# Tutorial One Solutions

[1] What do hubvietri.f and ueq0vietri.f give for the Green's function?

| | |
|---|---|
| hubvietri.f | G(0,1)=-0.10272 |
| ueq0vietri.f | G(0,1)=-0.10246 |
| hubvietri.f | G(0,2)=-0.00410 |
| ueq0vietri.f | G(0,2)=-0.00411 |

[2] Can you show that when you reduce $\Delta\tau$ (at fixed $\beta$) that the agreement between the two codes becomes perfect? The Trotter errors associated with the checkerboard breakup can thus be eliminated.

| | |
|---|---|
| hubvietri.f $L = 4,\ \ \Delta\tau = 0.12500$ | $G(0,1) = -0.10272$ |
| hubvietri.f $L = 8,\ \ \Delta\tau = 0.06250$ | $G(0,1) = -0.10252$ |
| hubvietri.f $L = 16, \Delta\tau = 0.03125$ | $G(0,1) = -0.10247$ |
| teq0vietri.f | $G(0,1) = -0.10246$ |

[3] Show (numerically) that for the special case of a 4x4 lattice there are no Trotter errors with the checkerboard breakup. That is, verify that the two codes agree perfectly in this case. Can you prove the result analytically?

For a 4x4 lattice with $t = 1, U = 0, \mu = -1$ I find

| | |
|---|---|
| hubvietri.f $L = 4,\ \ \Delta\tau = 0.12500$ | $G(0,1) = -0.10136$ |
| hubvietri.f $L = 8,\ \ \Delta\tau = 0.06250$ | $G(0,1) = -0.10136$ |
| hubvietri.f $L = 16, \Delta\tau = 0.03125$ | $G(0,1) = -0.10136$ |
| teq0vietri.f | $G(0,1) = -0.10136$ |

I will leave the analytic proof of the vanishing to you. I suspect it might be related to the fact that the 4x4 lattice is topologically equivalent to the 2x2x2x2 lattice (n=2 hypercube in 4 dimensions). See also Tutorial 2 Solutions.

[4] Check that the results in hubvietri.f for $G(0,1)$ are consistent with the results reported by the code for the kinetic energy. The code reports $G(0,1) = -0.10272$ and the kinetic energy as $KE = -0.82172$. The Green's function $\langle c_i c_j^\dagger \rangle$ contains only one of the two Hermitian conjugate terms in the kinetic energy. There are also two spatial directions $(x, y)$ in the kinetic energy. Finally, there are two spin species. Thus we must multiply the near-neighbor Greens function by 8 to bring it into agreement with the kinetic energy. This relation is satisfied.