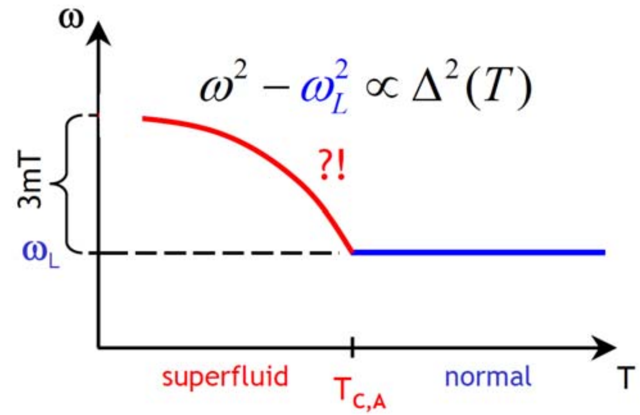


Cooper pairs



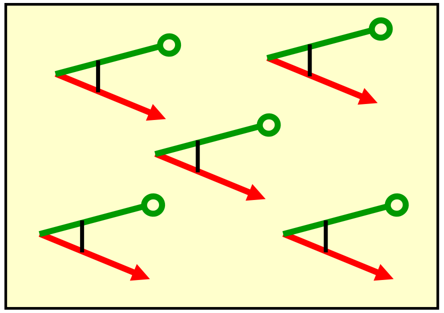
Fixed absolute orientation

Resolution of the NMR puzzle



Superfluid ³He - a quantum amplifier

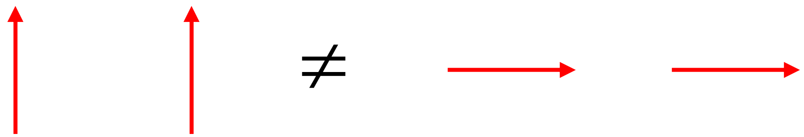
Cooper pairs in ^{3}He-A



Fixed absolute orientation

What determines the actual relative orientation of $\hat{d}, \hat{l}$?

→ Anisotropic spin-orbit interaction of nuclear dipoles:

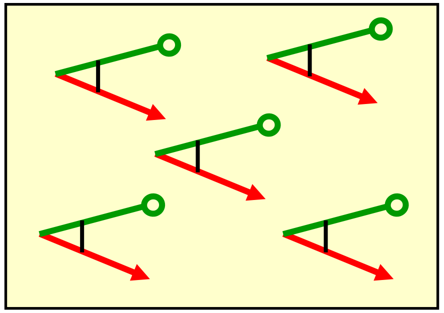


Dipole-dipole coupling of ^{3}He nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!

Superfluid ^{3}He - a quantum amplifier

Cooper pairs in ^{3}He-A



Fixed absolute orientation

- Long-range order in $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

Quantum $\Downarrow$ coherence

$\hat{d}, \hat{l}$ locked in all Cooper pairs

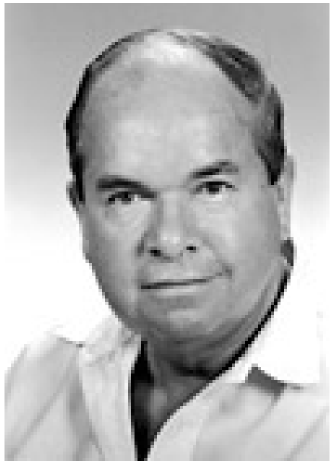
$\Downarrow$

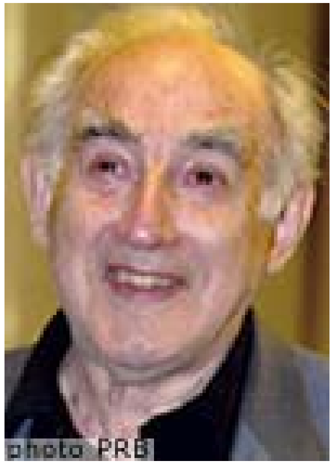
NMR frequency increases: $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$ Leggett (1973)

→ Nuclear dipole interaction macroscopically measurable

The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"



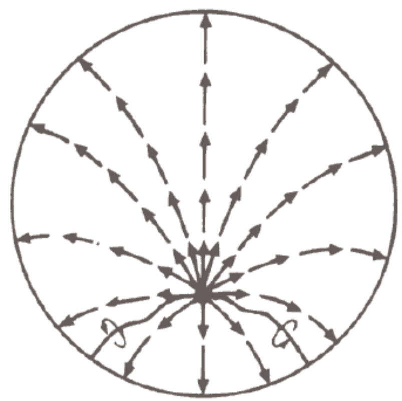


Alexei A. Abrikosov
USA and Russia

Vitaly L. Ginzburg
Russia

Anthony J. Leggett
UK and USA

Order parameter textures and topological defects

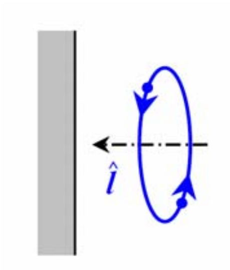


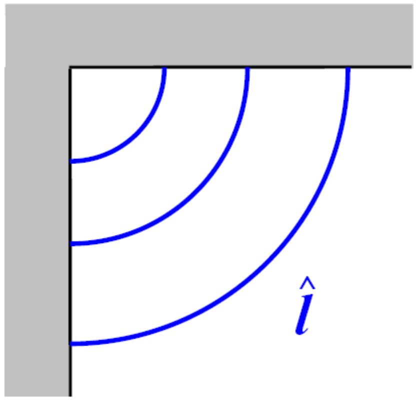
Order parameter textures

Orientation of anisotropy directions $\hat{d}, \hat{l}$ in ^{3}He-A ?

Magnetic field → $\hat{d}$

Walls → $\hat{l}$



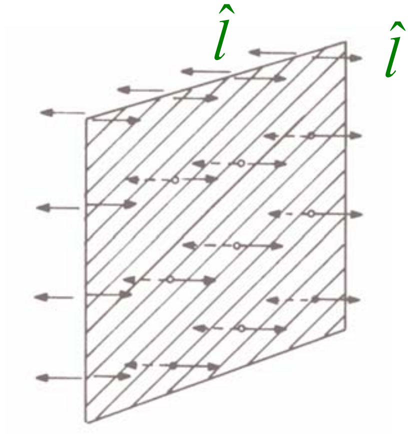


→ "Textures" in $\hat{d}, \hat{l}$ ↔ liquid crystals

→ Topologically stable defects: Classification by homotopy theory

Order parameter textures and topological defects

D=2: domain walls in $\hat{d}$ or $\hat{l}$

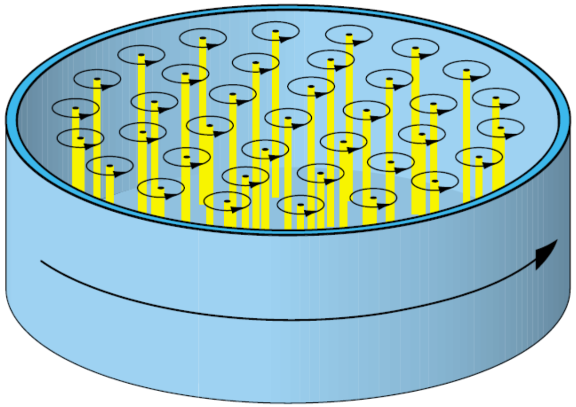


Single domain wall

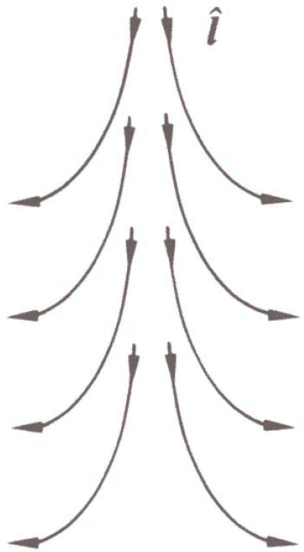
Domain wall lattice

Order parameter textures and topological defects

D=1: Vortices



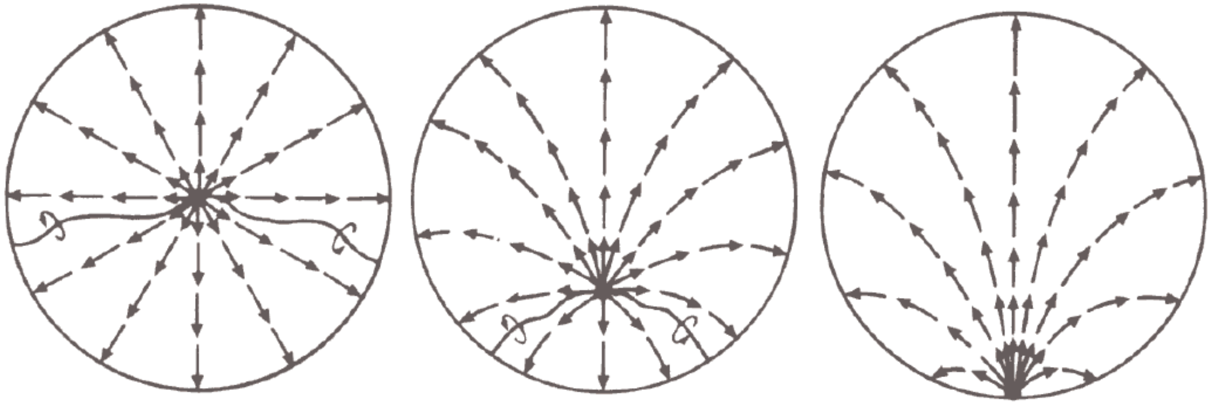
Vortex formation
(rotation experiments)



e.g., Mermin-Ho vortex
(non-singular)

Order parameter textures and topological defects

D=0: Monopoles



“Boojum” in $\hat{l}$-texture of ^{3}He-A
(geometric constraint)

Defect formation by, e.g.,
- rotation
- geometric constraints
- rapid crossing through phase transition

Big bang simulation

in the low temperature lab

BANG!

Universality in continuous phase transitions

BANG!

High symmetry, short-range order — $T>T_c$

	Spins:	Helium:	Universe:
$T>T_c$	para-magnetic	normal liquid	Unified forces and fields
$T=T_c$ — Phase transition			
$T<T_c$	ferromagnetic	superfluid	elementary particles, fundamental interactions
Defects:	domain walls	vortices, etc.	cosmic strings, etc. Kibble (1976)

$T=T_c$ Phase transition

Broken symmetry, long-range order — $T<T_c$

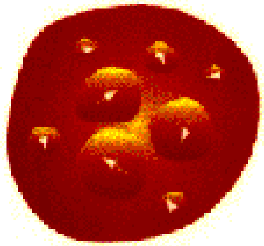
nucleation of galaxies?

Rapid thermal quench through 2. order phase transition Kibble (1976)

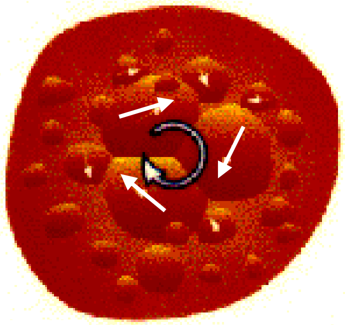
1. Local temperature $T \gg T_C$

Expansion + rapid cooling

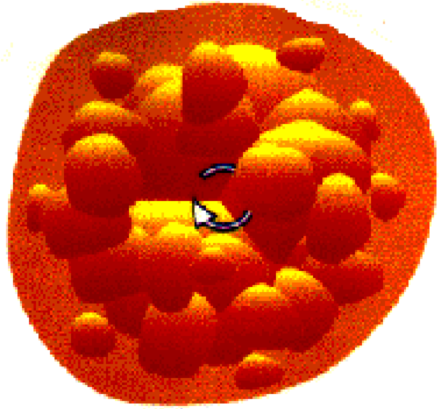
2. Nucleation of independently ordered regions



Clustering of ordered regions → Defects

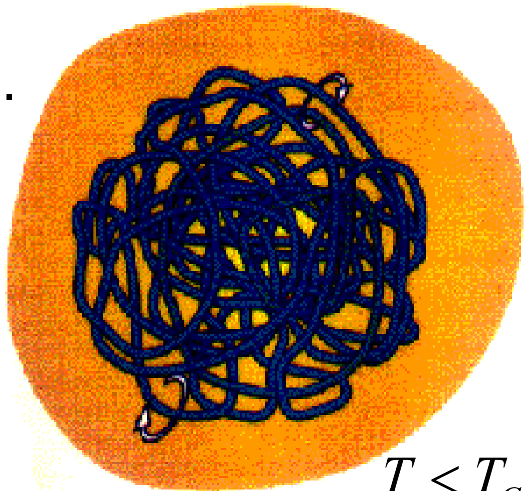


3.



Defects overlap

4.



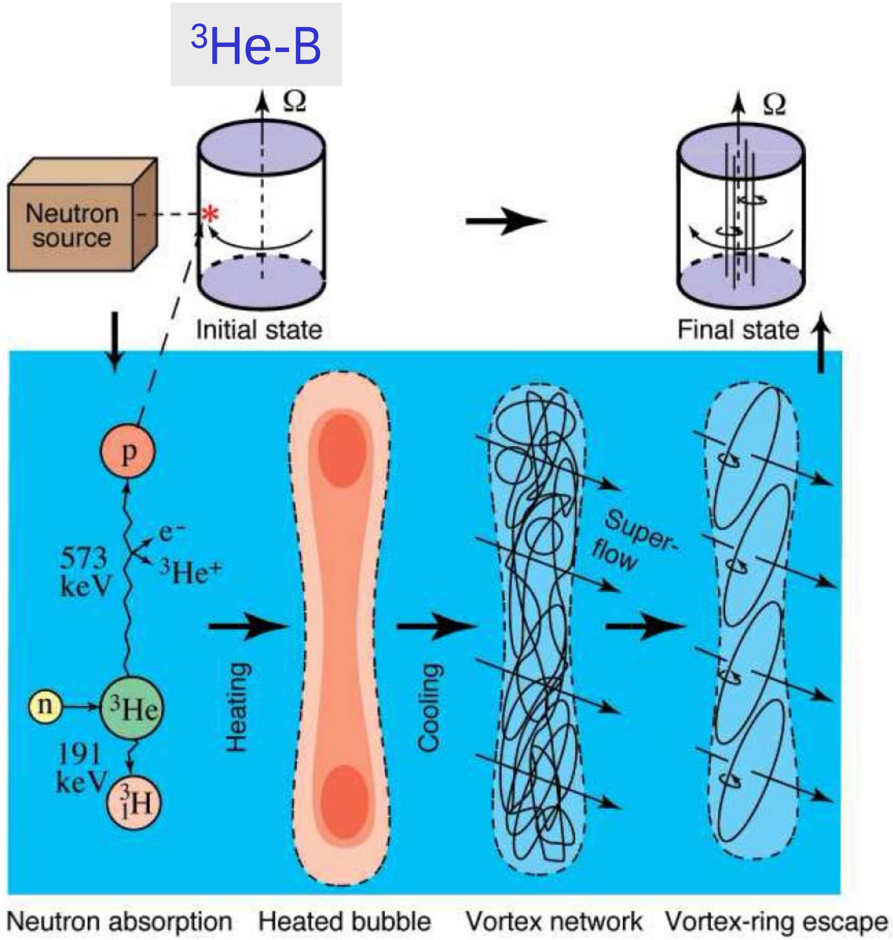
$T < T_C$: Vortex tangle

Estimate of density of defects Zurek (1985)

"Kibble-Zurek mechanism": How to test?

Big bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)

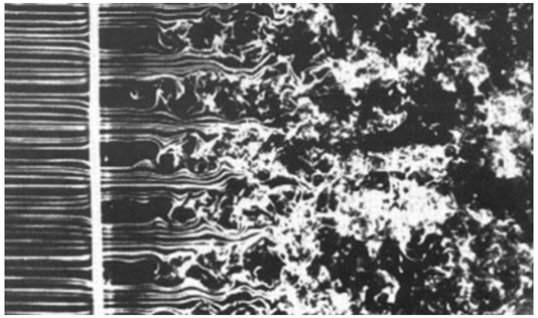


Measured vortex tangle density:
Quantitative support for Kibble-Zurek mechanism

Present research on superfluid ^{3}He: Quantum Turbulence

Classical Turbulence

Leonardo da Vinci (1452-1519)



Flow through grid

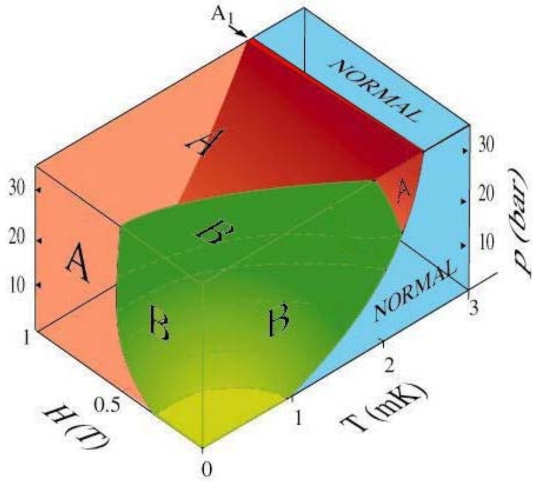
Quantum Turbulence = Turbulence in the absence of viscous dissipation (superfluid at T→0)

- Why are quantum and classical turbulence so similar?
- What provides dissipation in the absence of friction?

Test system: ^{3}He-B

Vinen, Donnelly: **Physics Today** (April, 2007)

Conclusion



Superfluid Helium-3:

- Anisotropic superfluid
  - 3 different bulk phases
  - Cooper pairs with internal structure

- Large symmetry group broken
  - Close connections to particle theory
  - Zoo of topological defects
  - Kibble-Zurek mechanism quantitatively verified