TUTORIAL 2

Chain of correlated fermions

Luttinger liquid properties

Jordan-Wigner transformation

- Introduce fermion operators on each site of the ring:

$$S_i^- = (-1)^i c_i \exp\left(i\pi \sum_{j=1}^{i-1} n_j\right)$$

- Use this transformation to map the XXZ chain on a t-V chain of spinless fermions

$$H_{\mathrm{t-V}} = -t \sum_i (c_{i+1}^\dagger c_i + H.C.) + V(n_i - \tfrac{1}{2})(n_{i+1} - \tfrac{1}{2})$$

Numerical implementation

- Let us assume a ½-filled band

$$S_i^Z = c_i^\dagger c_i - 1/2 \Rightarrow \text{total } S^Z = 0$$

- Adapt the XXZ code for Ne=2p+1 electrons on a L=4p+2 sites ring. Why assuming an odd # of particles ?

- Calculate the charge gap:

$$\Delta_c = E(N_e + 1) + E(N_e - 1) - 2E(N_e)$$

What happens for V/t>2 ?

Exact solution of t-V model

- Bethe's equations solved by Haldane (PRL 80):
- Metal-insulator transition at V/t=2 (KT)
- Phase separation for V/t<-2
- Luttinger liquid for -2<V/t<2:
  - Fermi velocity: $u = \frac{\pi \sin 2\lambda}{\pi - 2\lambda}$ & $\cos 2\lambda = -\frac{V}{2t}$
  - Drude weight: $$D = \frac{\pi^2 \sin 2\lambda}{4\lambda(\pi - 2\lambda)}$$
  - Luttinger exponent: $K = \frac{D}{u} = \pi/(4\lambda)$

Numerical computations of the LL parameters (at e.g. V/t=1)

- Investigate the finite size scaling of the GS energy and the Fermi velocity; deduce the central charge c=1 of the model.
- Show that the inverse compressibility is:

$$\kappa^{-1} = L\Delta_c$$

- Calculate the LL parameter K from:

$$\kappa^{-1} = \frac{K}{\pi u}$$

Consistency check

- Introduce a flux Φ through the ring.

  Hint: add a phase $\phi = \frac{2\pi}{L}\Phi$ in the hopping

- Calculate the charge stiffness:

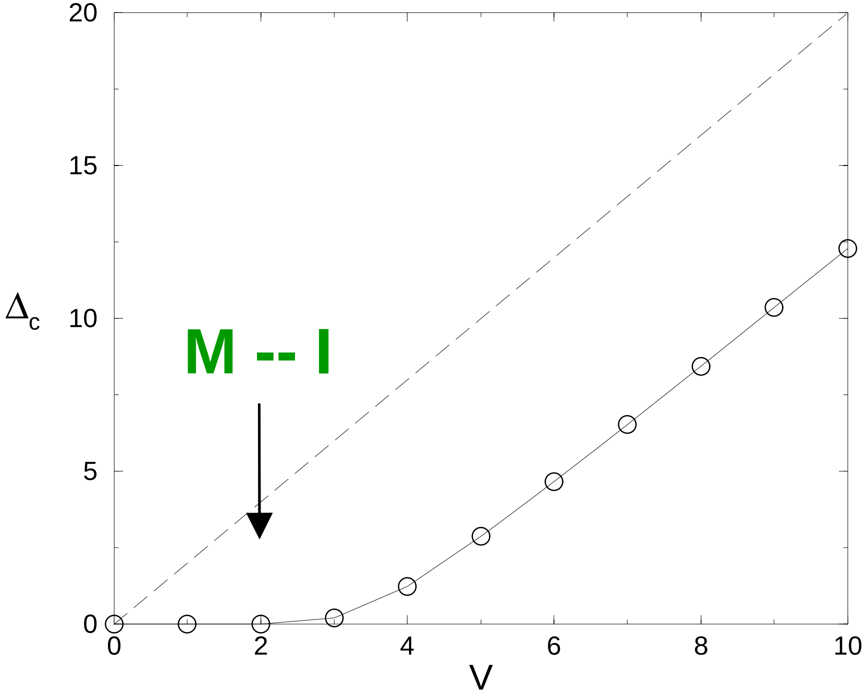
$$D = \frac{1}{2}\frac{\partial^2(E_0/L)}{\partial\phi^2}$$

- Deduce the value of K from:

$$D = uK$$

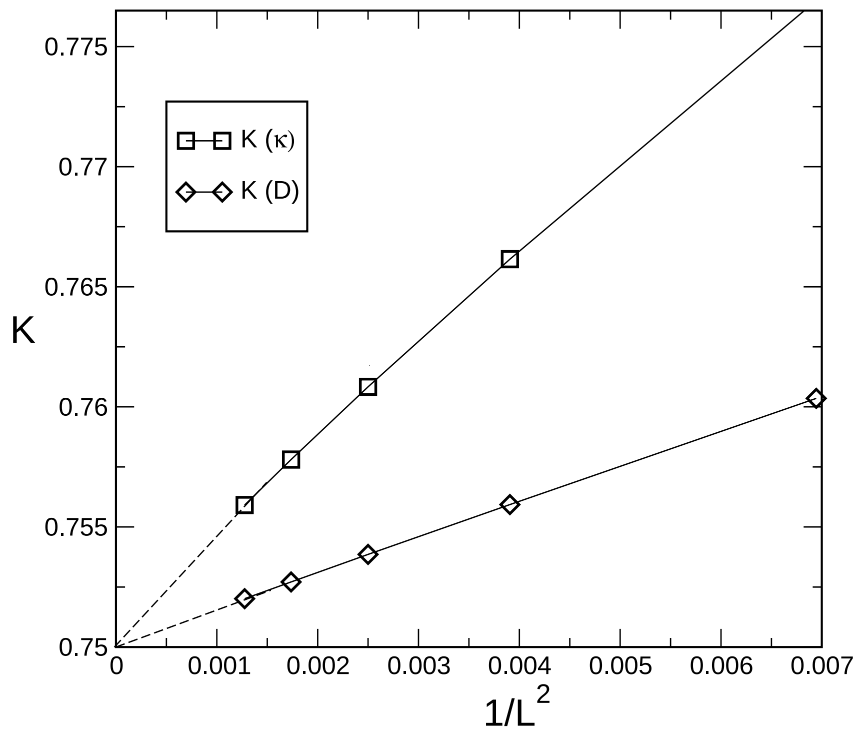
Numerical results
(S. Capponi, PhD, Toulouse, 1999)

Metal-Insulator transition



Charge gap vs V (extrapolation)

Finite size scaling of K



LL regime (V/t=1)

Comparison with Bethe Ansatz

Luttinger liquid exponent:

Numerical extrapolation (circles)

vs

exact results (full line)