

Center for  
Electronic Correlations and Magnetism  
University of Augsburg

Theory of correlated fermionic condensed matter

## 2. Electronic correlations - from models to materials

b. Merging DMFT with density functional theory (LDA+DMFT)  
and application to transition-metal oxides

XIV. Training Course in the Physics of Strongly Correlated Systems  
Salerno, October 6+7, 2009

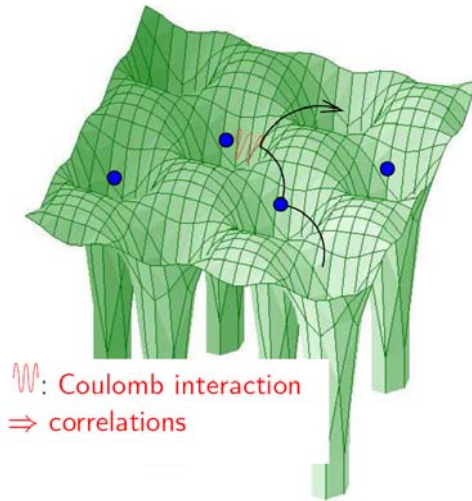
Dieter Vollhardt

*Supported by Deutsche Forschungsgemeinschaft through SFB 484*

# Outline:

- Merging of density functional theory with DMFT (“LDA+DMFT“)
- Application to transition-metal oxides

# Correlated Electron Materials: LDA+DMFT

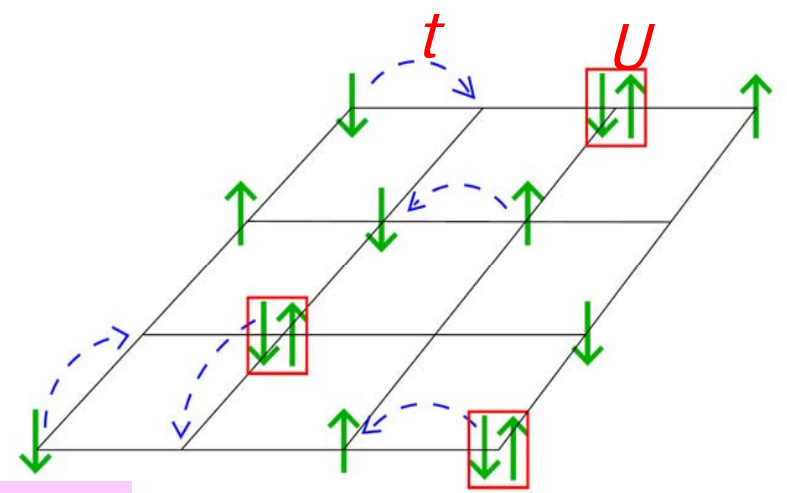
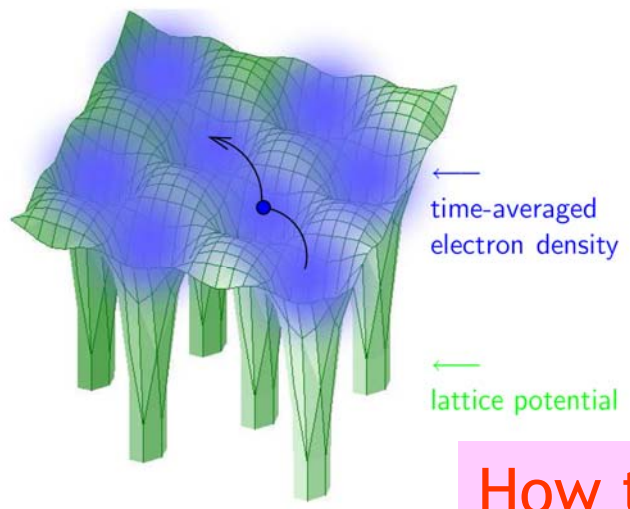


Held©2004

### DFT/LDA

### Model Hamiltonians

<ul style="list-style-type: none"> <li>+ material specific: “ab initio”</li> <li>- fails for strong correlations</li> <li>+ fast code packages</li> </ul>	<ul style="list-style-type: none"> <li>- input parameters unknown</li> <li>+ systematic many-body approach</li> <li>- computationally expensive</li> </ul>
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How to combine?

# Computational scheme for correlated electron materials:

Material specific electronic structure  
(Density functional theory: LDA, GW, ...)

+

Local electronic correlations  
(Many-body theory: DMFT)



LDA+DMFT

Anisimov, Poteryaev, Korotin, Anokhin, Kotliar (1997)

Lichtenstein, Katsnelson (1998)

Nekrasov, Held, Blümer, Poteryaev, Anisimov, DV (2000)

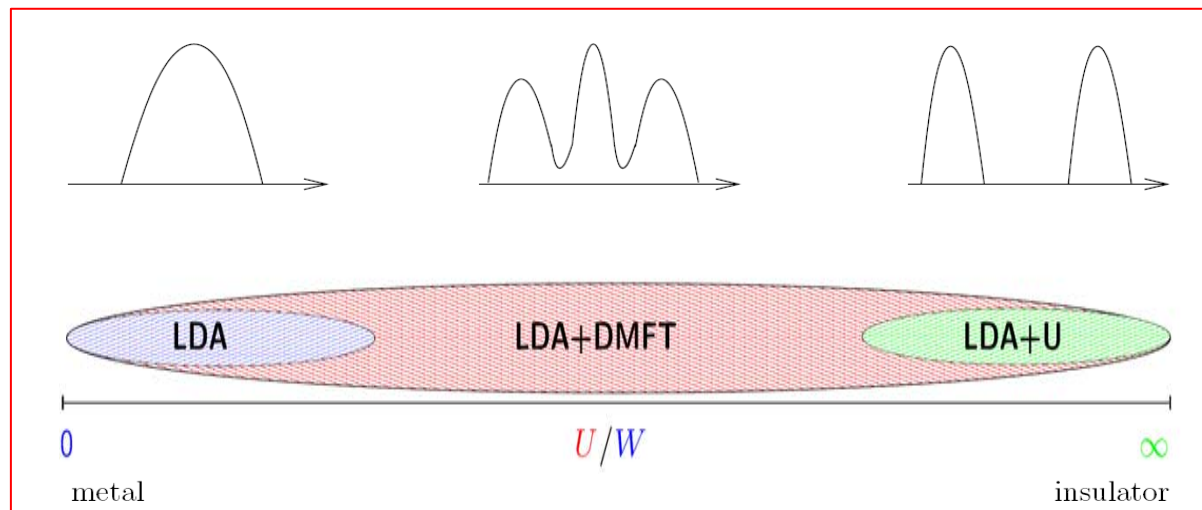
*Physics Today*, March 2004; Kotliar, DV

# Computational scheme for correlated electron materials:

Material specific electronic structure  
(Density functional theory: LDA, GW, ...)

+

Local electronic correlations  
(Many-body theory: DMFT)



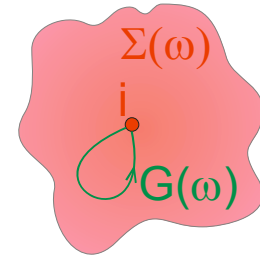
# LDA+DMFT (simplest version)

- 1) Calculate LDA band structure:  $\epsilon_{lm'l'm'}(k) \rightarrow \hat{H}_{LDA}$
- 2) Supplement LDA by local Coulomb interaction (only for correlated bands)

$$\hat{\mathcal{H}} = \underbrace{\sum_{\mathbf{k}lm'l'm'\sigma} \epsilon_{lm'l'm'}(\mathbf{k}) \hat{c}_{klm\sigma}^\dagger \hat{c}_{kl'm'\sigma}}_{\text{LDA}} - \underbrace{\sum_{\substack{i=i_d, \\ l=l_d}} \sum_{m\sigma} \Delta\epsilon_d \hat{n}_{ilm\sigma}}_{\text{double counting correction}}$$

$$+ \underbrace{\sum_{\substack{i=i_d, \\ l=l_d}} \sum_{m\sigma, m'\sigma'}' \frac{U_{mm'}^{\sigma\sigma'}}{2} \hat{n}_{ilm\sigma} \hat{n}_{ilm'\sigma'}}_{\text{local Coulomb interaction}} - \underbrace{\sum_{\substack{i=i_d, \\ l=l_d}} \sum_{m\sigma, m'\sigma'}' J_{mm'} \hat{c}_{ilm\sigma}^\dagger \hat{c}_{ilm'\sigma'}^\dagger \hat{c}_{ilm'\sigma} \hat{c}_{ilm\sigma'}}_{\text{Hund's rule coupling}}$$

3) Solve self-consistently with an impurity solver,  
e.g., QMC: LDA+DMFT(QMC)



(i) Effective single impurity problem

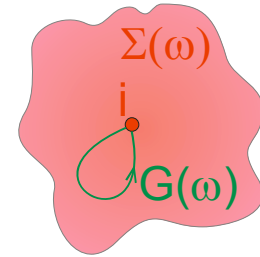
$$G = -\frac{1}{Z} \int \mathcal{D}[\psi\psi^*] \psi\psi^* e^{\psi^* [G^{-1} + \Sigma]\psi - U\psi^*\psi\psi^*\psi + J\psi^*\psi\psi^*\psi}$$

(ii)  $k$ -integrated Dyson equ. (orbital degeneracy)

$$G(\omega) = \int d\varepsilon \frac{N^{LDA}(\varepsilon)}{\omega - \Sigma(\omega) - \varepsilon}$$



3) Solve self-consistently with an impurity solver,  
 e.g., QMC: LDA+DMFT(QMC)



(i) Effective single impurity problem

$$G = -\frac{1}{Z} \int \mathcal{D}[\psi\psi^*] \psi\psi^* e^{\psi^* [G^{-1} + \Sigma] \psi - U\psi^*\psi\psi^*\psi + J\psi^*\psi\psi^*\psi}$$

(ii)  $k$ -integrated Dyson equ. (general)

$$G_{mm'}^{\sigma}(\omega) = \frac{1}{V_B} \int d^3k \left[ (\omega - \Sigma^{\sigma}(\omega)) \delta_{m,m'} - \left( H_{LDA}^{0\text{ eff}}(\mathbf{k}) \right)_{m,m'} \right]^{-1}$$

# LDA+DMFT(X): Search for the “best” impurity solver X

Hubbard I

IPT

NCA

ED

QMC (Hirsch-Fye)

NRG

Recent:

PQMC

DDMRG

CT-QMC

# Application of LDA+DMFT

# Spectral function (“interacting DOS”) in DMFT

k-integrated spectral function  
→ PES

$$A(\omega) = -\frac{1}{\pi} \text{Im} \mathbf{G}(\omega)$$

k-resolved spectral function  
→ ARPES

$$\mathbf{G}(\mathbf{k}, \omega) = [\omega - \Sigma(\omega) - \mathbf{H}_{LDA}^0(\mathbf{k})]^{-1}$$

Matrices in orbital space

$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} \text{Tr} \mathbf{G}(\mathbf{k}, \omega)$$

## LDA+DMFT: Collaborators

### Augsburg

K. Byczuk  
V. Eyert  
K. Held  
G. Keller  
M. Kollar  
J. Kuneš  
I. Leonov  
T. Pruschke  
X. Ren

### Ekaterinburg

V. I. Anisimov  
D. E. Kondakov  
A. V. Kozhevnikov  
A. V. Lukoyanov  
I. A. Nekrasov  
Z. Pchelkina  
S. L. Skornyakov

### UC Davis

R. T. Scalettar

### Lawrence Livermore

A. K. McMahan

### MPI Stuttgart

O. K. Andersen

### Osaka

S. Suga  
A. Sekiyama

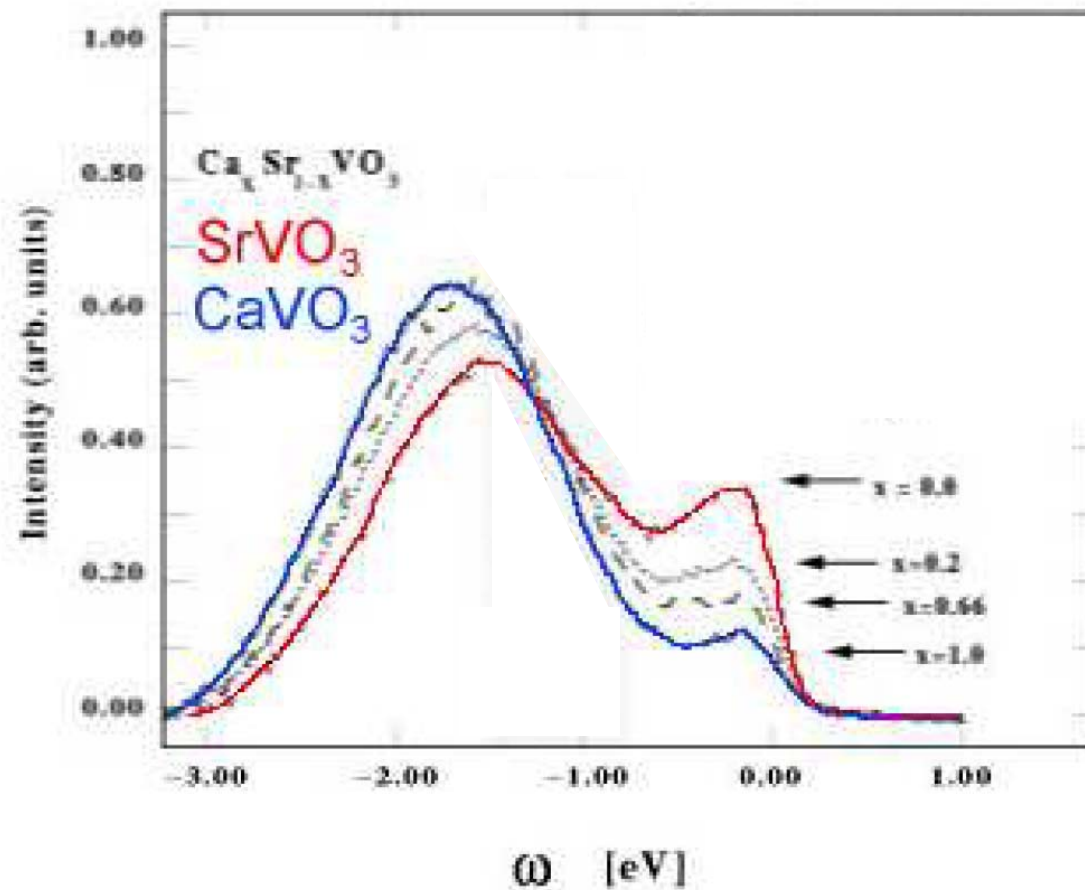
### Ann Arbor

J. W. Allen  
S.-K. Mo

1. Application:  
3d<sup>1</sup> system (Sr,Ca)VO<sub>3</sub>

# 3d<sup>1</sup> system: (Sr,Ca)VO<sub>3</sub>

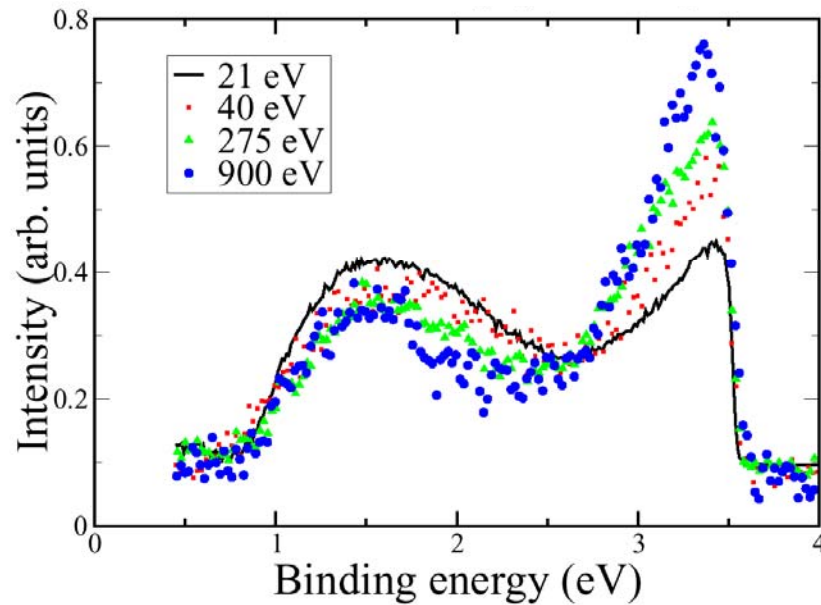
Photoemission spectroscopy (PES)



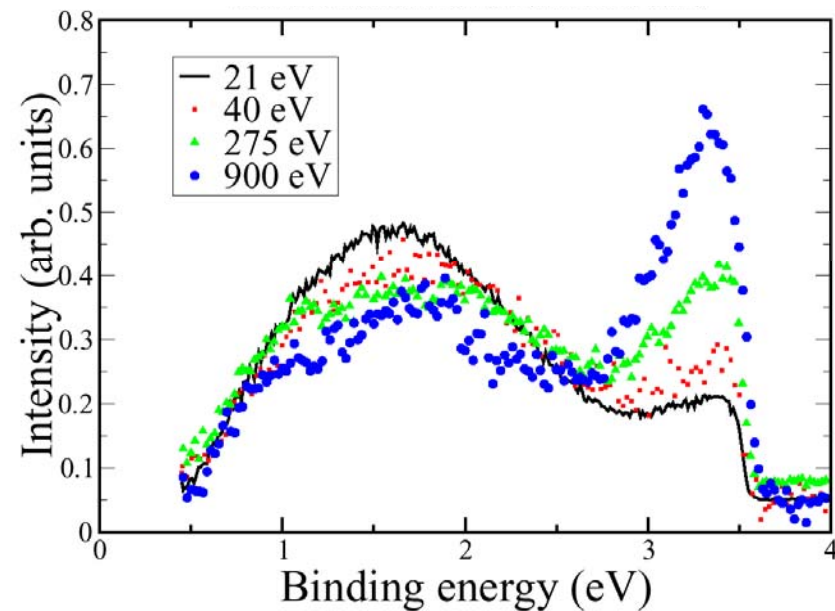
Inoue *et al.*, PRL (1995)

# Experiment

## Photoemission spectra at high photon energies



SrVO<sub>3</sub>



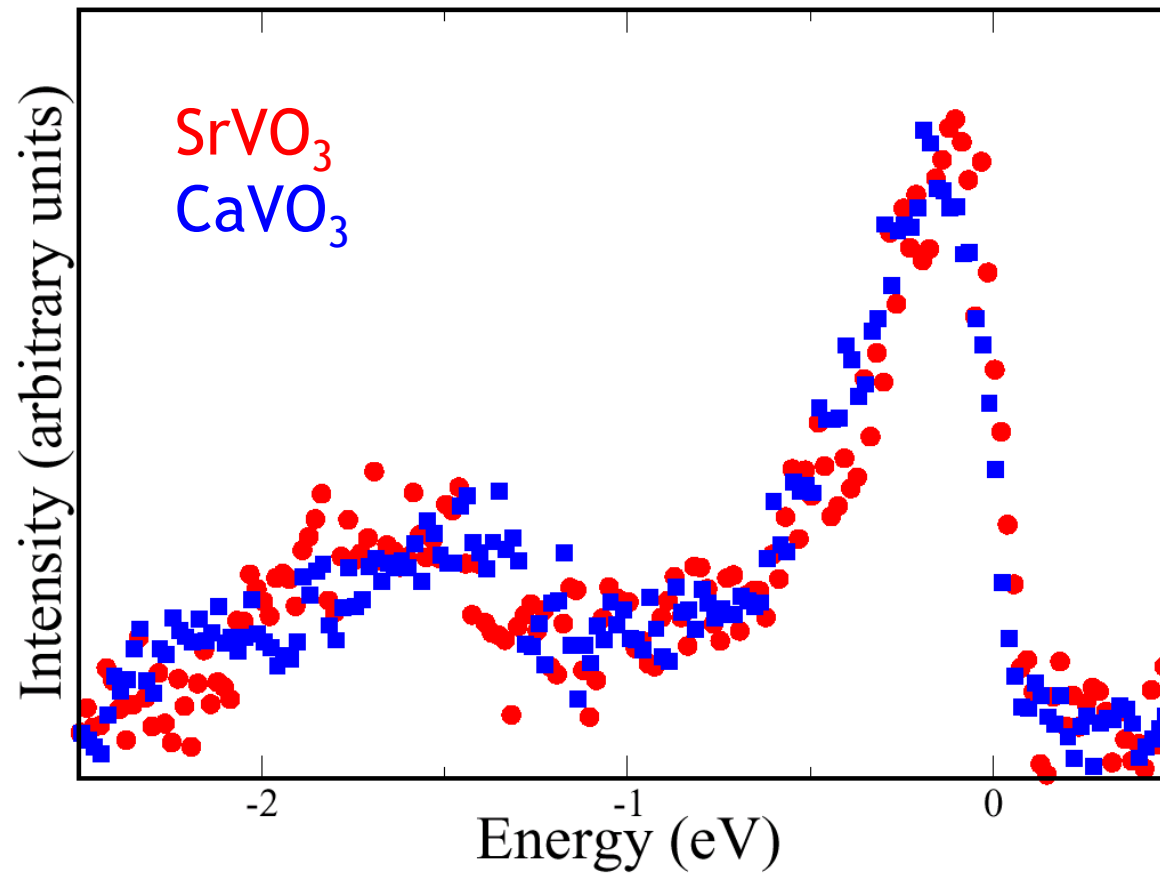
CaVO<sub>3</sub>

Osaka - Augsburg - Ekaterinburg collaboration: Sekiyama *et al.*; PRL (2004)



## Experiment

### Photoemission spectra at high photon energies

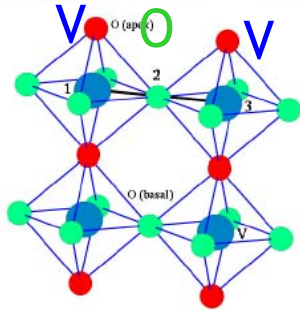
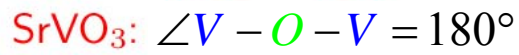


Osaka - Augsburg - Ekaterinburg collaboration: Sekiyama *et al.*; PRL (2004)

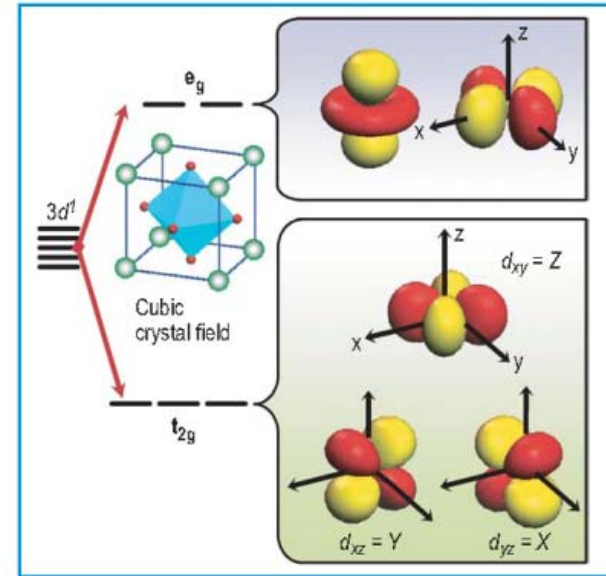
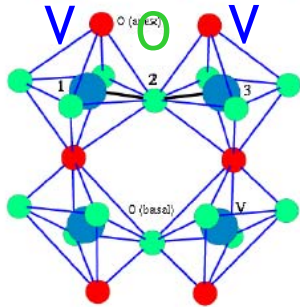
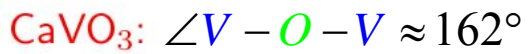
# Theory

## Electronic structure

### Crystal structure



orthorhombic distortion

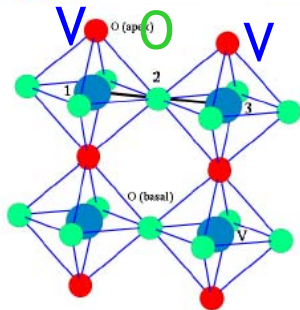


# Theory

## Electronic structure

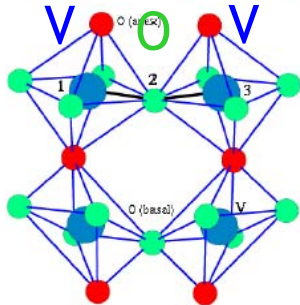
### Crystal structure

$\text{SrVO}_3$ :  $\angle \text{V} - \text{O} - \text{V} = 180^\circ$

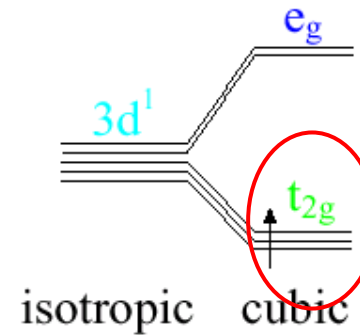


orthorhombic distortion

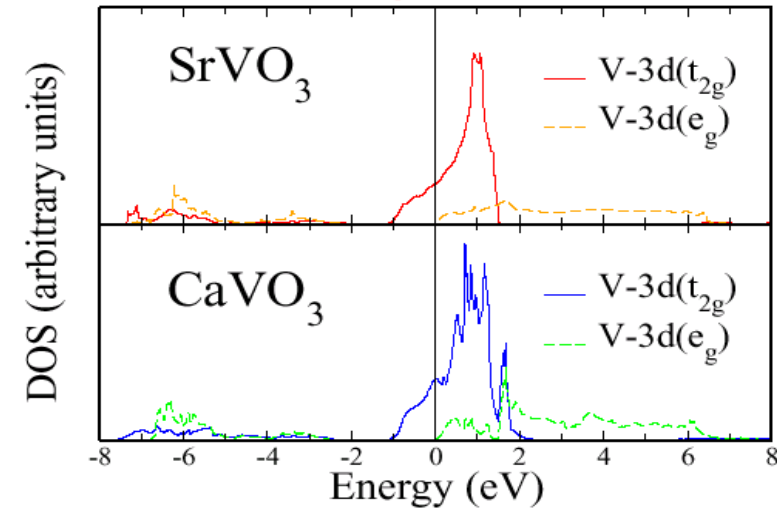
$\text{CaVO}_3$ :  $\angle \text{V} - \text{O} - \text{V} \approx 162^\circ$



### Band scheme



### LDA density of states



No correlation effects/spectral transfer

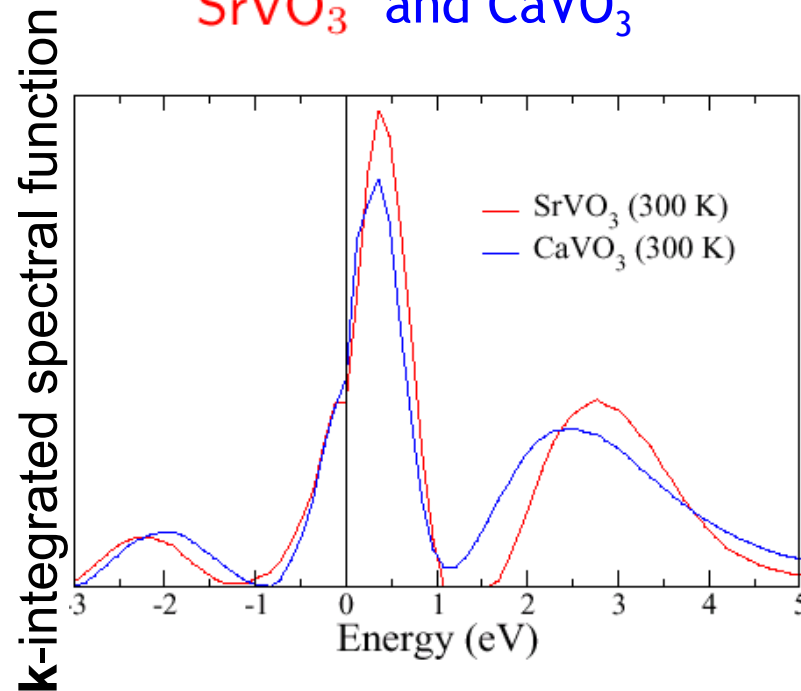
# LDA+DMFT results

$$A(\omega) = -\frac{1}{\pi} \text{Im} \mathbf{G}(\omega)$$

Correlation induced  
spectral transfer  
→ 3-peak structure

constrained LDA:  
U=5.55 eV, J=1.0 eV

SrVO<sub>3</sub> and CaVO<sub>3</sub>

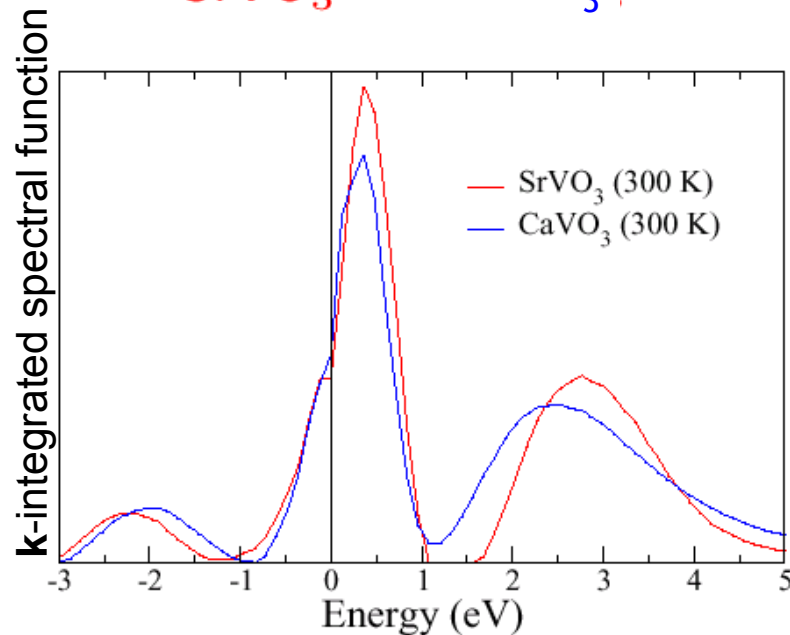


Strongly correlated paramagnetic metal  
(Mott-Hubbard system)

Osaka - Augsburg - Ekaterinburg collaboration: Sekiyama *et al.*; PRL (2004)

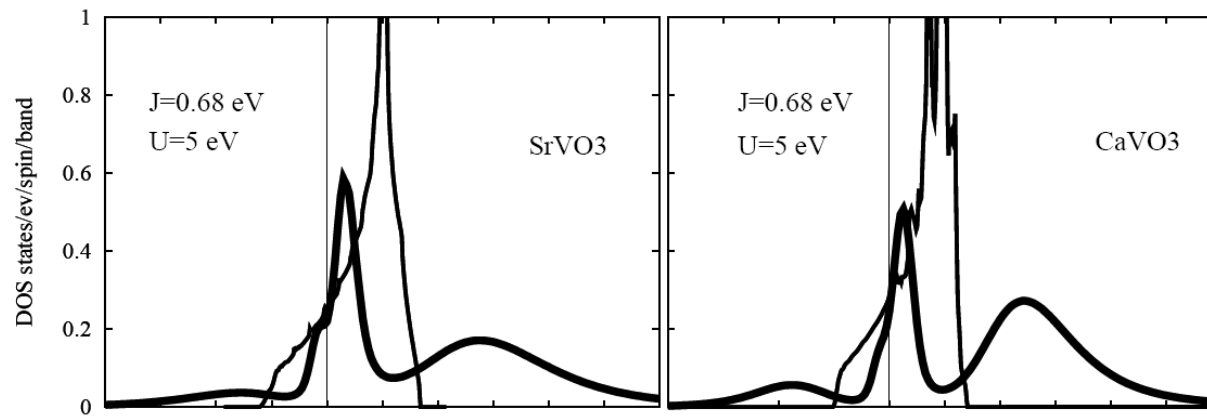
# LDA+DMFT results

$\text{SrVO}_3$  and  $\text{CaVO}_3$



Constrained LDA:  
 $U=5.55$  eV,  $J=1.0$  eV

Osaka - Augsburg - Ekaterinburg  
collaboration: Sekiyama *et al.*,  
PRL **93**, 156402 (2004)



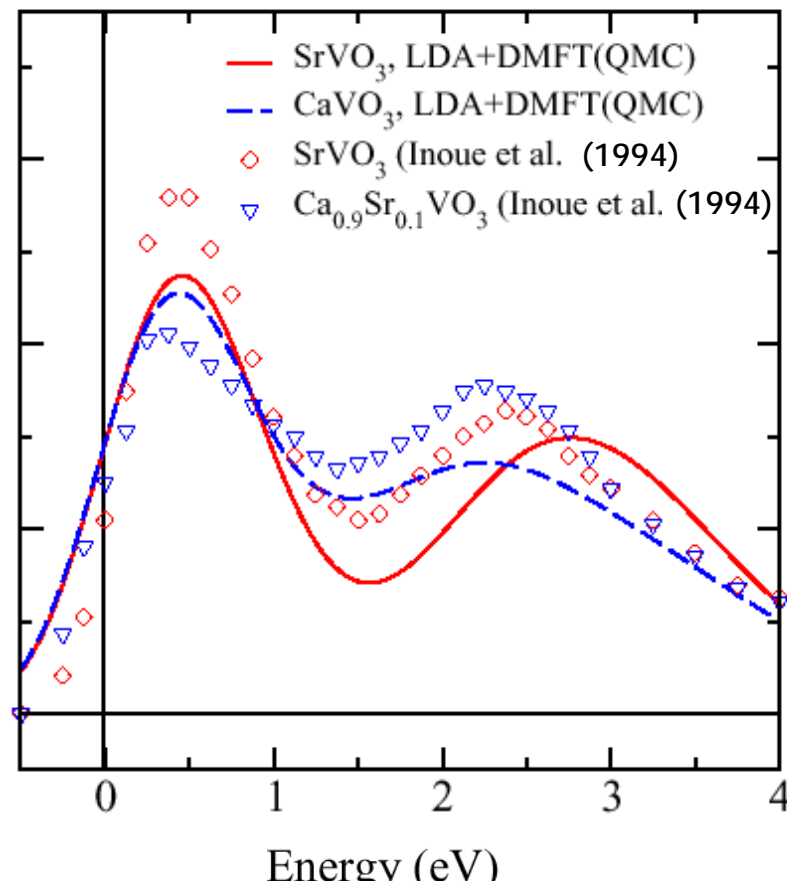
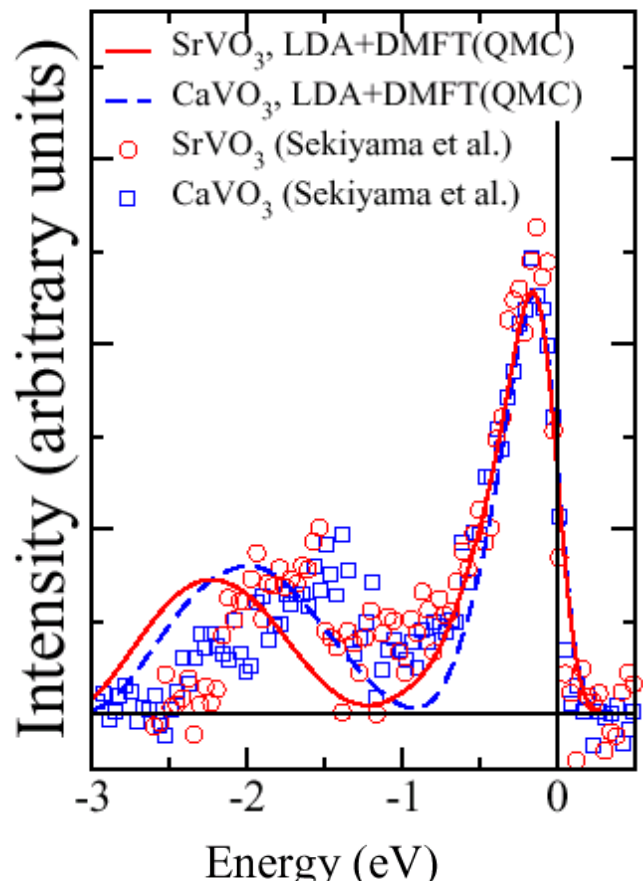
Pavarini, Biermann,  
Poteryaev, Lichtenstein,  
Georges, Andersen  
(2004)

# Comparison with experiment

Osaka - Augsburg -Ekaterinburg  
collaboration,  
Sekiyama *et al.*, PRL (2004)  
Nekrasov *et al.*, PRB (2005)

- (i) bulk-sensitive high-resolution photoemission spectra (PES)
- (ii)  $1s$  x-ray absorption spectra (XAS)

Measurement at O K-edge:  
no symmetry breaking of V  $2p$   
shell in final state (XAS  $\approx$  IPES)

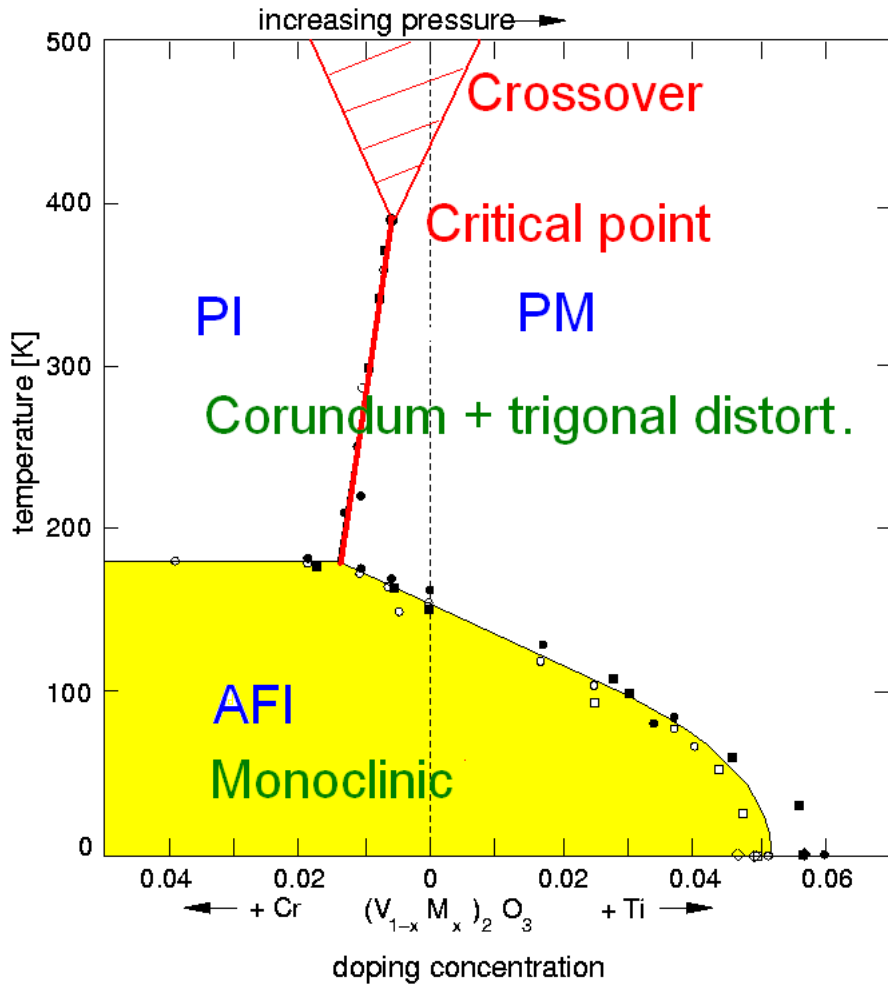


Confirmation of three-peak structure in correlated bulk systems

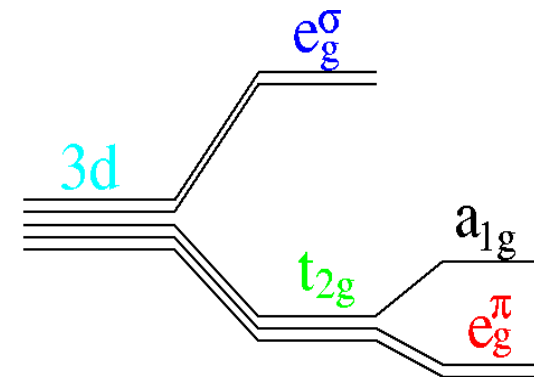
2. Application:

3d<sup>2</sup> system: V<sub>2</sub>O<sub>3</sub>

# 3d<sup>2</sup> system: V<sub>2</sub>O<sub>3</sub>



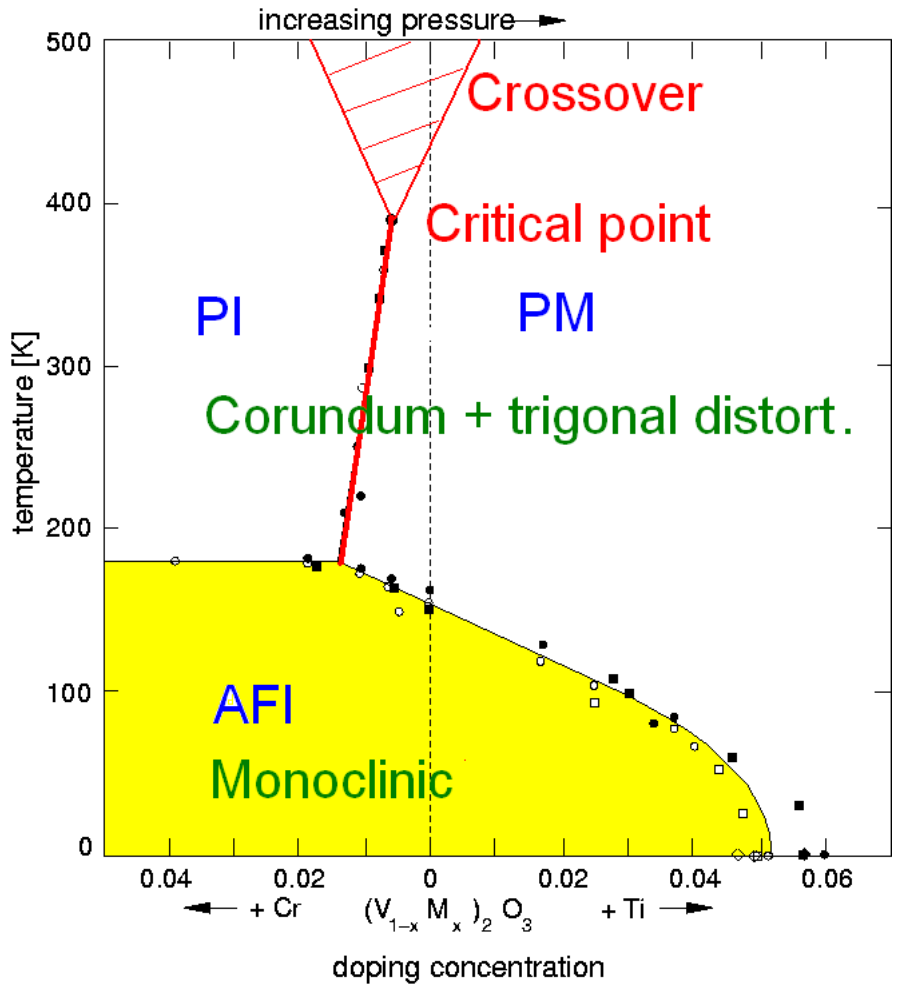
← increasing interaction U



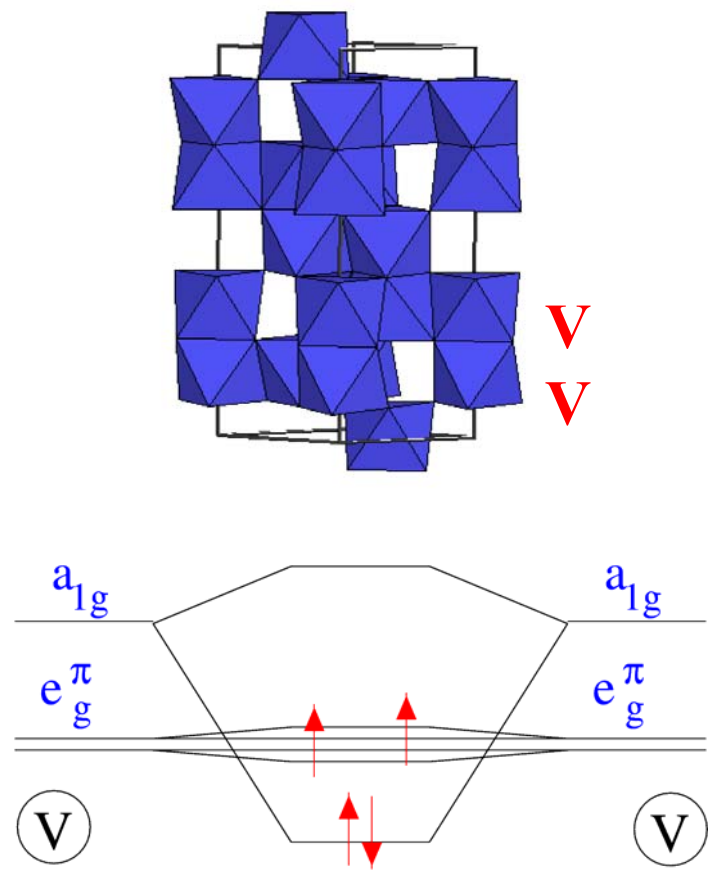
isotropic cubic trigonal



# 3d<sup>2</sup> system: V<sub>2</sub>O<sub>3</sub>



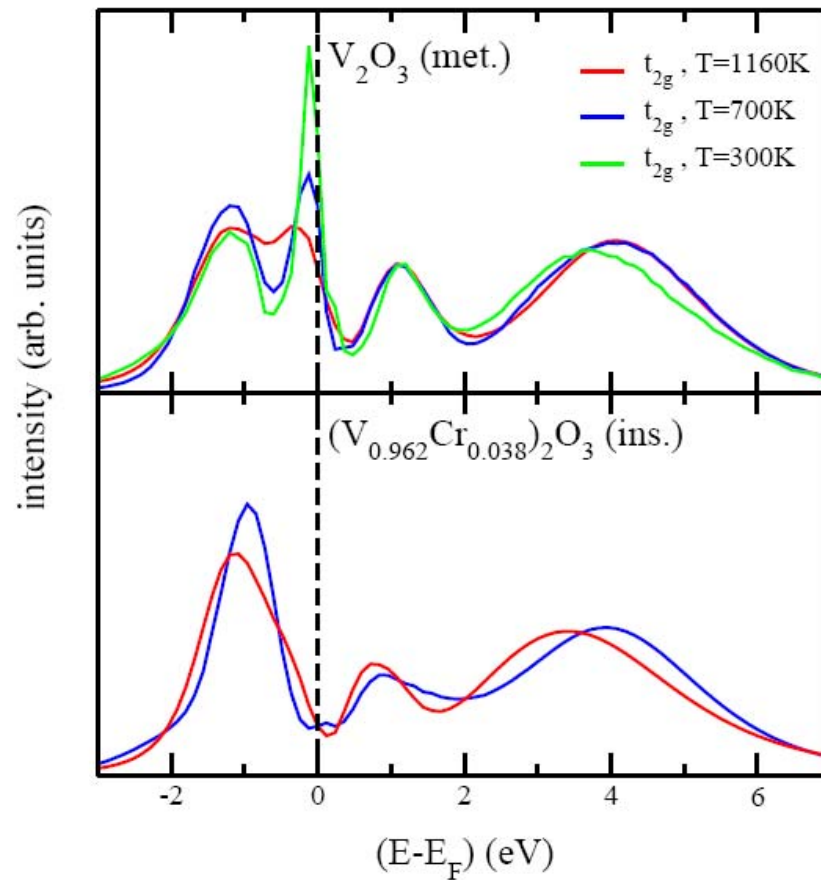
← increasing interaction U



Castellani, Natoli, Ranninger (1978)

$a_{1g}$  singlet

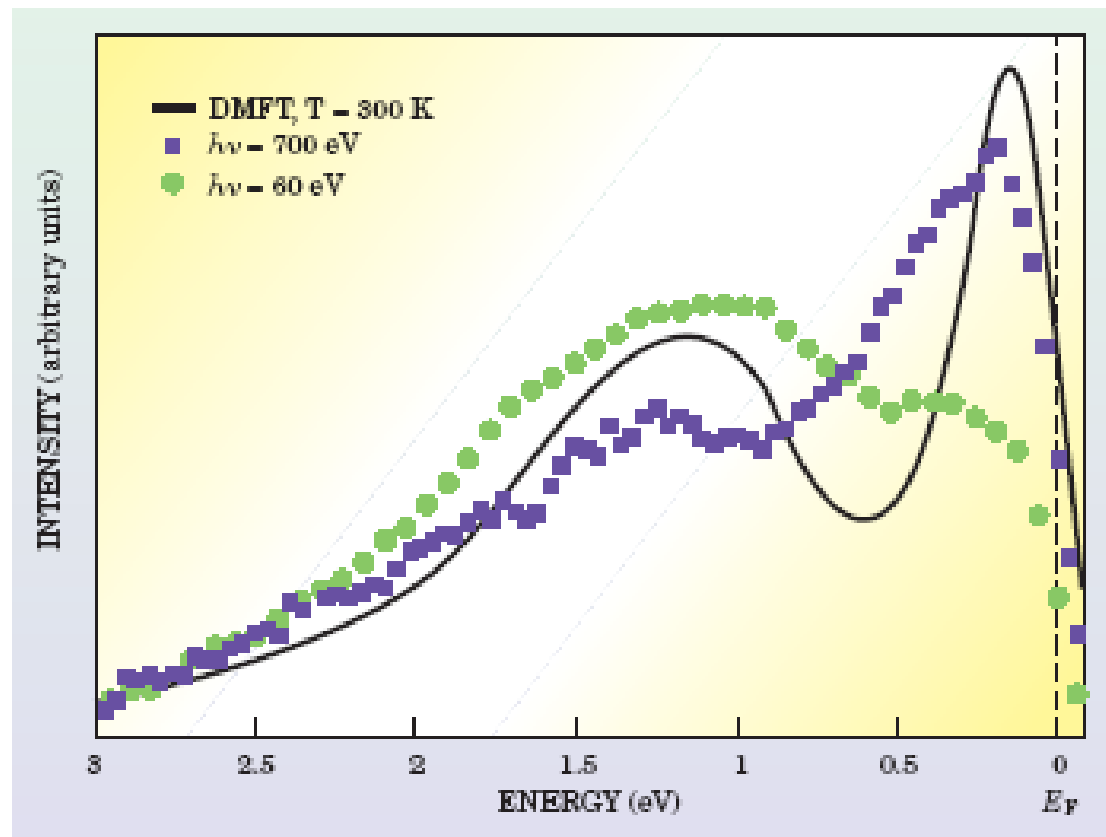
# $V_2O_3$ : LDA+DMFT Spectra



$U=5.0$  eV,  $J=0.93$  eV

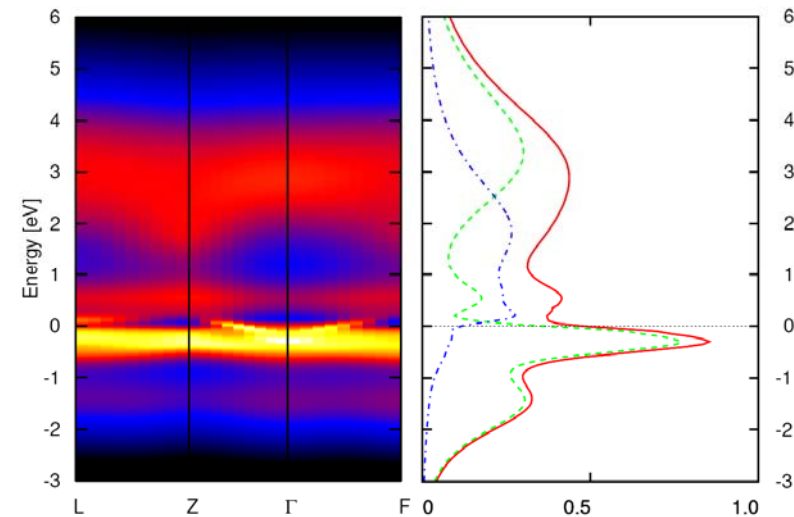
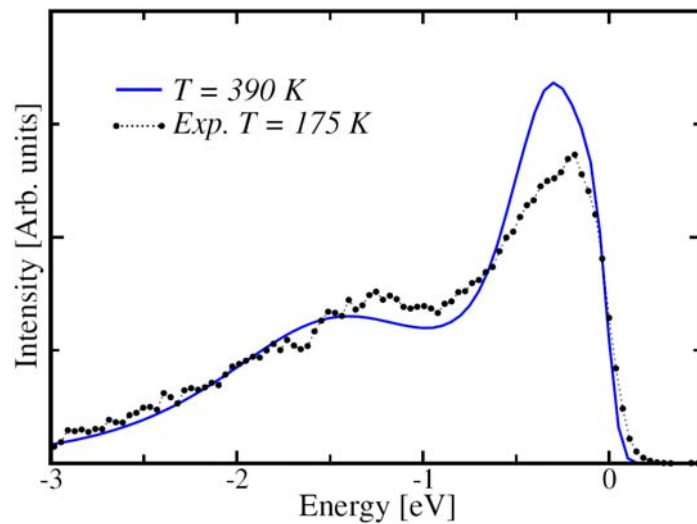
Held, Keller, Eyert, DV, and Anisimov, PRL (2001),  
Keller, Held, Eyert, DV, and Anisimov; PRB (2004)

# Metallic $V_2O_3$ : Photoemission Spectra



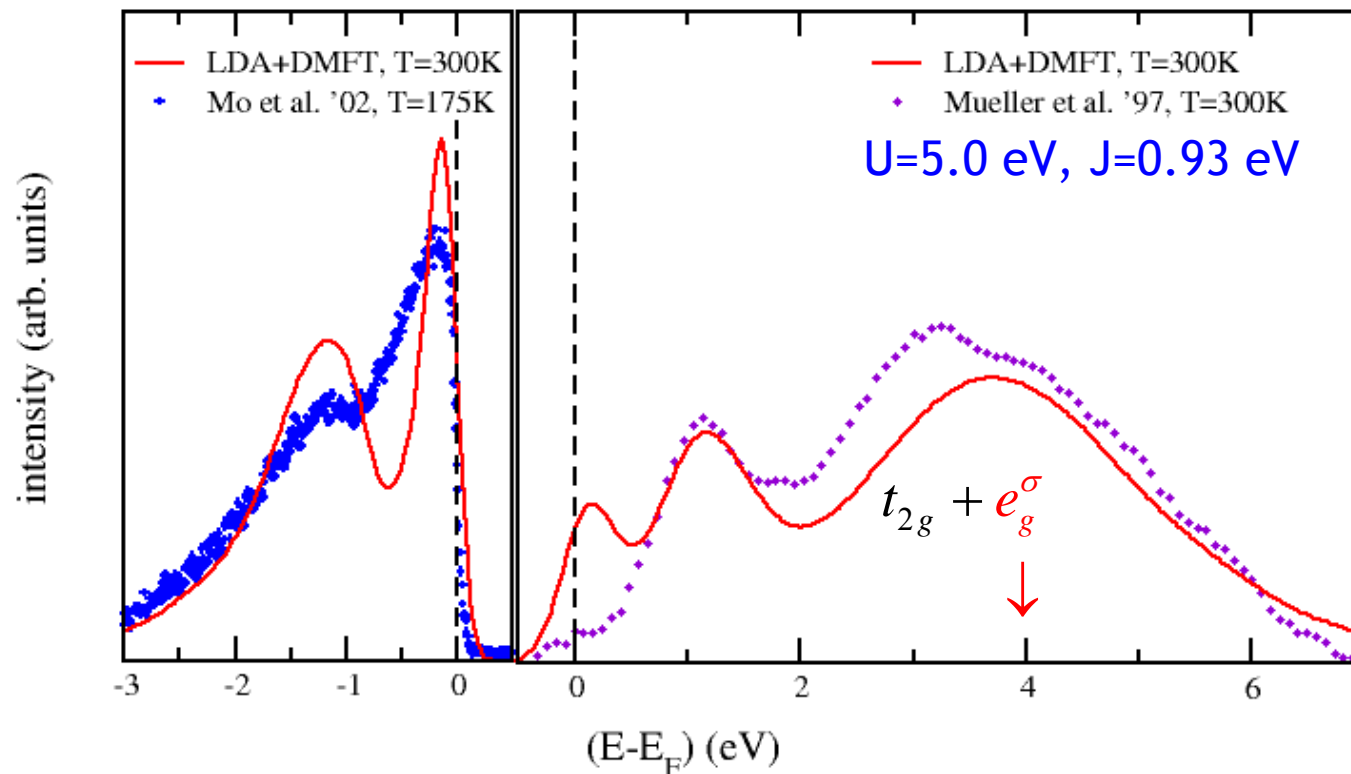
Ann Arbor - Osaka - Augsburg - Ekaterinburg collaboration; Mo *et al.*, PRL (2003)

# Metallic $V_2O_3$ : Photoemission Spectra in LDA+DMFT and Experiment



Poteryaev, Tomczak, Biermann, Georges, Lichtenstein, Rubtsov, Saha-Dasgupta, Andersen (2007)

# Metallic $V_2O_3$ : Photoemission and XAS Spectra in Theory and Experiment

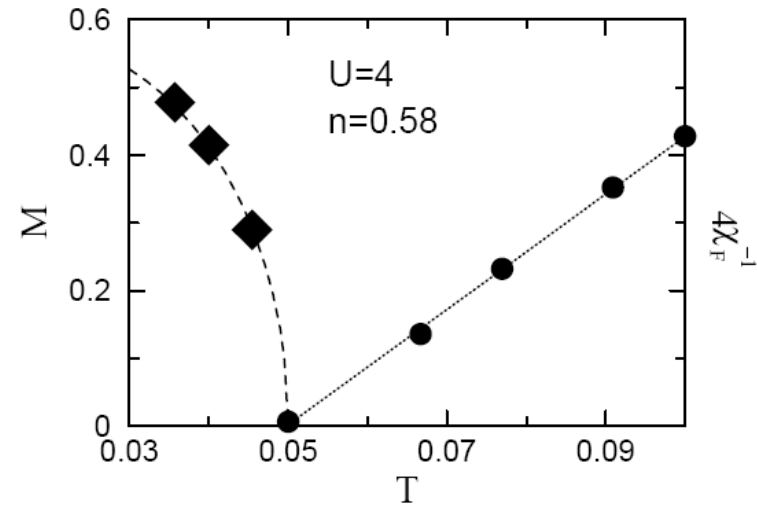
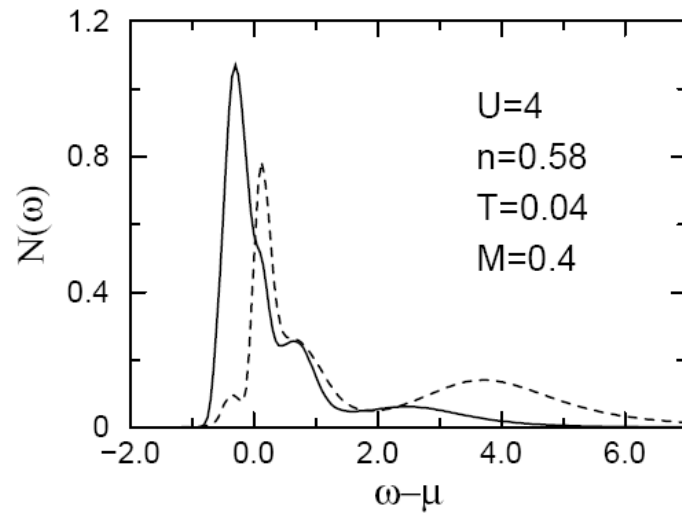


Ann Arbor - Osaka - Augsburg - Ekaterinburg collaboration; Mo *et al.*, PRL (2003)

### 3. Application: Ferromagnetic Materials

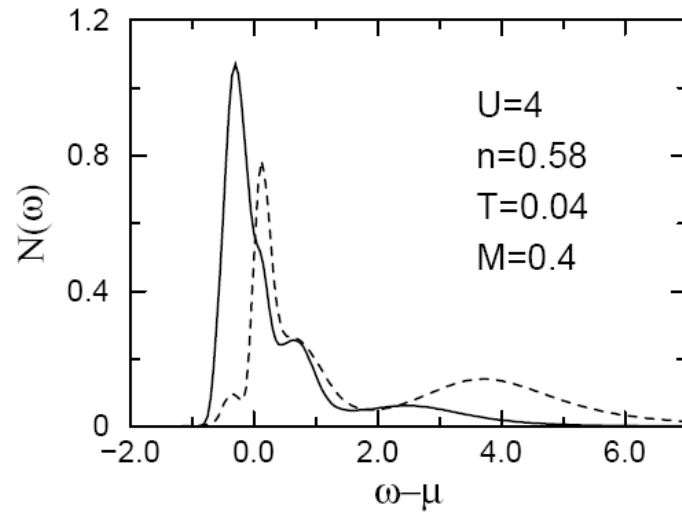
# DMFT: Ferromagnetism in the one-band Hubbard model

Ulmke (1998)

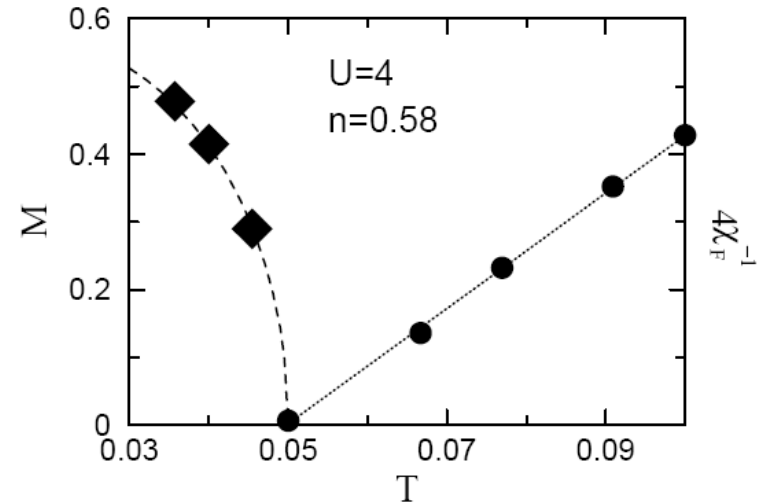


Generalized fcc lattice ( $Z \rightarrow \infty$ )

# DMFT: Ferromagnetism in the one-band Hubbard model

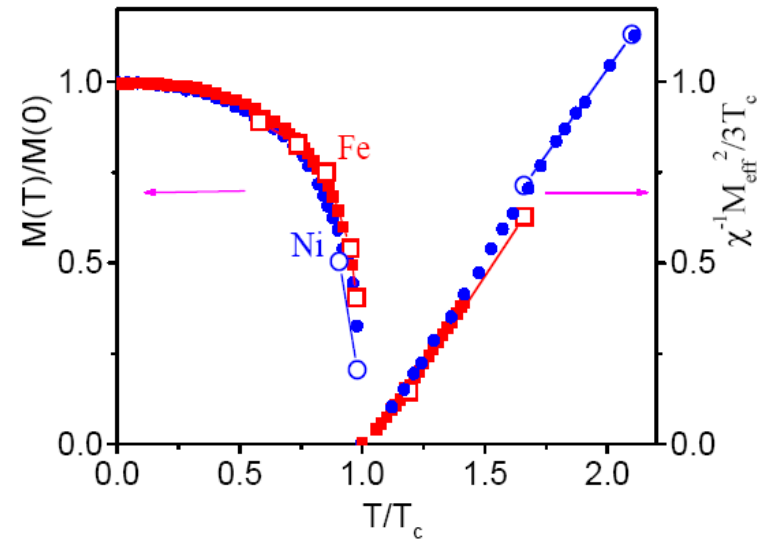
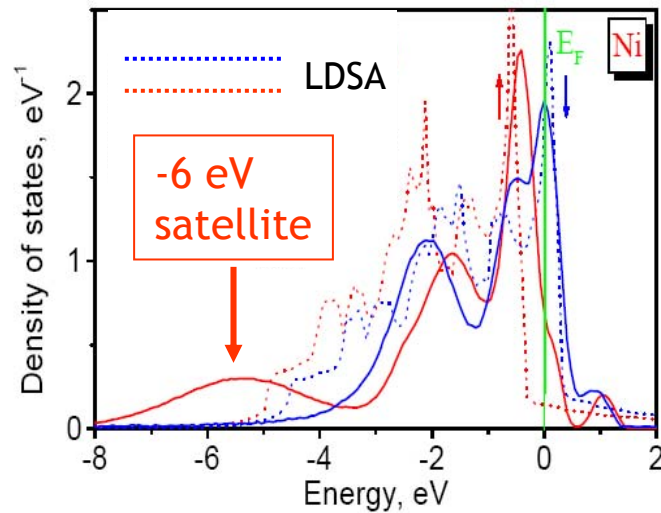


Ulmke (1998)



# LDA+DMFT for ferromagnetic Ni

Lichtenstein, Katsnelson, Kotliar (2004)



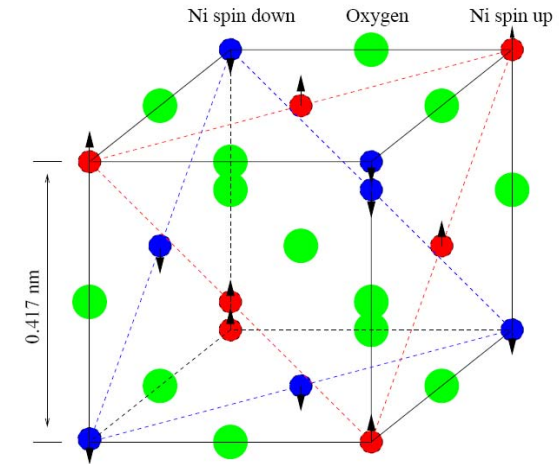


## 4. Application: Mott-Hubbard vs. charge-transfer insulator

NiO

# NiO: Mott-Hubbard vs. Charge-Transfer Insulator

## NiO (Bunsenite)



- Rock-salt structure
- Type II antiferromagnet (AF),  $T_N = 523$  K
- Moment  $\sim 1.8 \mu_B$
- Insulator with large energy gap  $\sim 4$  eV; persists up to  $> 1000$  K

LDA: metallic ground state

LSDA: AF, but

- gap + moment too small
- gap due to AF

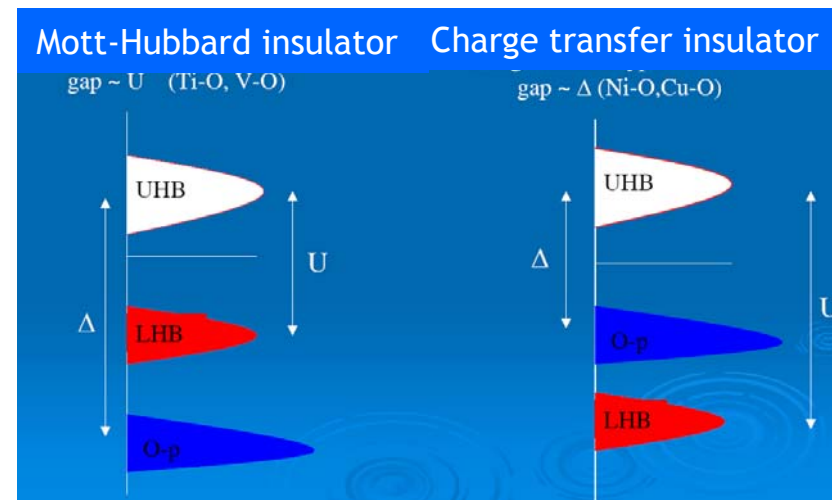
# NiO

“Surprising properties of materials with incompletely filled 3d bands”

Boer, Verwey (1937)  
Peierls, Mott (1937)

- Antiferromagnet,  $T_N = 523$  K
- Insulator with gap  $\sim 4$  eV; persists up to  $> 1000$  K

NiO: Really a prototypical Mott-Hubbard insulator?



Mott (1949)  
Hubbard (1963, 1965)

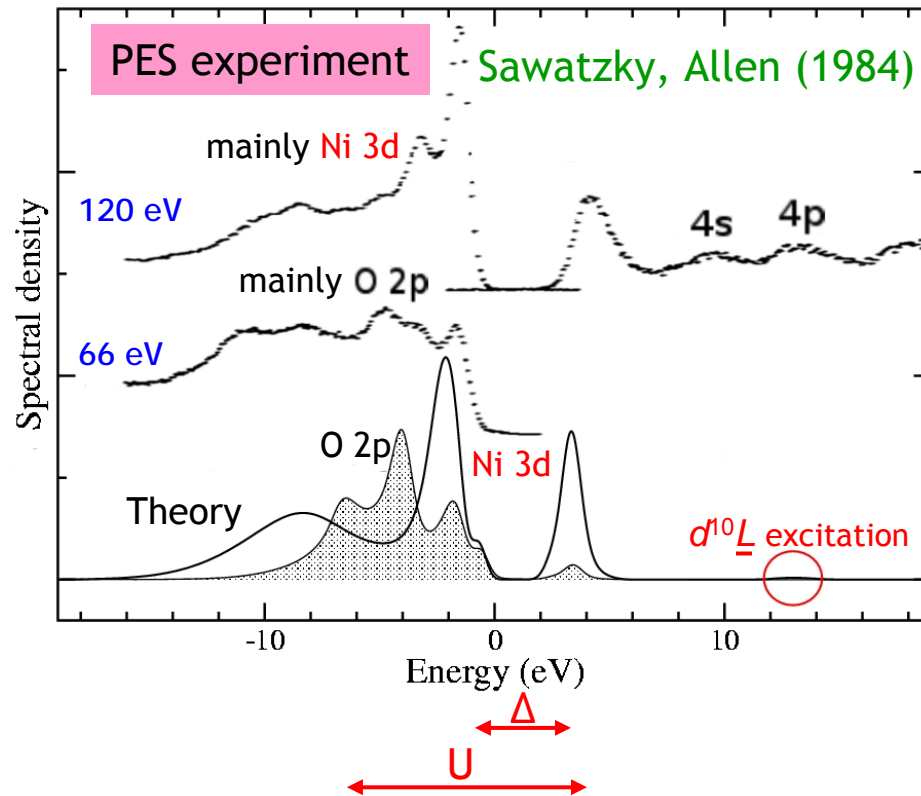
Zaanen, Sawatzky,  
Allen (1985)

→ Need to include correlated Ni-3d + O-2p states (p-d hybridization)

# NiO: LDA+DMFT in Wannier function basis

p-d hybridization in LDA+DMFT  $\rightarrow$  8-band Hamiltonian

Kuneš, Anisimov, Lukoyanov, DV; PRB (2007)



- charge-transfer gap clearly seen
- correct position of p-band
- valence band: p-d character
- conduction band: d-character
- lower Hubbard at -9 eV

p-d hybridization is essential

## NiO: LDA+DMFT in Wannier function basis

p-d hybridization in LDA+DMFT  $\rightarrow$  8-band Hamiltonian

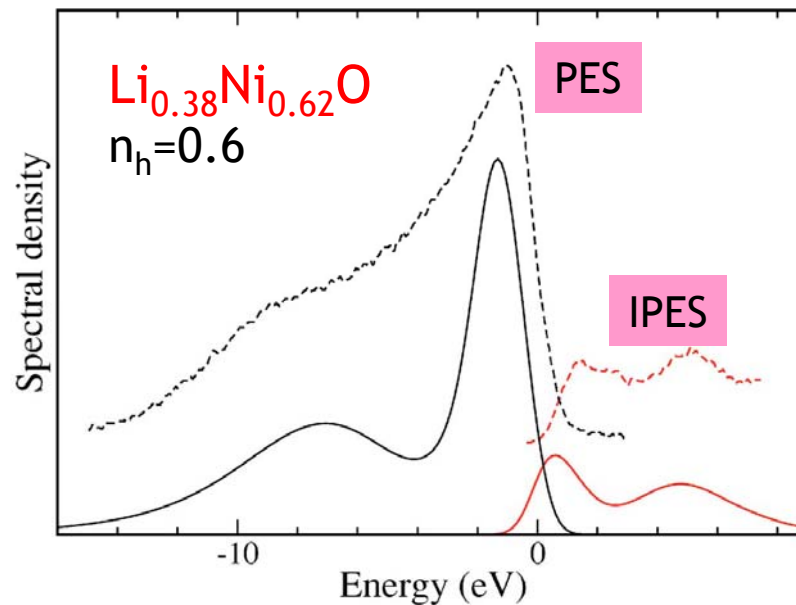
Kuneš, Anisimov, Lukoyanov, DV; PRB (2007)

Hole doping of NiO



van Elp, Eskes, Kuiper, Sawatzky (1992)

$$x\text{Ni}^{2+} \rightarrow \text{Li}^{1+} \Rightarrow n_h \approx \frac{x}{1-x} \text{ per Ni site}$$



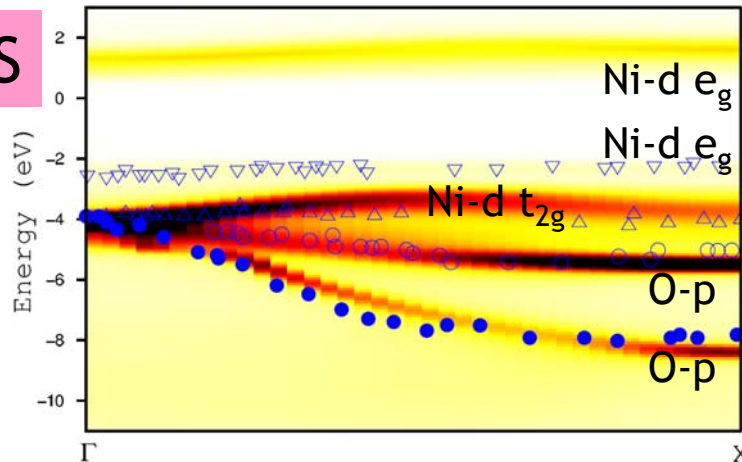
Spectral transfer  $\rightarrow$  Mott gap filled

## NiO: LDA+DMFT in Wannier function basis

p-d hybridization in LDA+DMFT  $\rightarrow$  8-band Hamiltonian

Kuneš, Anisimov, Lukoyanov, DV; PRB (2007)

ARPES



Exp.:

Shen *et al.* (1990,1991)

Theory:

Kuneš, Anisimov, Skornyakov,  
Lukoyanov, DV; PRL (2007)

- Ni-d bands only weakly dispersive
- O-p bands dispersive
- Result of d-correlations + p-d hybridization

5. Application:  
Correlation-induced structural transformation

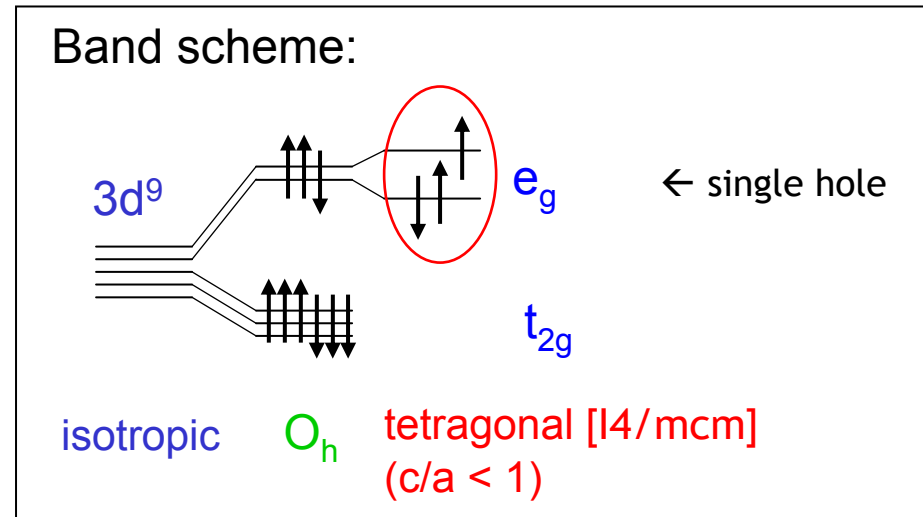
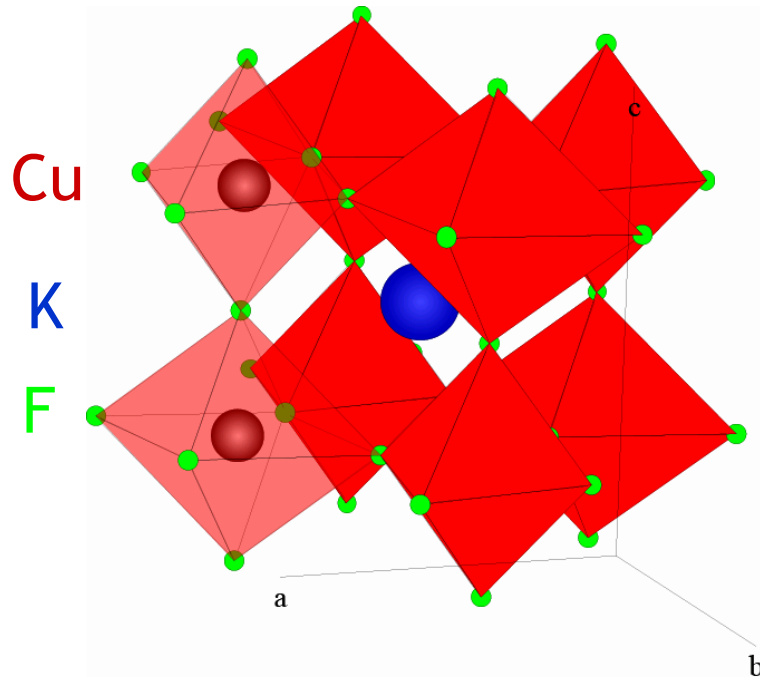


# KCuF<sub>3</sub>: Prototypical Jahn-Teller system

Room temperature crystal structure:

Kugel, Khomskii (1982)

Lichtenstein, Anisimov, Zaanen (1995)



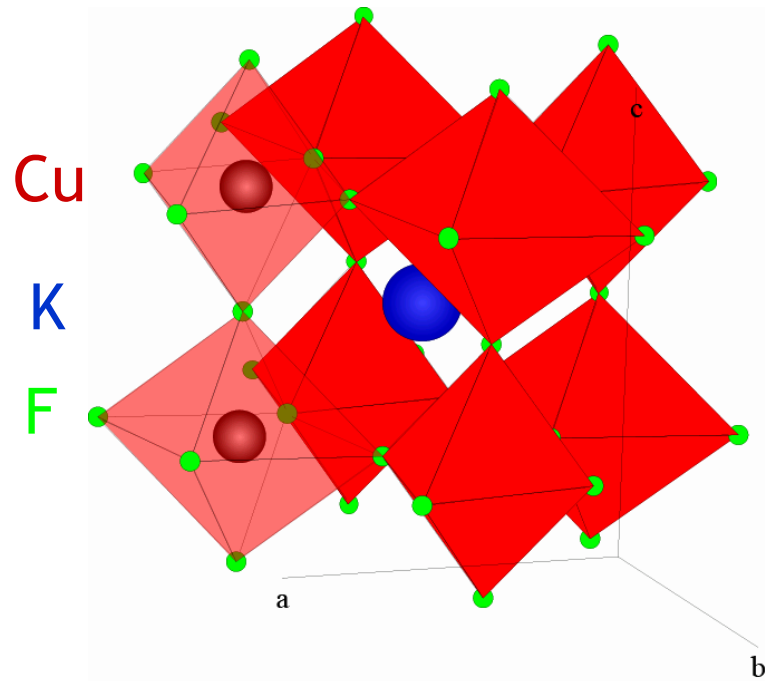
- insulating, pseudo-cubic perovskite
- $T_{\text{Neel}} \sim 38$  K

Cooperative JT distortion = spontaneous lifting of orbital degeneracy  
→ orbital order → structural relaxation with symmetry reduction



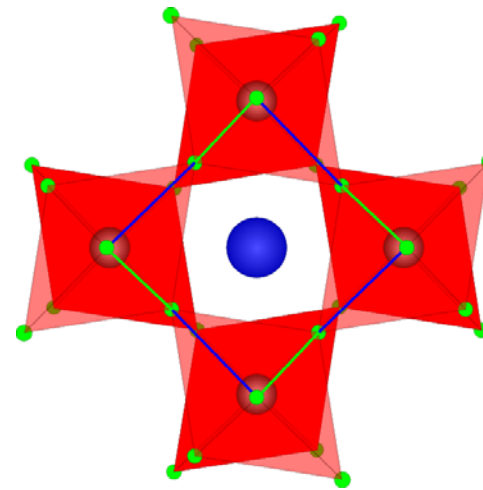
# KCuF<sub>3</sub>: Prototypical Jahn-Teller system

Room temperature crystal structure:



Kugel, Khomskii (1982)

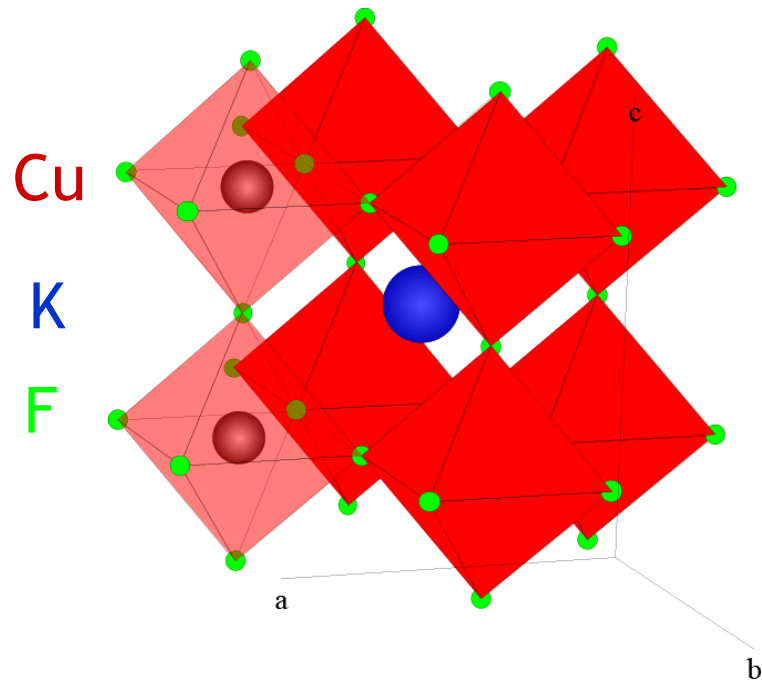
Liechtenstein, Anisimov, Zaanen (1995)



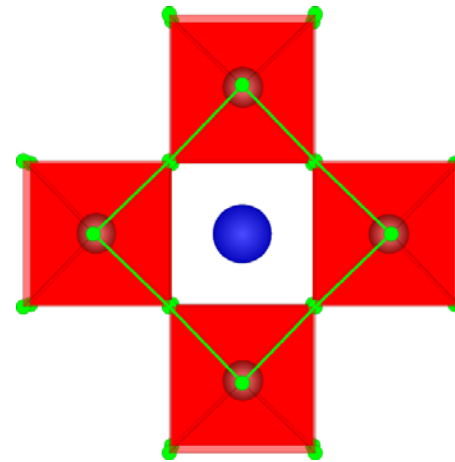
Cooperative JT distortion

# KCuF<sub>3</sub>: Prototypical Jahn-Teller system

Room temperature crystal structure:



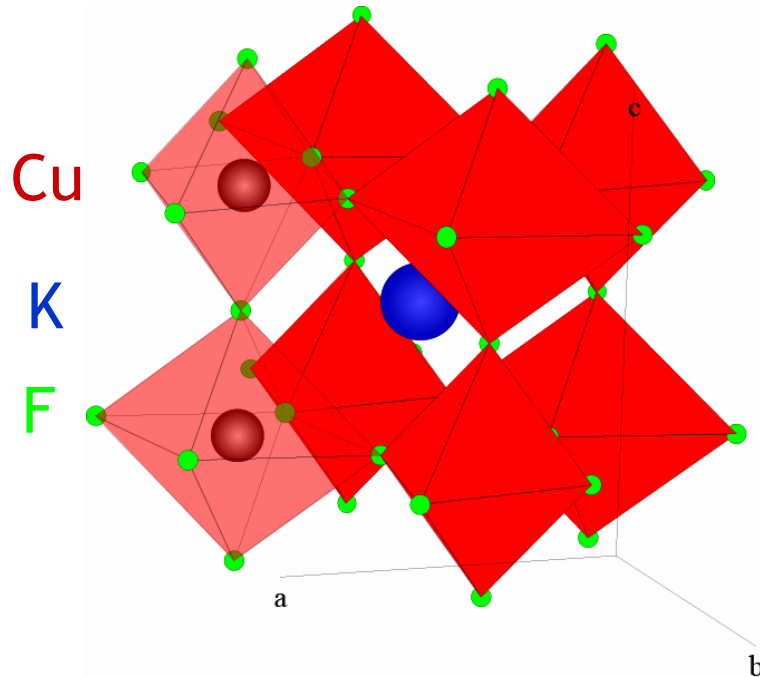
Kugel, Khomskii (1982)  
Liechtenstein, Anisimov, Zaanen (1995)



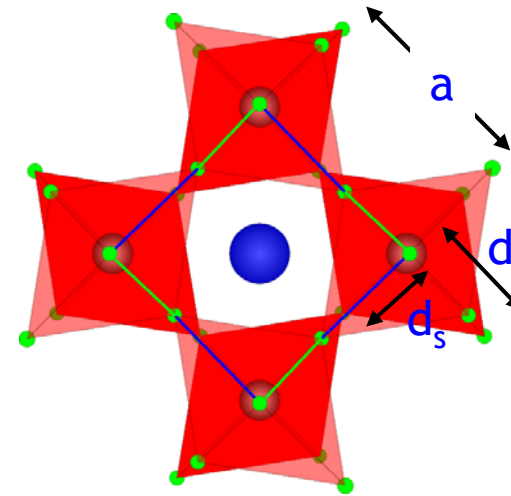
Undistorted structure

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$$\text{JT-distortion } \delta_{\text{JT}} = (d_l - d_s) / a$$

$T > T_N \sim 38 \text{ K}$ : Correlated paramagnetic insulator with strong JT distortion

LDA/GGA+U predicts magnetic LRO

How to determine - cooperative JT-distortion ?  
- correct orbital order ?

# KCuF<sub>3</sub>: GGA+DMFT results

Leonov, Bingelli, Korotin, Anisimov,  
Stojic, DV; PRL (2008)

Implementation with plane-wave pseudo-potentials

$$U = 7.0 \text{ eV}, J = 0.9 \text{ eV}$$

## GGA:

- **metallic** solution
- very shallow minimum of energy at  $\delta_{JT} = 2.5\%$   
→  $\delta_{JT} = 0$  for  $T > 100 \text{ K}$   
(no orbital order)

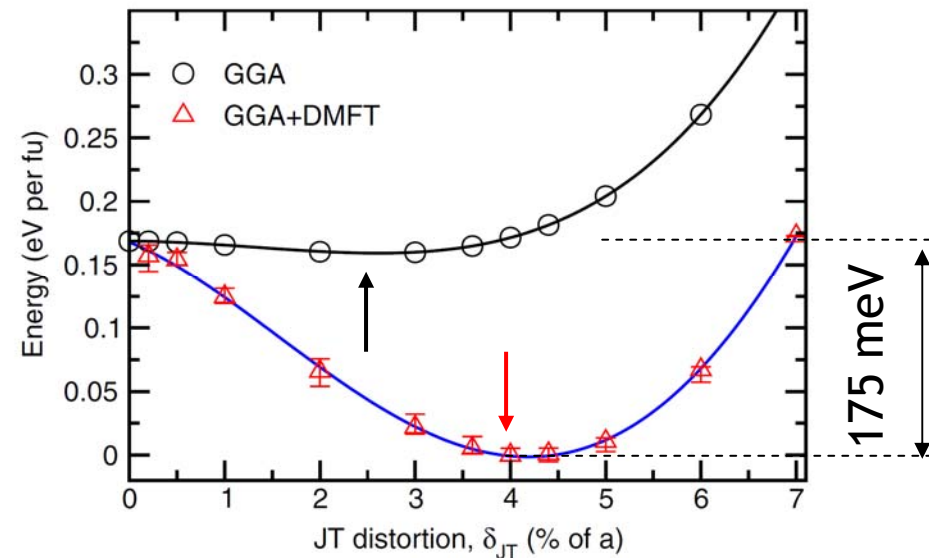
Inconsistent with experiment

## GGA+DMFT:

- **paramagnetic** insulator
- $\delta_{JT}^{\text{opt}} = 4.13\%$  → JT distortion **persists** up to **1000 K** (melting)
- **AF orbital order**

Good agreement with experiment at 300 K

## Total energy

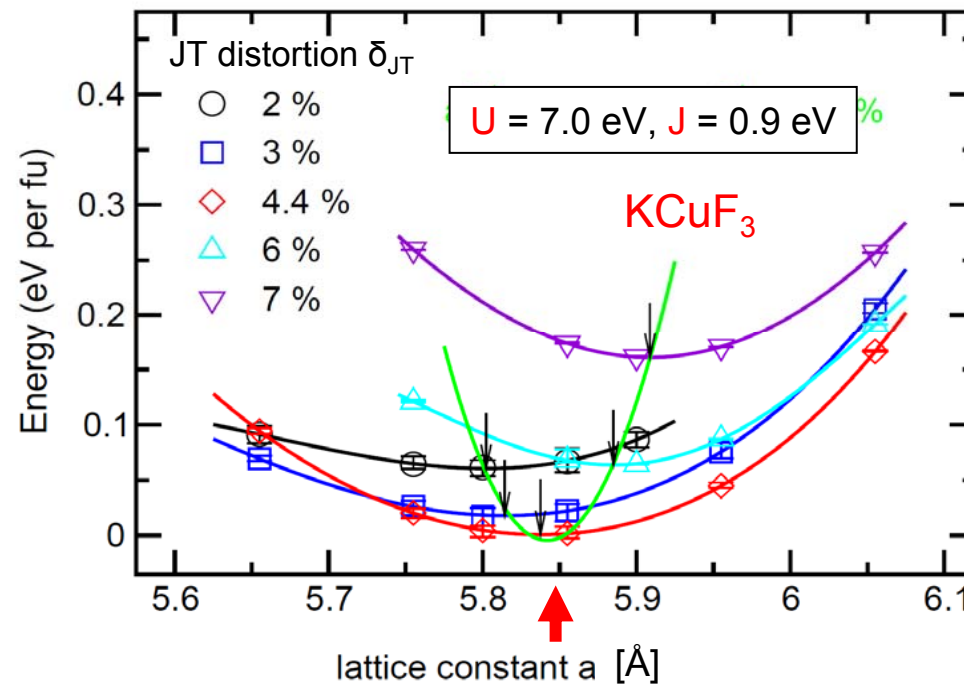


→ Structural transformation caused by **electronic correlations**

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# Beyond DMFT

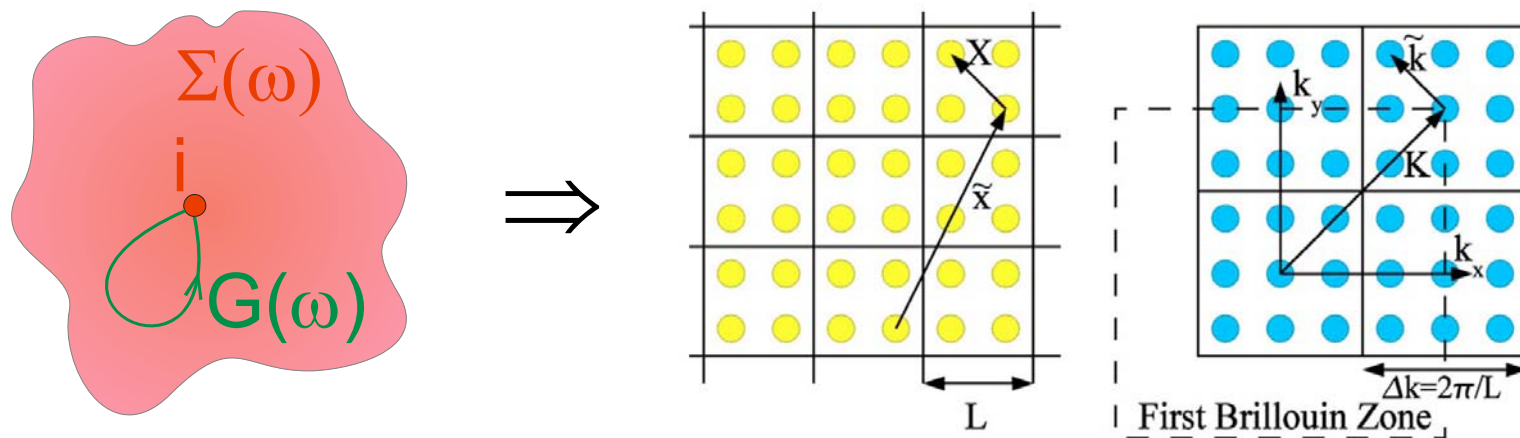
## Cluster Extensions

- Dynamical cluster approx. (DCA)
- Cluster DMFT (CDMFT)
- Self-energy functional theory

Hettler *et al.* (1998, 2000)

Kotliar *et al.* (2001)

Potthoff (2003)

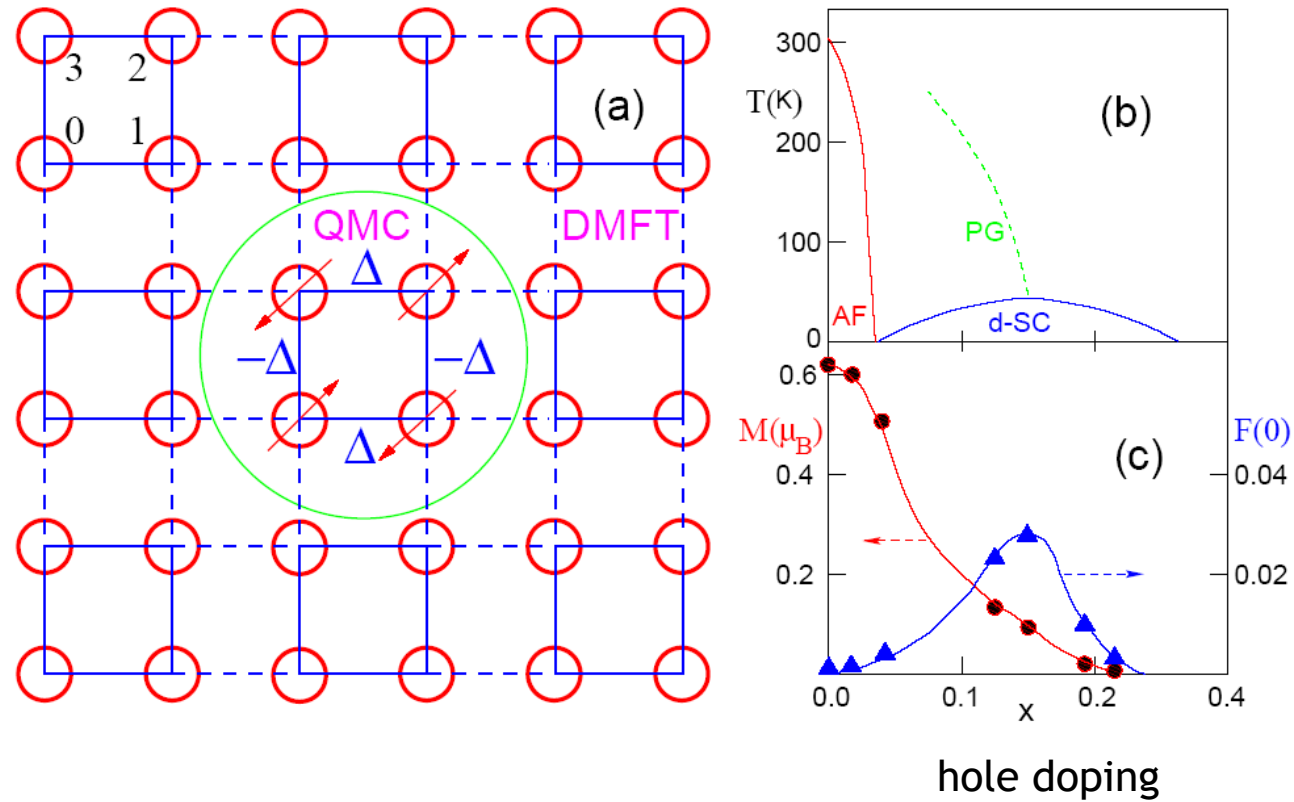


## Dynamical vertex approximation (DVA)

Local + non-local self-energy diagrams from local irred. vertex

Toschi, Katanin, Held (2006)

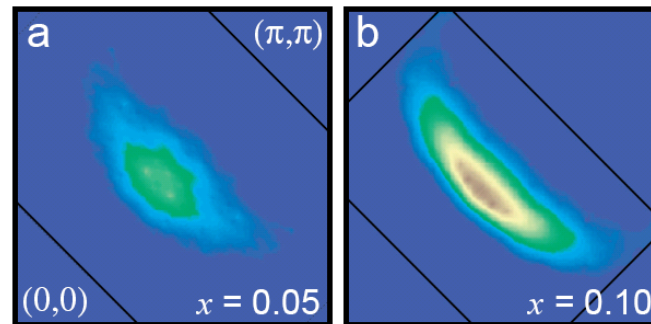
# Antiferromagnetic $d$ -wave $2 \times 2$ periodically repeated cluster



Lichtenstein, Katsnelson (2000)

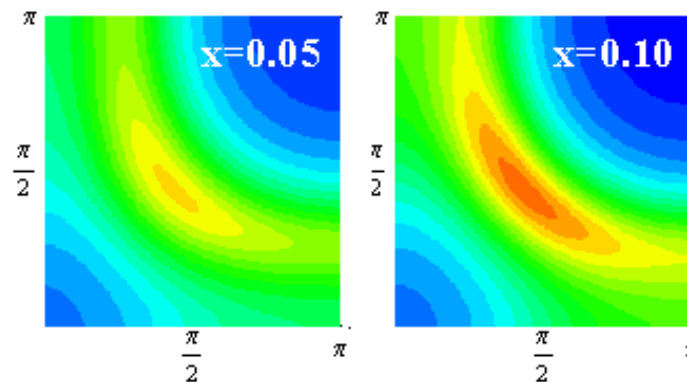
# Comparison with Experiments in Cuprates: Spectral Function $A(\mathbf{k}, \omega \rightarrow 0)$ vs. $\mathbf{k}$

hole doped



Shen *et al.* (2004)

2x2 CDMFT



Civelli, Capone, Kancharla,  
Parcollet, Kotliar (2005)



## Long-term goal

Explanation + prediction of general properties of complex correlated electron materials, e.g.,

