Spin & doped ladders

Spin liquid

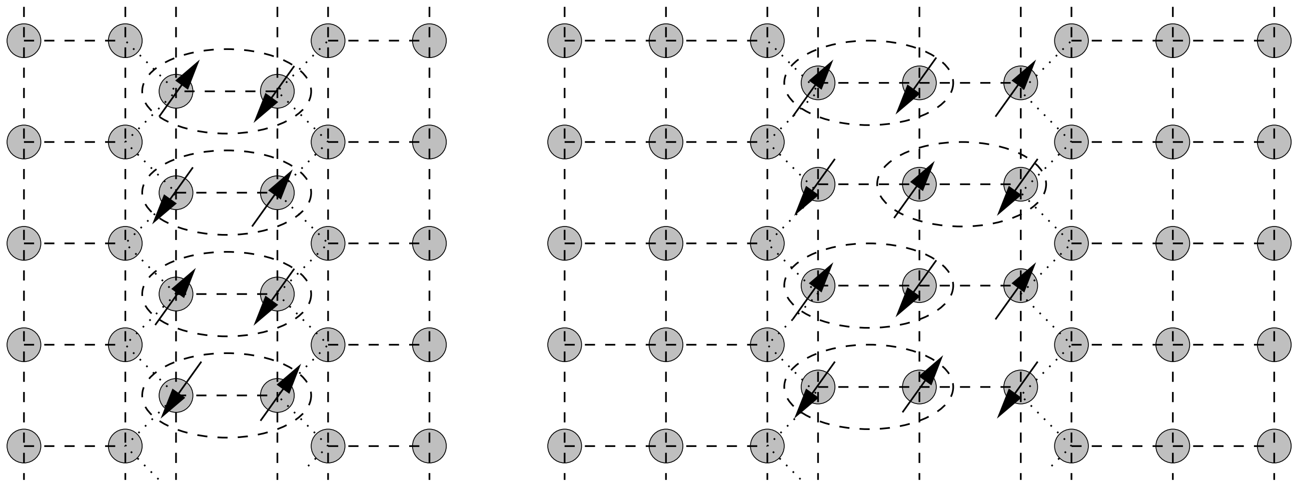
& superconducting properties

Outline

- **Ladder geometry**: some materials & physical properties of spin ladders
- **Doping**: microscopic models & field theory (bosonisation), numerical results, phase diagram, magnetic properties …
- **Pairing** mechanism & beyond

Ladder geometry

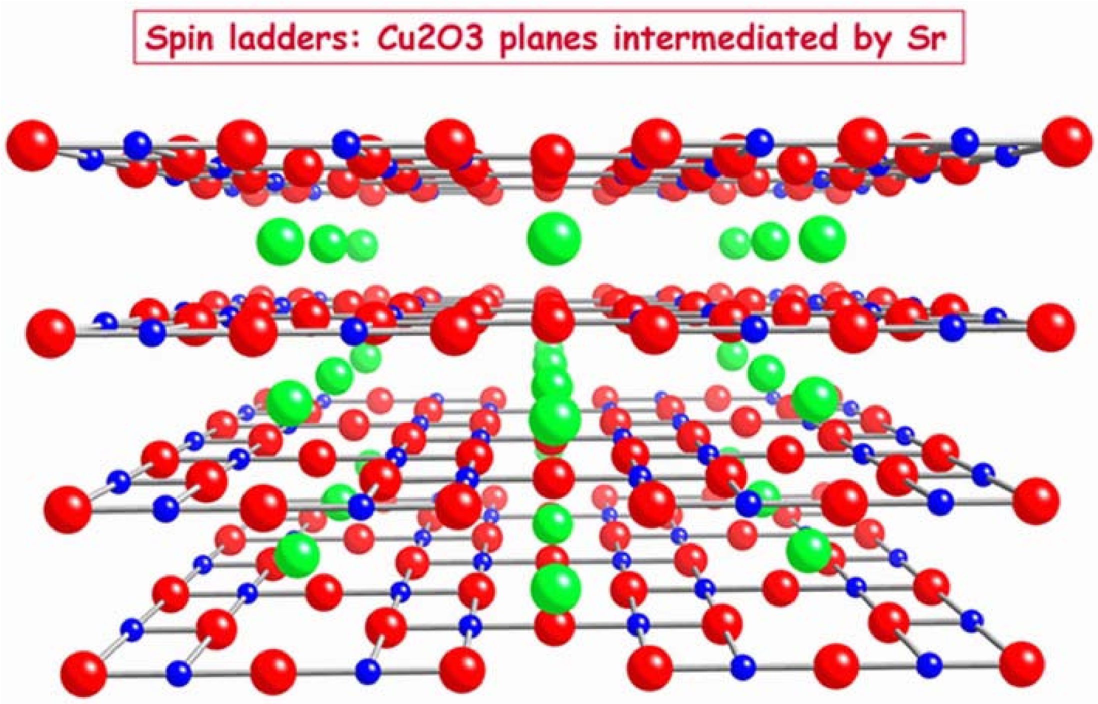
Two-leg ladders **Three-leg ladders**

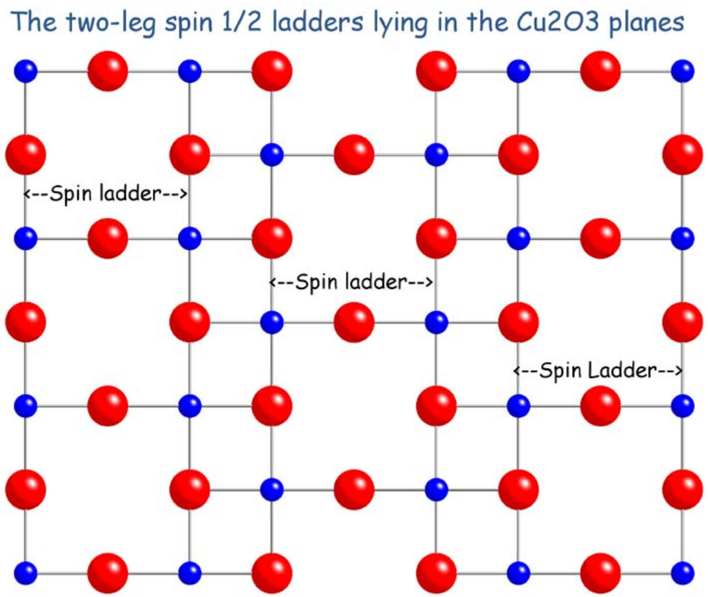


Spin ladders: each site carries a **S=1/2** (typically Cu-II atom)

Even & odd-leg ladders have different magnetic properties …

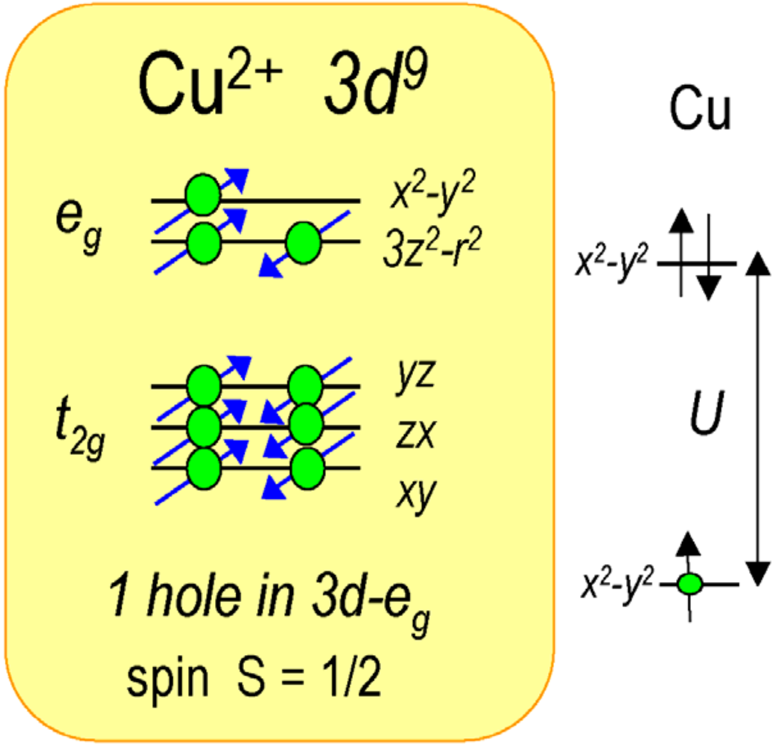
A typical spin ladder: $SrCu_2O_3$



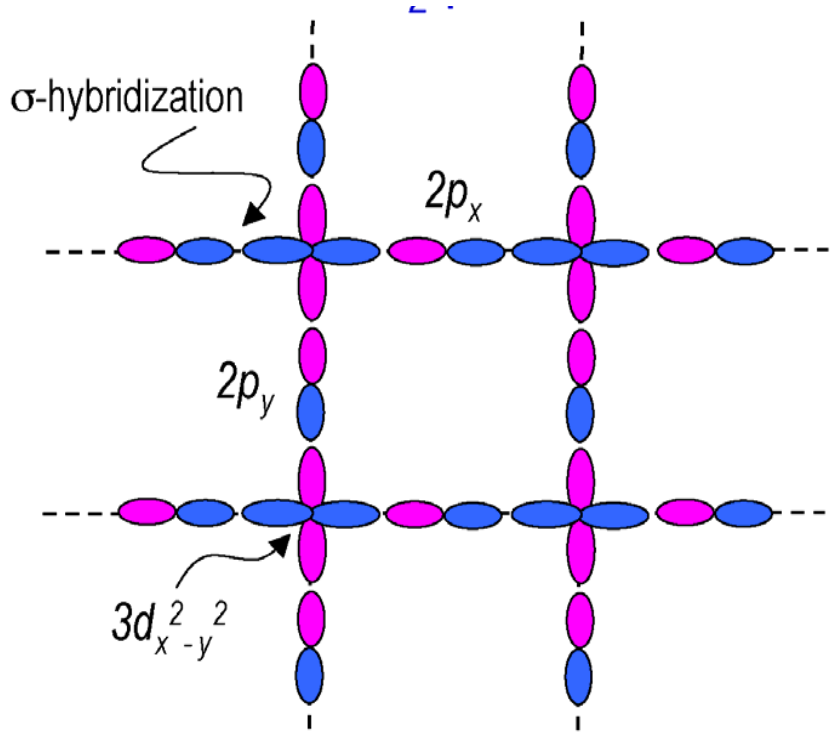


Derived from 2D high-Tc cuprate structure

Copper oxide Mott insulator
Heisenberg AF super-exchange

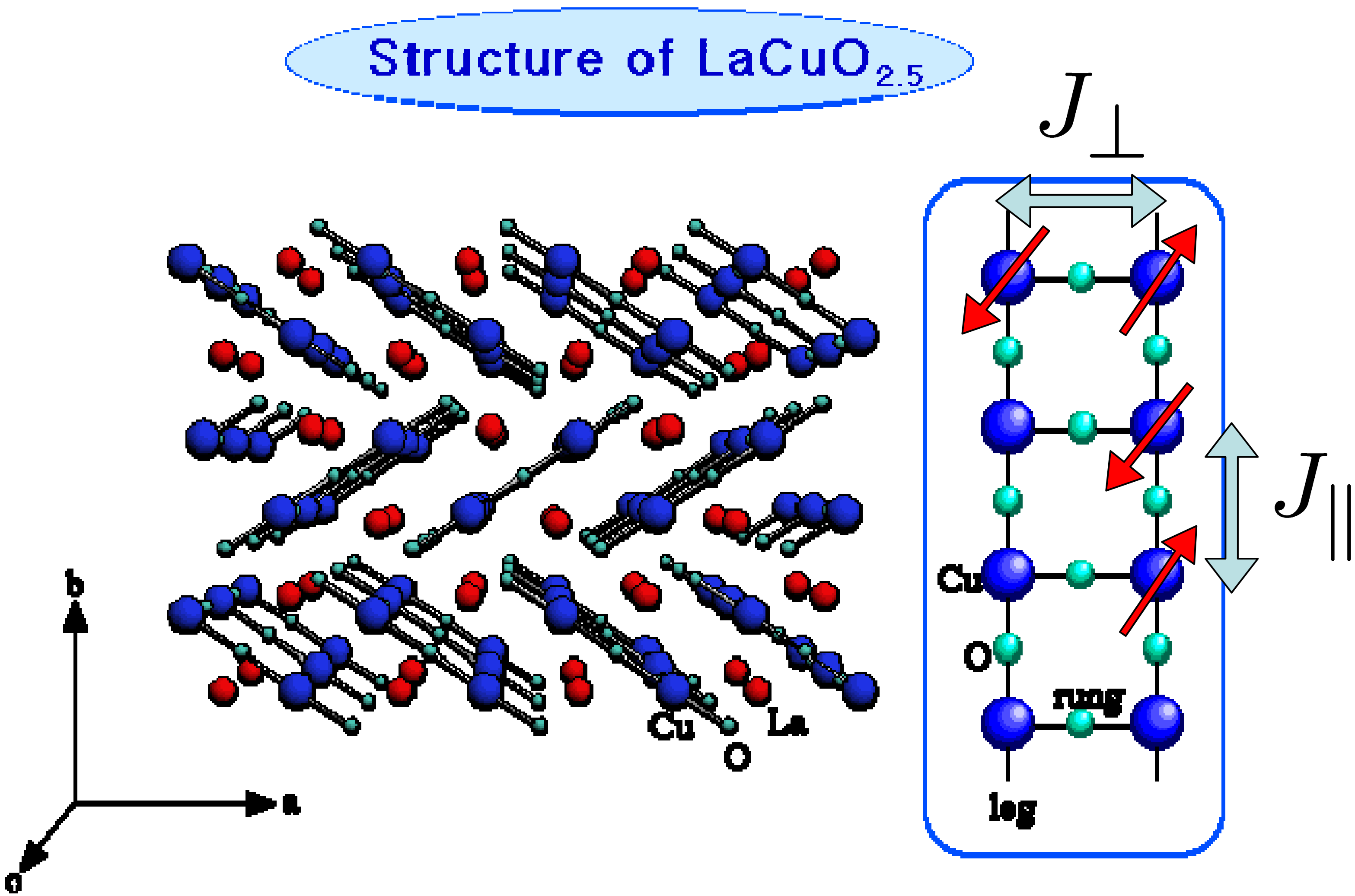


On-site repulsion U



AF exchange J
between copper spins

A ladder with 3D couplings



$Cu_2(C_5H_{12}N_2)_2Cl_4$

Chaboussant et al., 1998

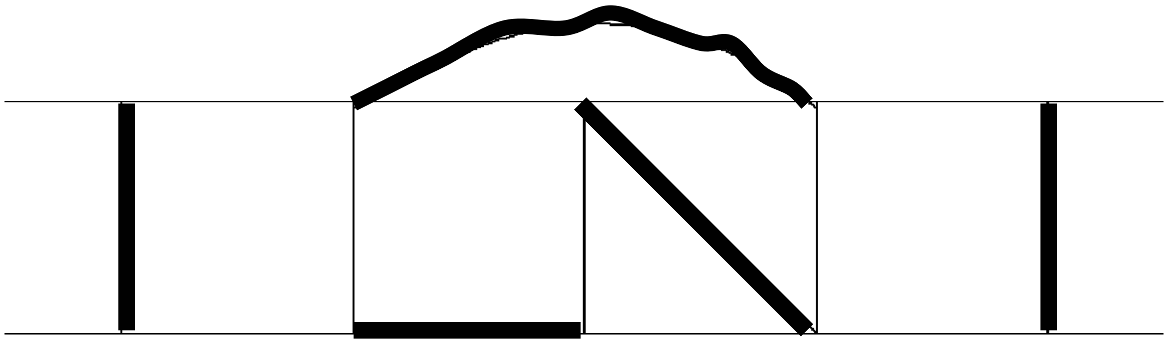
A few simple theoretical considerations

From the strong & weak coupling limits…

A RVB-like spin liquid

Anderson 87

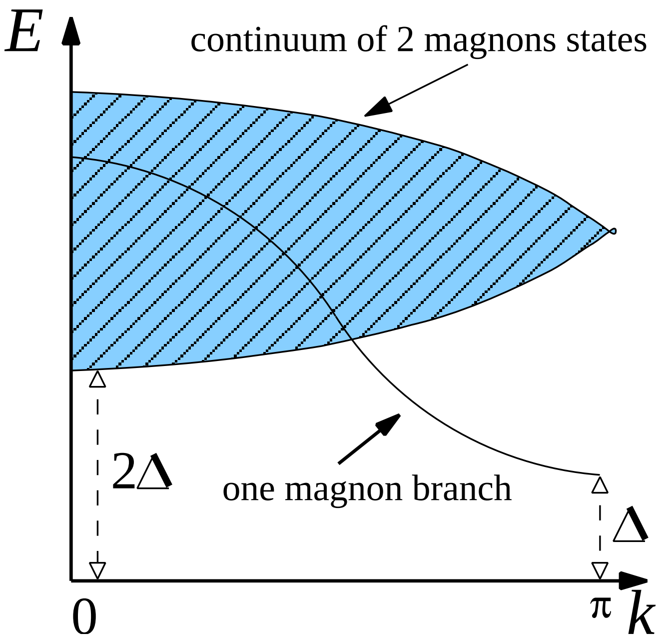
(a)
RVB



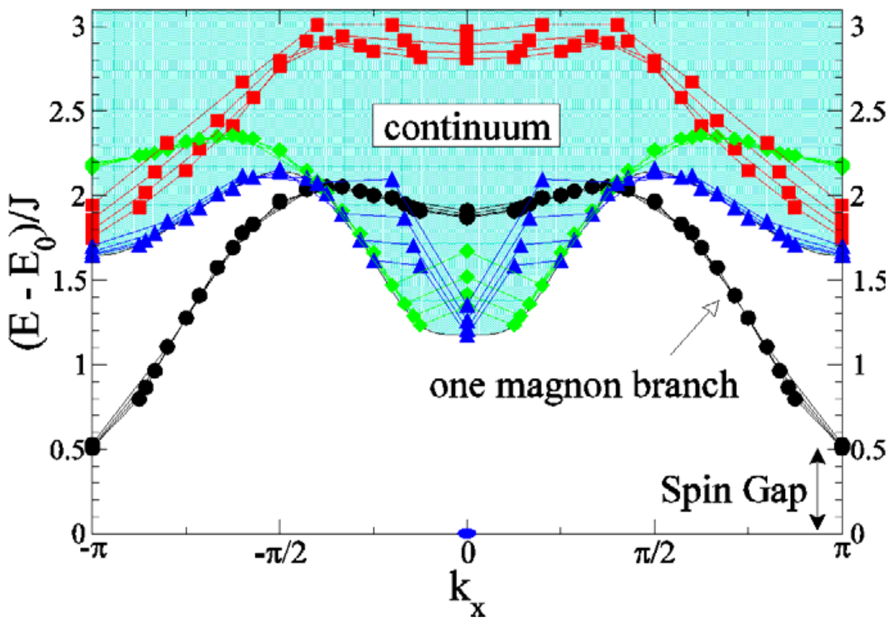
Resonating Valence Bond
= linear superposition of **short-range** VB configurations

$$\text{—} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - \uparrow\downarrow\rangle)$$

Strong coupling limit



Exact diag. (G. Roux et al.)



- For isolated rungs: cost $J_\perp$ to excite a triplet
- At finite $J_{||}$ magnon can propagate to gain kinetic enegy:

$$\omega_{\mathrm{mag}}(k) = J_\perp + J_{||}\cos k$$

The spin gap is robust !

- For even # of chains
  → **finite spin gap**

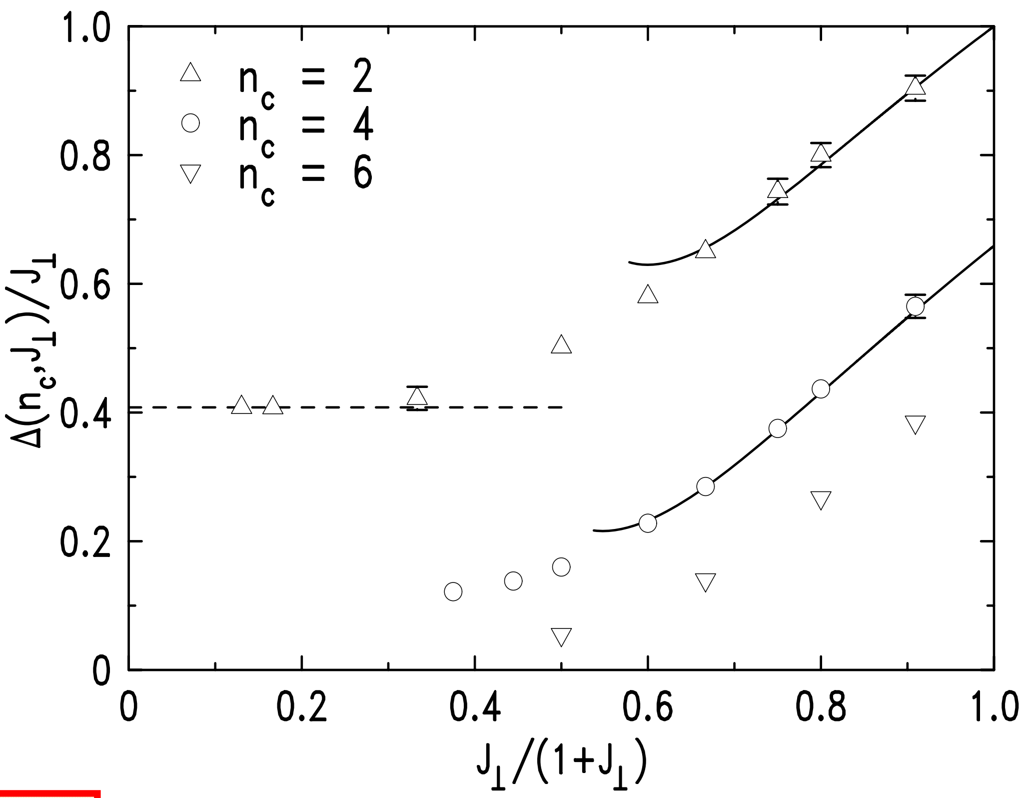
- 2-leg ladder & strong coupling:

$$\Delta \sim J_\perp - J_{||}$$

- 2-leg ladder & weak coupling:

$$\Delta \sim 0.41 J_\perp$$

QMC determination of the spin gap

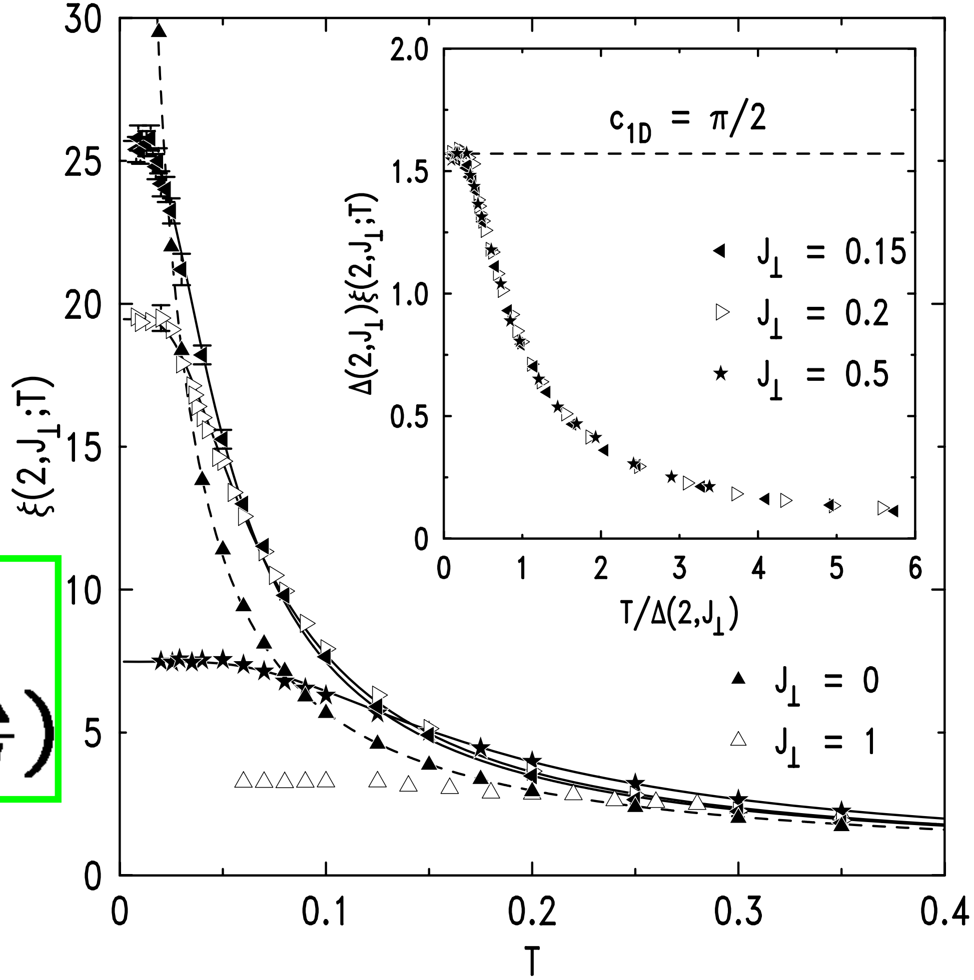


Greven et al., PRL 96

Spin correlation length vs T
Scaling behavior

Weak coupling result

$$\Delta(J_\perp)\xi(J_\perp,T) = c_{10} + \frac{T}{\Delta}\exp\left(-\frac{\Delta}{T}\right)$$



Greven et al. (QMC)

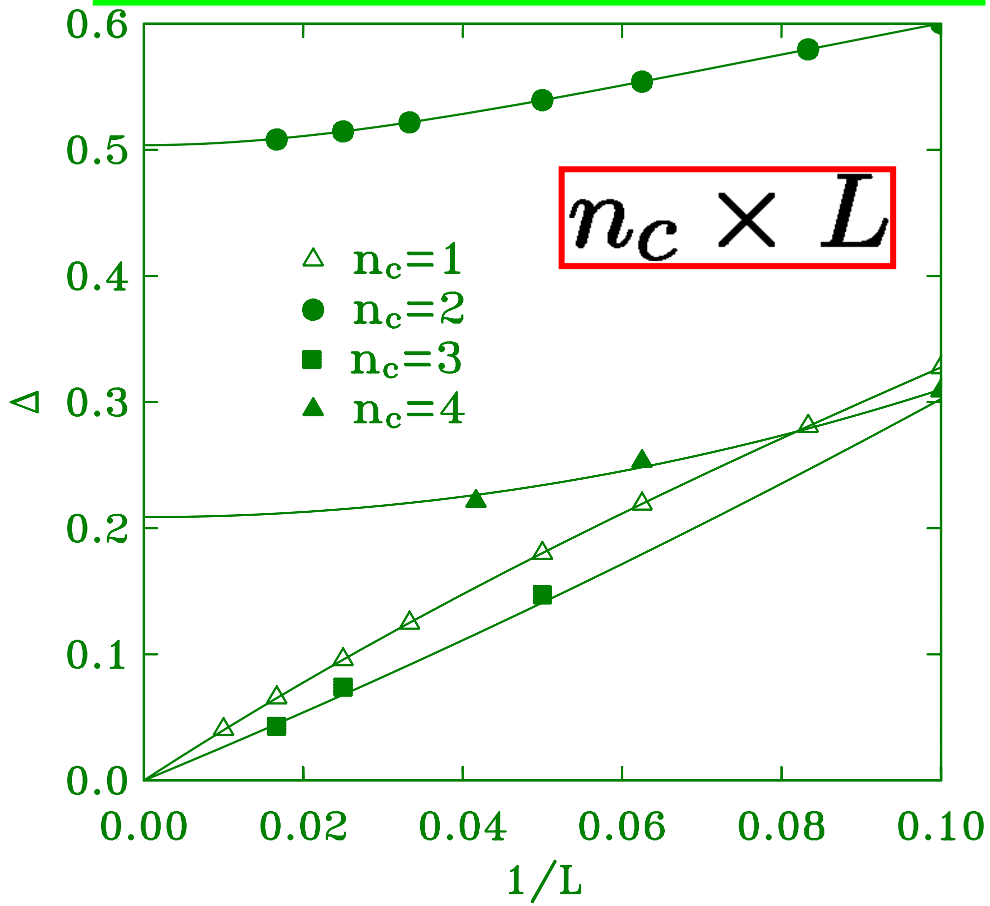
Even-odd effects !!

- GS (& low-T low-energy) properties depend on the # of legs

- For nc even: finite gap

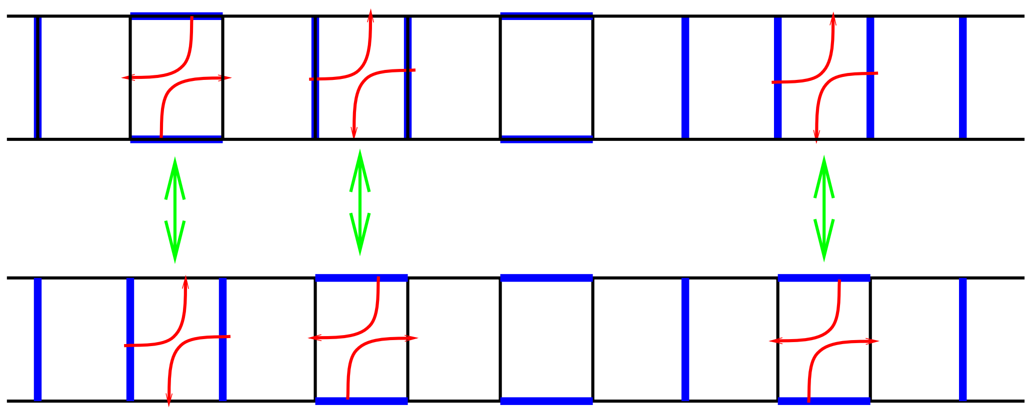
- For nc odd: gapless

Scaling of spin gap (DMRG)



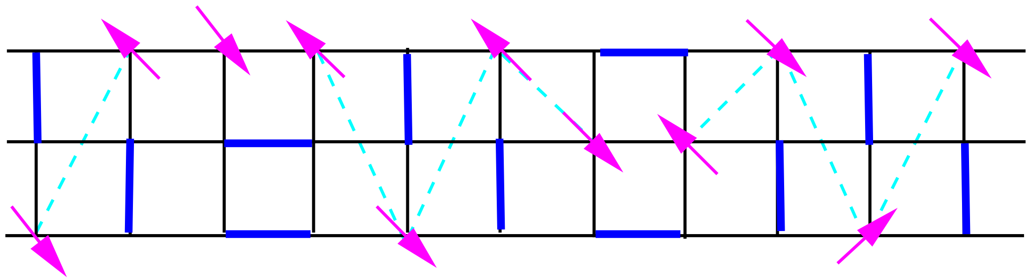
White et al. (1994)

Simple qualitative argument



(a)

“spin pairing”



(b)

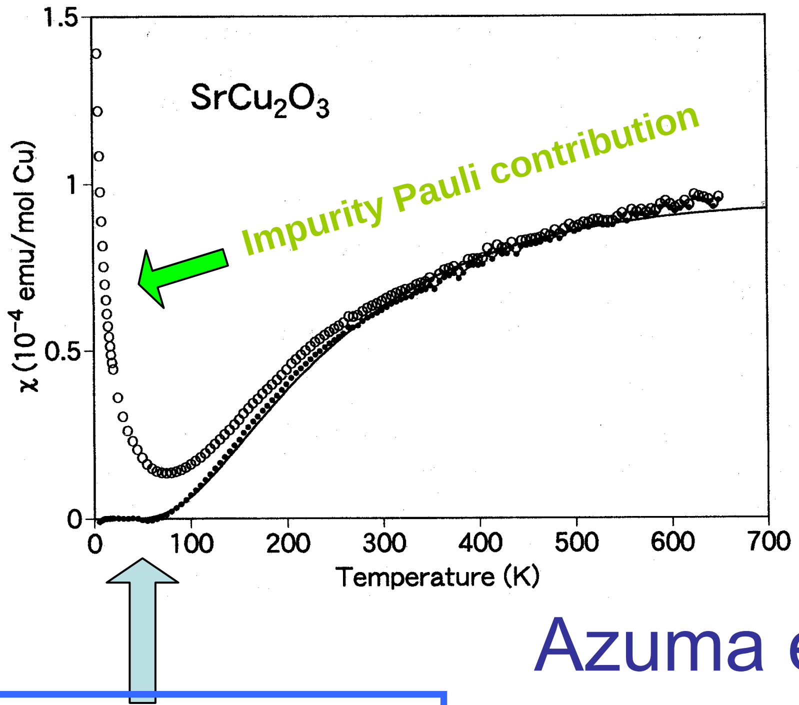
Similar to 1D chain

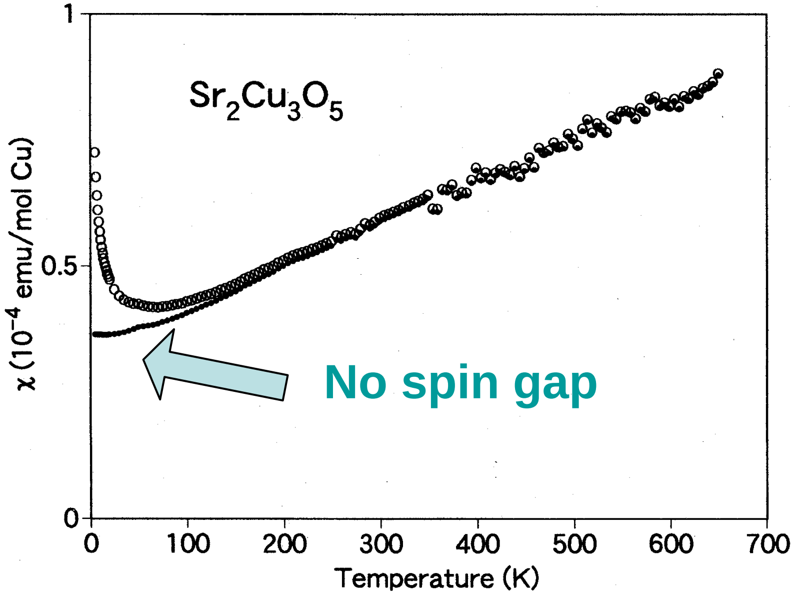
Spin gap revealed by experiments

Susceptibility: 2-leg vs 3-leg ladders

$$\chi(T) \sim \frac{1}{\sqrt{T}} \exp\left(-\frac{\Delta}{T}\right)$$

Single magnon branch contribution (2-leg ladder)
Troyer et al. 96





Azuma et al., PRL 94

Spin gap signature

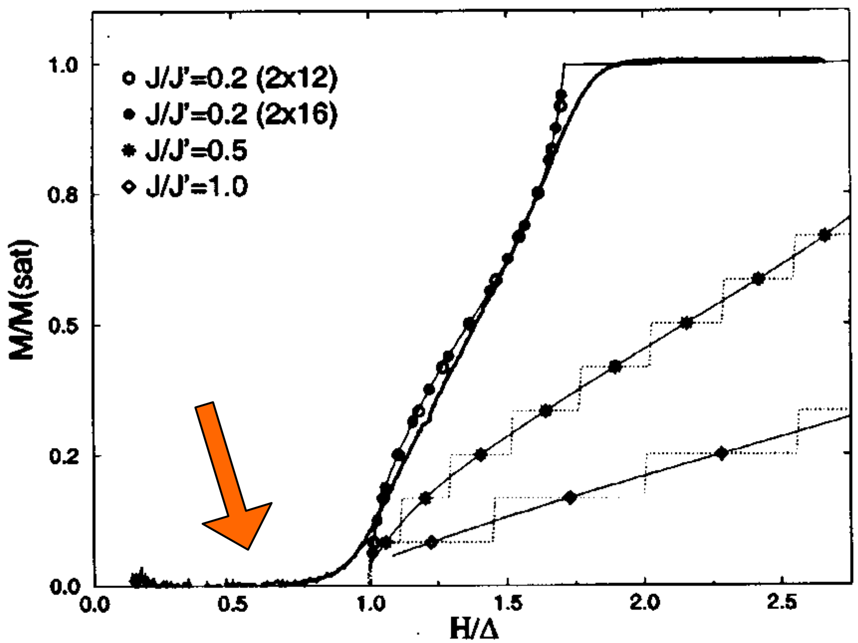
Thermodynamic properties

Exemple of Cu(HpN)Cl (2-leg)

<u>Magnetization curve</u>

Comparison with Lanczos

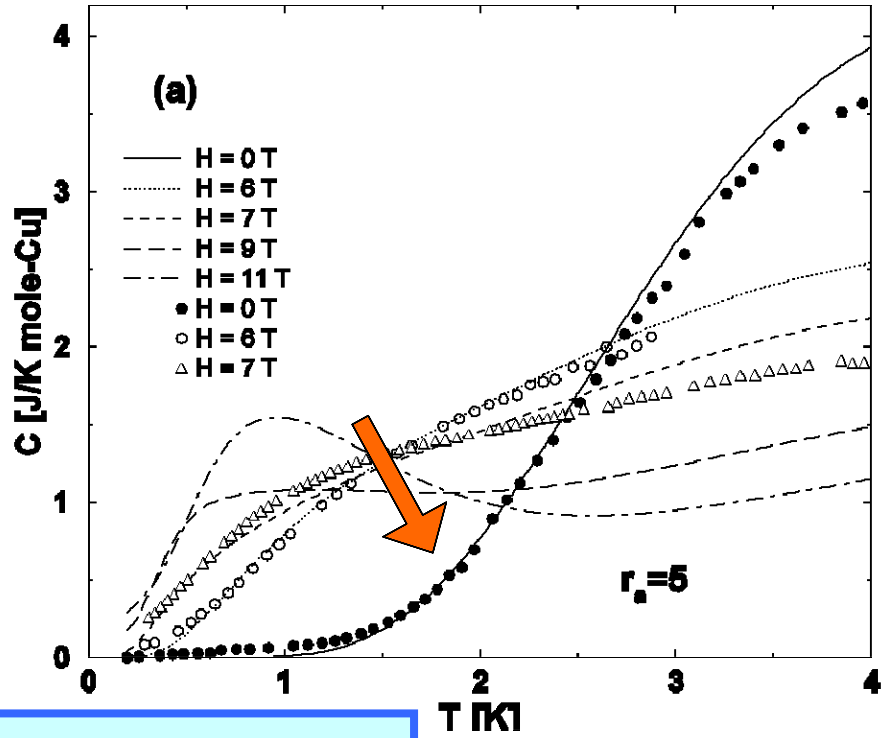
Hayward et al., 1996



<u>Specific heat vs T</u>

Comparison with full diagonalization

Calemczuk et al., 1998



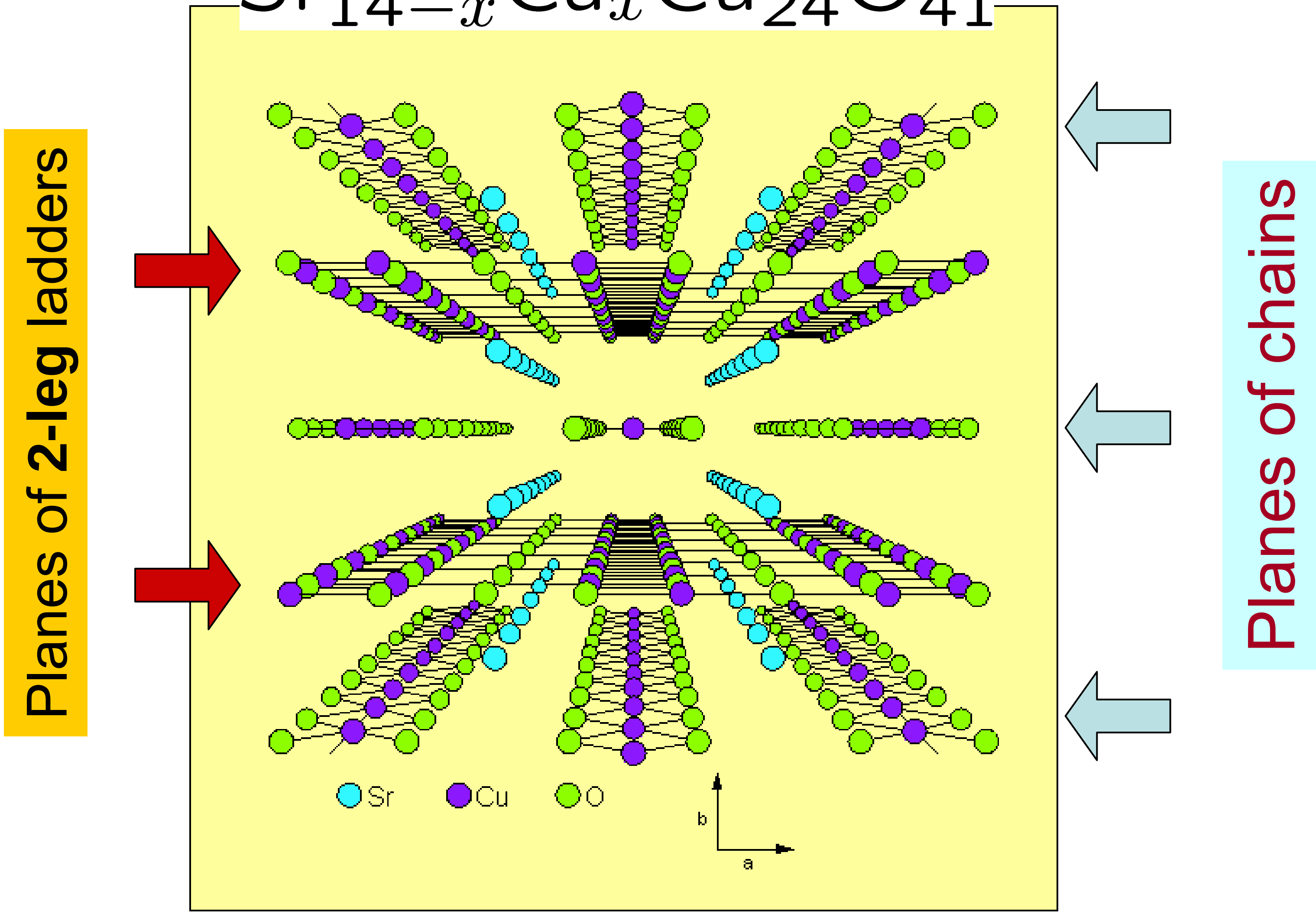
Clear signatures of spin gap !

Doping spin ladders

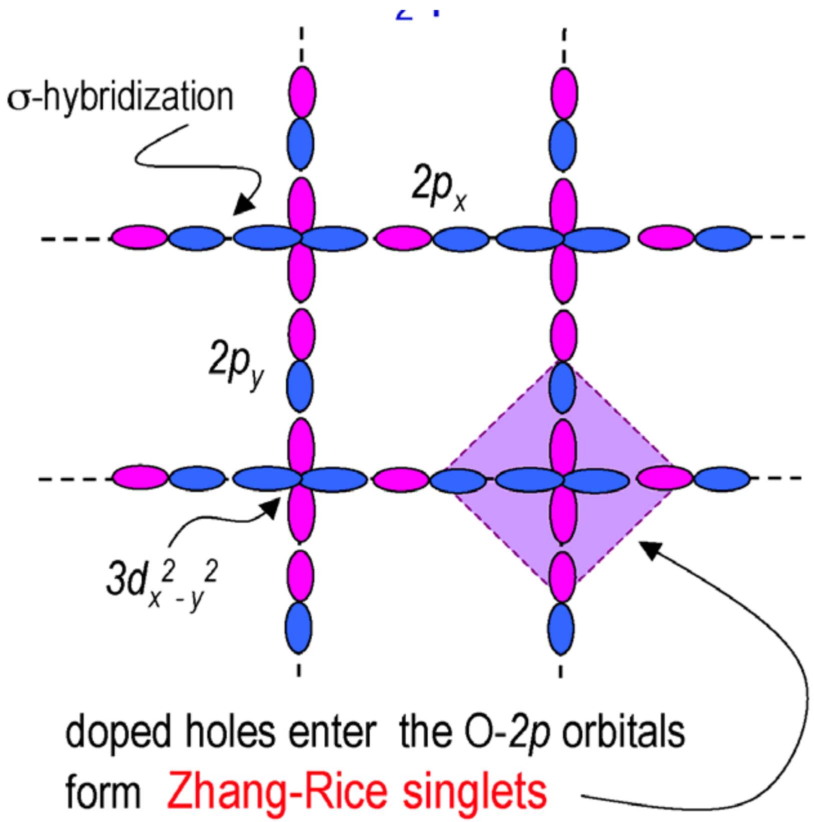
Towards a superconducting state?

The "telephone number" compound: A superconducting ladder

$Sr_{14-x}Ca_xCu_{24}O_{41}$



Doping the ladders: Zhang-Rice singlets



Basic model for lightly doped system:

t-J Hamiltonian

(or large-U Hubbard model)

$$H = -t \sum_{\langle i,j\rangle,s} \left\{ c_{is}^{+} \left(1 - n_{i,-s}\right)\left(1 - n_{j,-s}\right) c_{js} + hc. \right\} + J \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j$$

Conformal Field theory

Prediction for long wavelength (low-energy) physics

- **CnSm** phases (Balents & Fisher)

  - n =0,1 or 2 **charge** modes
  - m=0,1 or 2 **spin** modes

- Insulating (Mott) spin gap phase: C0S0 phase

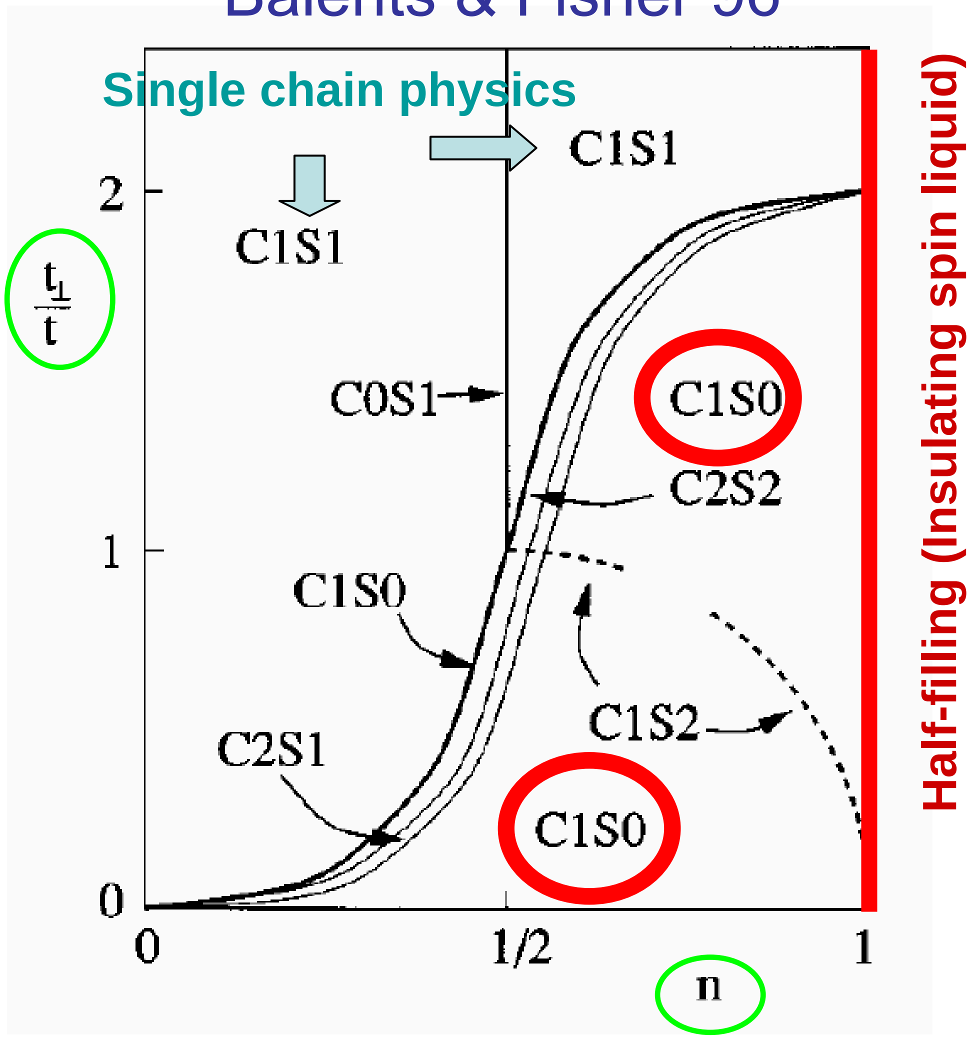
- If spin gap robust under doping: **C1S0** phase:

  - Spin correlations: $\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \sim \exp\left(-r_{ij}/\xi\right)$
  - Charge correlations: $\sim (1/r_{ij})^{2K_\rho}$
  - Superconducting correlations: $\sim (1/r_{ij})^{1/2K_\rho}$

Weak coupling approach

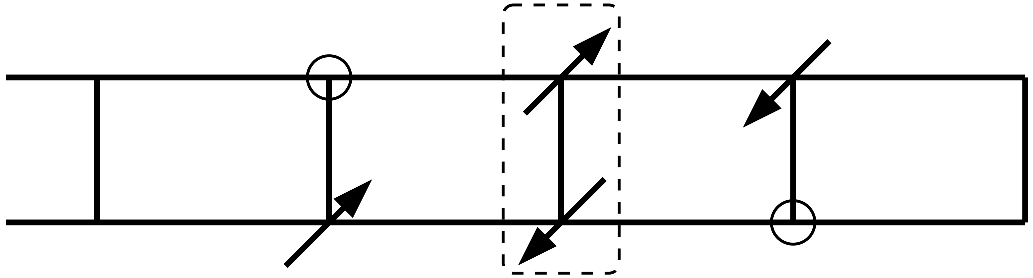
- Anisotropic Hubbard ladder & small U
- Weak coupling RG: C1S0 phase stable

Balents & Fisher 96

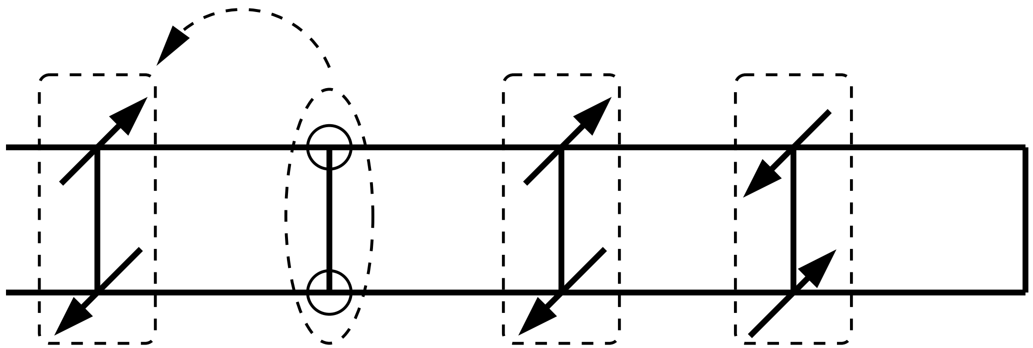


Strong coupling argument

Separated holes
→ unpaired spins



Rung hole pair
Spin gap survises

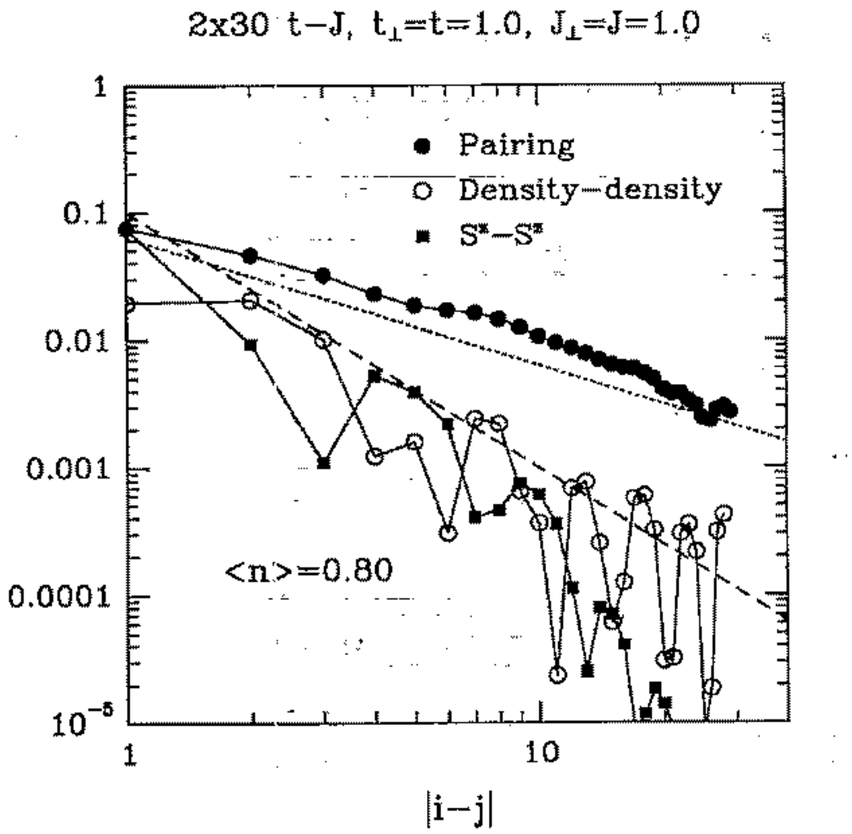


Balance between kinetic & magnetic energies
→ at large rung coupling: $\Delta_{\mathrm{binding}} \sim J_{\perp}$

Numerical results (t-J)

Hayward PRL 95

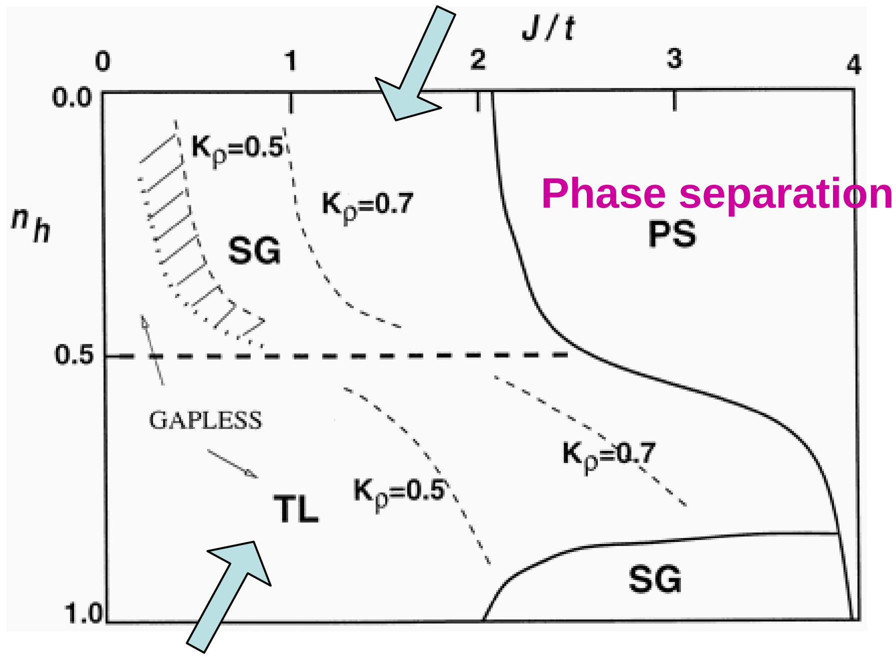
Correlations vs distance (DMRG)



Dominant pairing correlations

Phase diagram (ED)

C1S0 superconductor



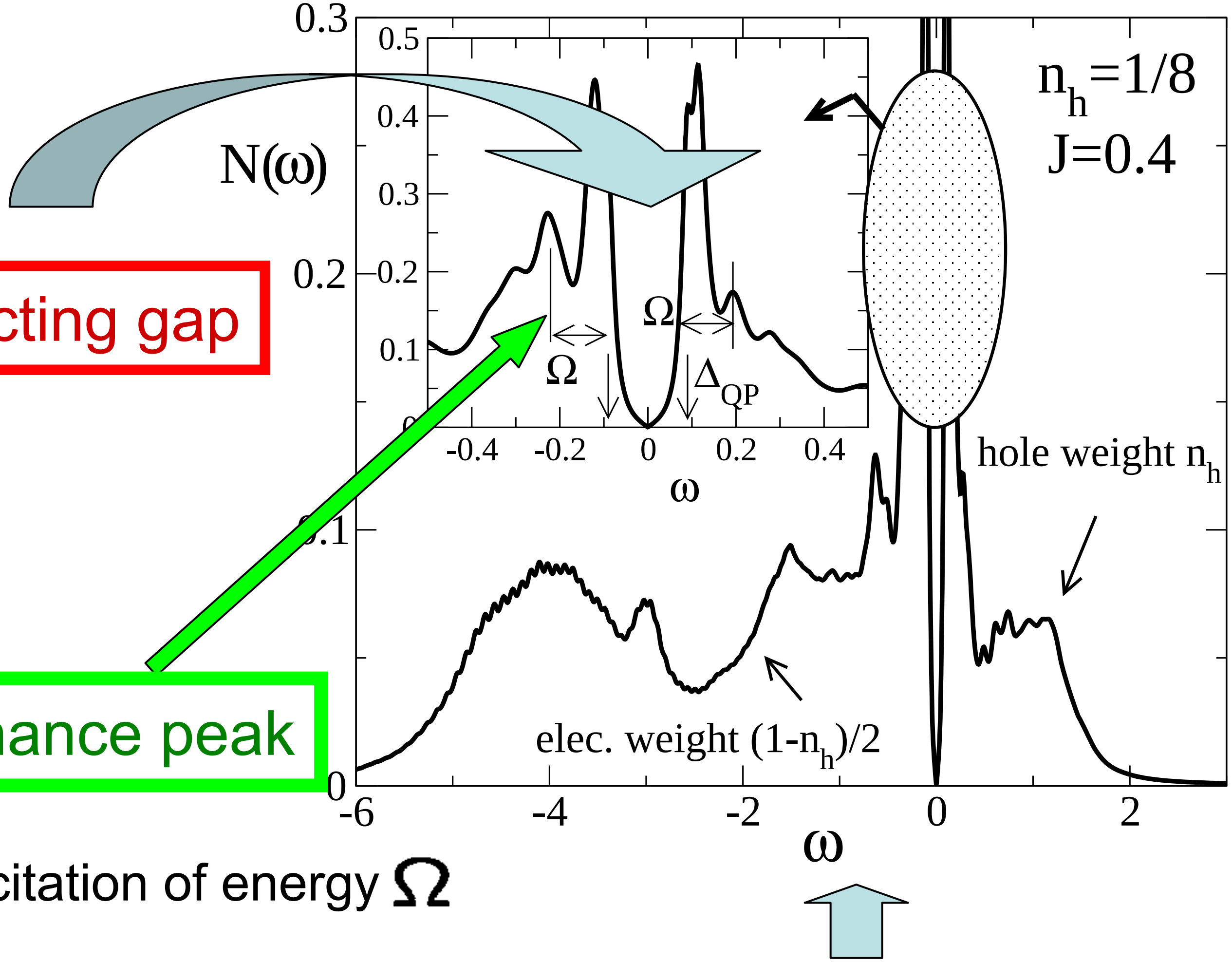
Phase separation

C1S1

Tunneling density of states

Poilblanc et al. 2004
(Lanczos)

Superconducting gap



Magnon resonance peak

Low energy triplet excitation of energy Ω

Lower Hubbard band

Getting more insight into the nature of the (spin-fluctuation driven) pairing interaction ?

Unconventional d-wave pairing

"Order parameter": $F_{ij} = \langle \Delta_{ij}^{\dagger} \rangle$ with pair creation operator $\Delta_{ij}^{\dagger} = c_i^{\dagger} c_j^{\dagger}$

Numerics: pairing correlations $\langle \Delta_{ij}^{\dagger} \Delta_{kl} \rangle$

$F_{\text{rung}} = F_{i,1;i,2}$ and $F_{\text{leg}} = F_{i,a;i+1,a}$

have opposite signs

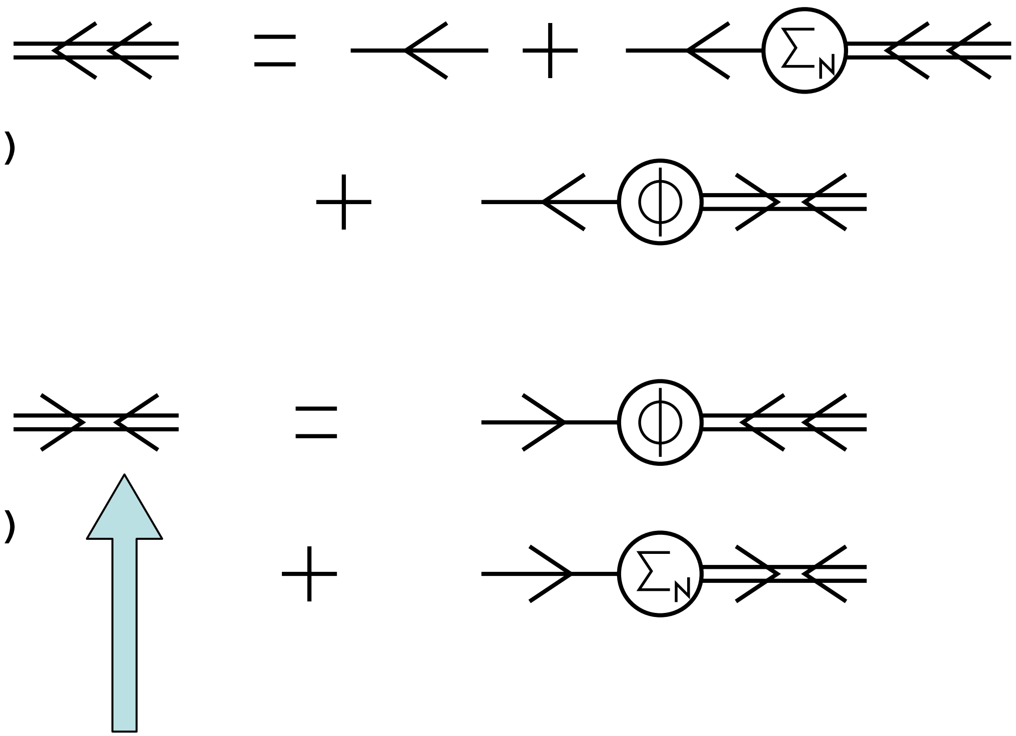
↓

Typical of d-wave

Solving Dyson equations numerically ?

(a)

(b)



Superconducting Green function

$$F(\mathbf{k},\omega) = u_{\mathbf{k}} v_{\mathbf{k}} \delta(\omega - E_{\mathbf{k}}) \ \text{(BCS)}$$

- Compute G & F Green functions
- Invert Dyson's equation to extract the pairing function:

$$\phi(\mathbf{k},\omega)$$

Superconducting Green function (Lanczos)

Generalize calculation of dynamical correlation to **grand-canonical ensemble**

$$F(q, q_y(= 0, \pi), \omega)$$

How to compute F ??

$\Rightarrow$ off-diagonal correlations !!

$F(\mathbf{q},\omega) = \tilde{F}_{\mathbf{q}}(\omega + i\epsilon) + \tilde{F}_{\mathbf{q}}(-\omega + i\epsilon)$

$\tilde{F}_{\mathbf{q}}(z) = \langle N - 2| c_{-\mathbf{q},-\sigma} \frac{1}{z - H + E_{N-1}} c_{\mathbf{q},\sigma} |N\rangle$

$\rightarrow$ new type of correlation:

$$\tilde{C}(z) = \langle \Psi_1 | \frac{1}{z' - H} | \Psi_2 \rangle$$

where $|\Psi_1\rangle = c_{\mathbf{q},\sigma}|N\rangle$ & $|\Psi_2\rangle = c^{\dagger}_{-\mathbf{q},\sigma}|N-2\rangle$

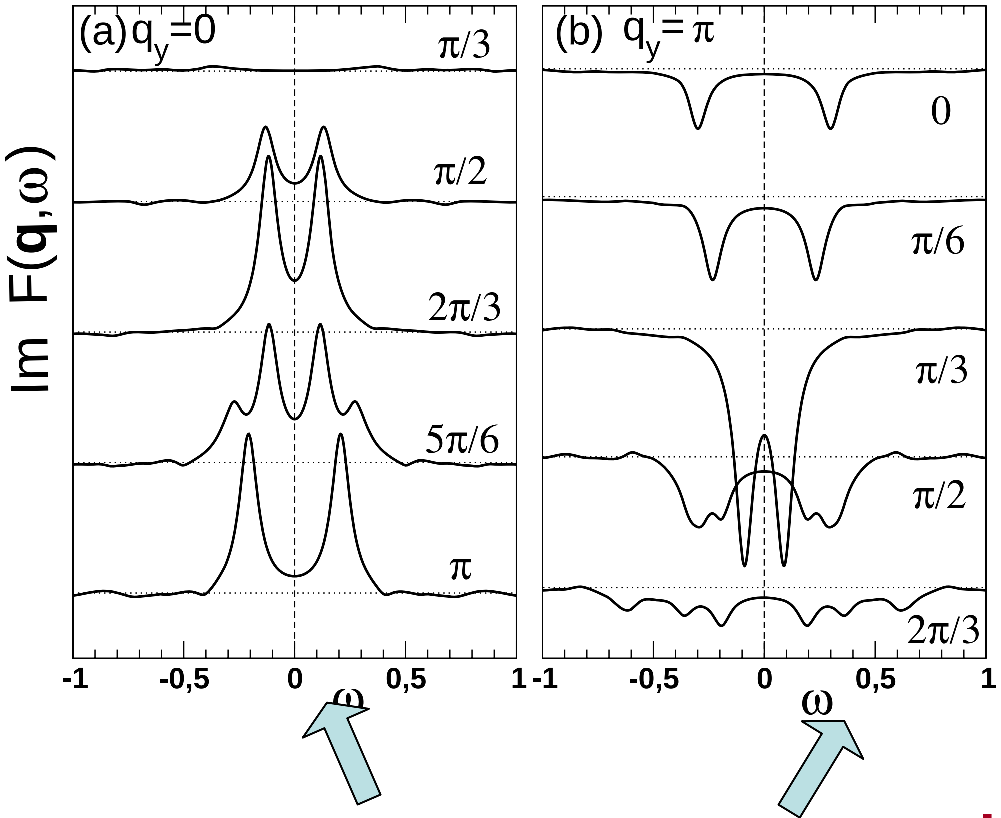
$\rightarrow$ Compute diagonal correlators using $|\Psi_1\rangle$, $|\Psi_2\rangle$

& $\frac{1}{\sqrt{2}}(|\Psi_1\rangle \pm i|\Psi_2\rangle)$!!

$\rightarrow$ express F in terms of G_{10}, G_{01}, $G_{1,i}$ & $G_{1,-i}$

Dynamics of d-wave pairing

Poilblanc, Scalapino, Capponi, PRL 2003



$$\mathrm{Im} F(q, q_y = 0, \omega)$$

and

$$\mathrm{Im} F(q, q_y = \pi, \omega)$$

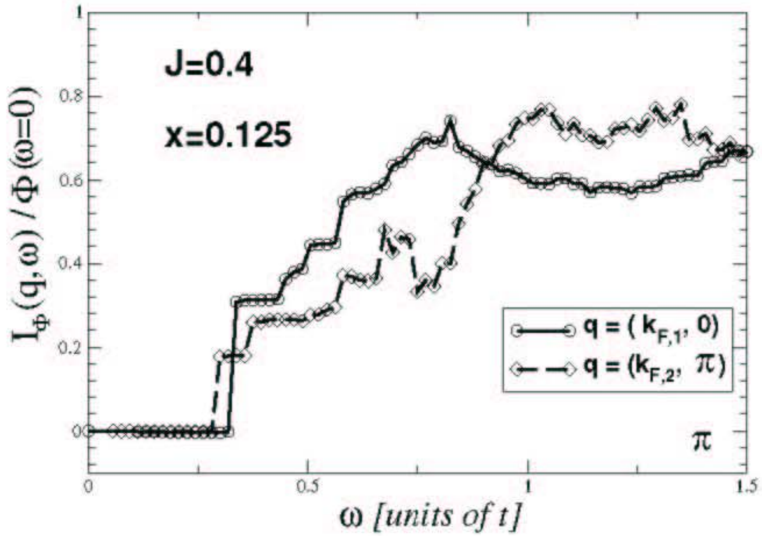
<u>opposite signs !</u>

Deviations from BCS
due to **retardation**

Change of sign !

=> **d-wave symmetry**

Retardation in SC pairing $\Phi(\mathbf{k},\omega)$



Pairing function

$$\Phi(\mathbf{k},\omega) = Z(\mathbf{k},\omega)\Delta(\mathbf{k},\omega)$$

Dispersion relation

$$\Downarrow$$

$$\Phi(\mathbf{k},0) = \Phi_{\rm stat} + 2\int_0^{\omega=\infty} d\omega' \, {\rm Im}\Phi(\mathbf{k},\omega')/\omega'$$

Integrated ${\rm Im}\Phi$ vs energy cutoff ω (w.r.t. total integral)

Summary/conclusion

- Ladders exhibit fascinating behaviors observable experimentally (except few experiments on doped ladders, yet)
- Simple system to investigate non-conventional superconductivity
- Not addressed here: role of disorder, magnetic field induced QCP, role of spin anisotropy, …