Local quantum criticality in confined fermions on optical lattices

Marcos Rigol

Institut für Theoretische Physik III

Universität Stuttgart

email: mrigol@theo3.physik.uni-stuttgart.de

* Local Quantities and the Phase Diagram
* Local Order Parameter
    · Local Quantum Criticality and Universality
* Momentum Distribution Function
* Conclusions

Collaborators

A. Muramatsu (Universität Stuttgart)

G. G. Batrouni (INLN, Nice)

R. T. Scalettar (UC Davis)

Interacting Bosons Confined in Optical Lattices

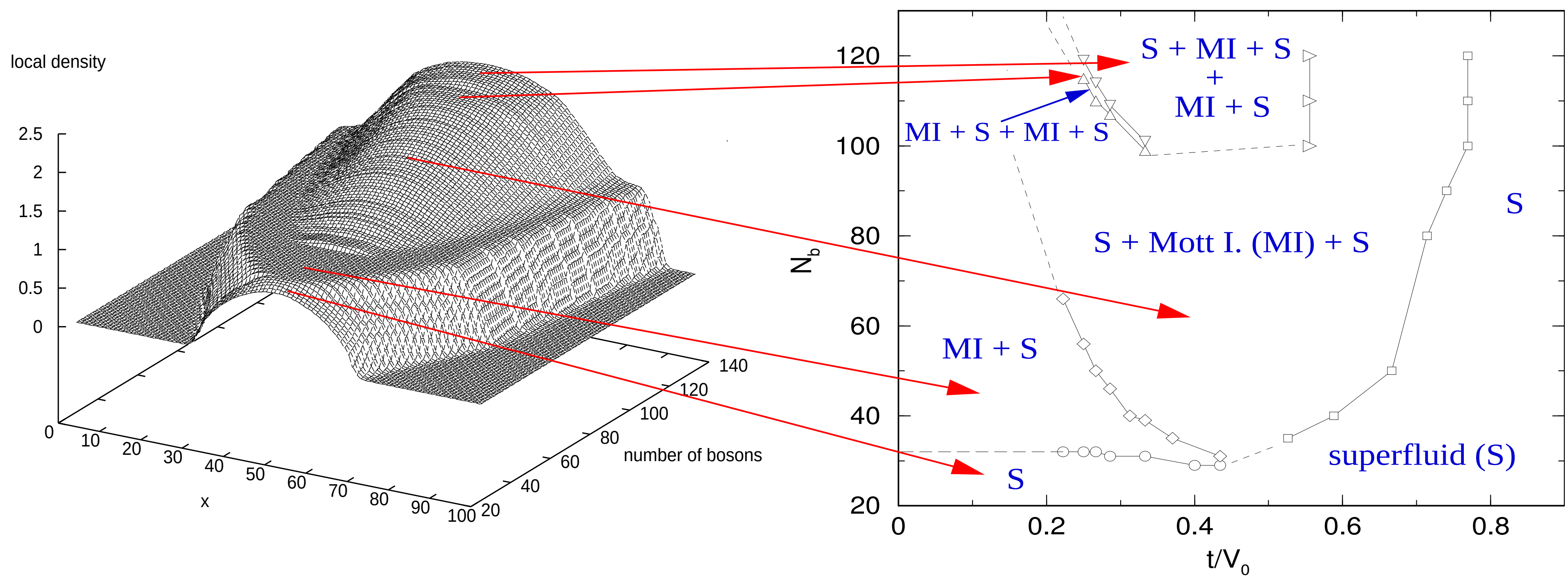
G. G. Batrouni, V. Rousseau, R. T. Scalettar, M. Rigol, A. Muramatsu, P. J. H. Denteneer, and M. Troyer, Phys. Rev. Lett. **89**, 117203 (2002)

Hubbard Hamiltonian

$$H = -t\sum_{\mathbf{i}}(a^\dagger_{\mathbf{i}}a_{\mathbf{i}+1} + a^\dagger_{\mathbf{i}+1}a_{\mathbf{i}}) + V_0\sum_{\mathbf{i}} n_{\mathbf{i}}(n_{\mathbf{i}}-1) + V_c\sum_{\mathbf{i}}(i-N/2)^2 n_{\mathbf{i}}$$

Method: World Line Quantum Monte Carlo

Local Density and Phase Diagram



Interacting Fermions Confined in Optical Lattices

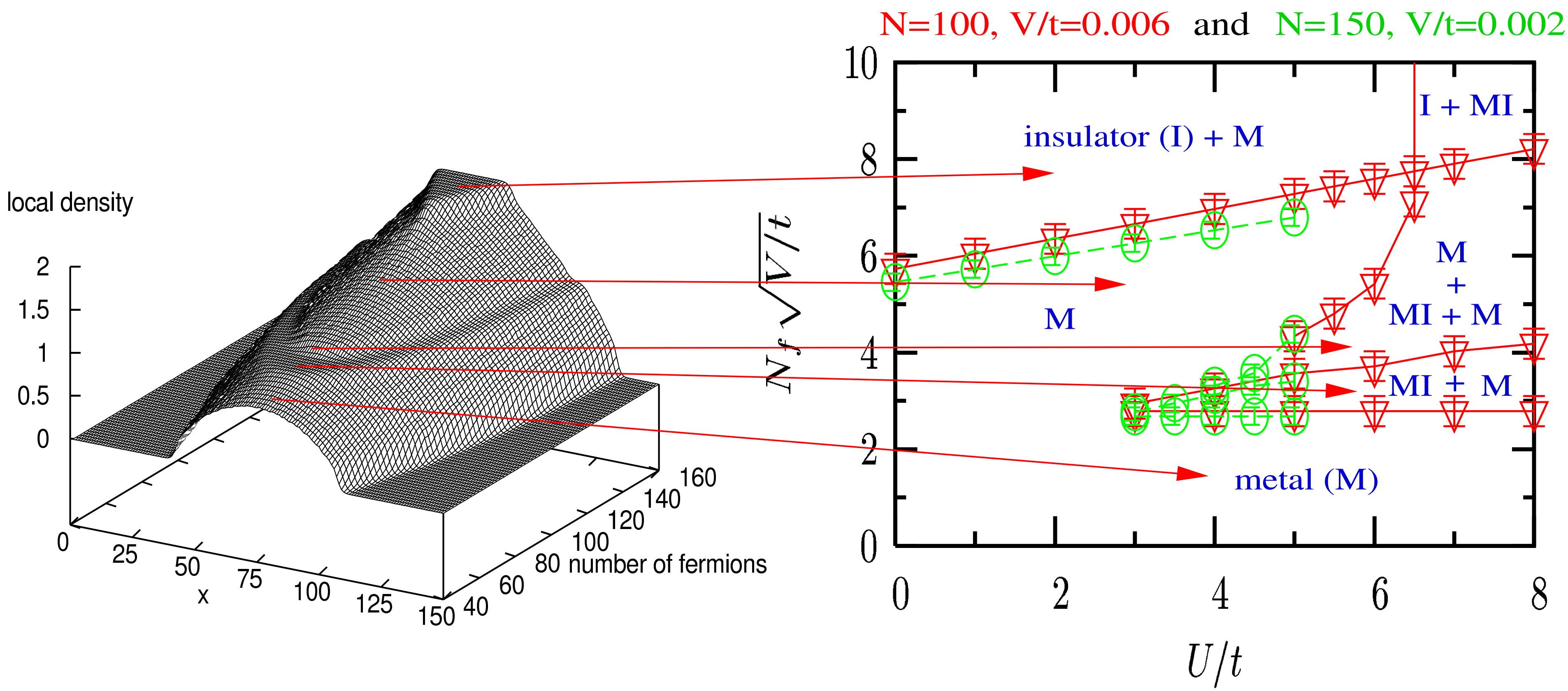
M. Rigol, A. Muramatsu, G. G. Batrouni, and R. T. Scalettar, Phys. Rev. Lett. **91**, 130403 (2003).

Hubbard Hamiltonian

$$H = -t \sum_{i\sigma} \left(c^\dagger_{i\sigma} c_{i+1\sigma} + c^\dagger_{i+1\sigma} c_{i\sigma} \right) + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) + V \sum_i (i - N/2)^2 \, n_i$$

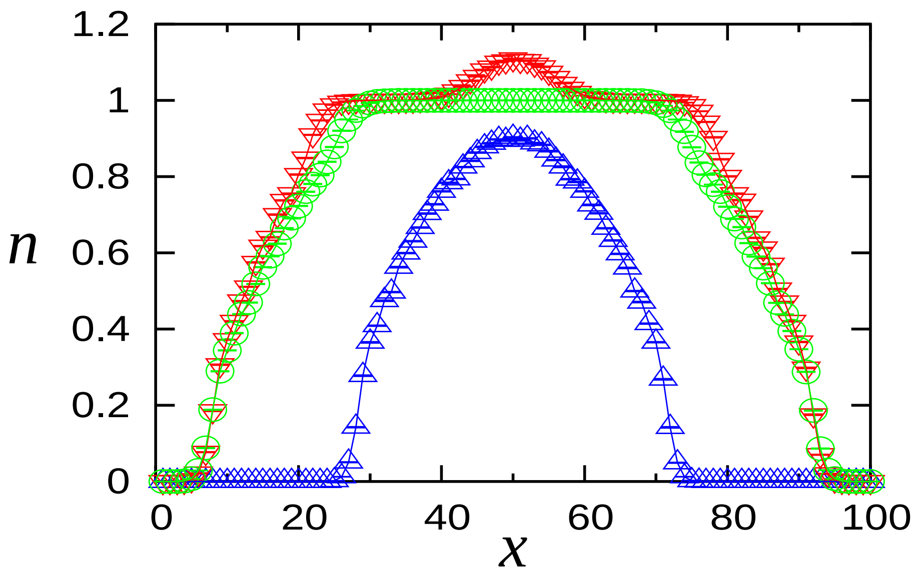
Method: Determinantal Quantum Monte Carlo

Local Density and Generic Phase Diagram

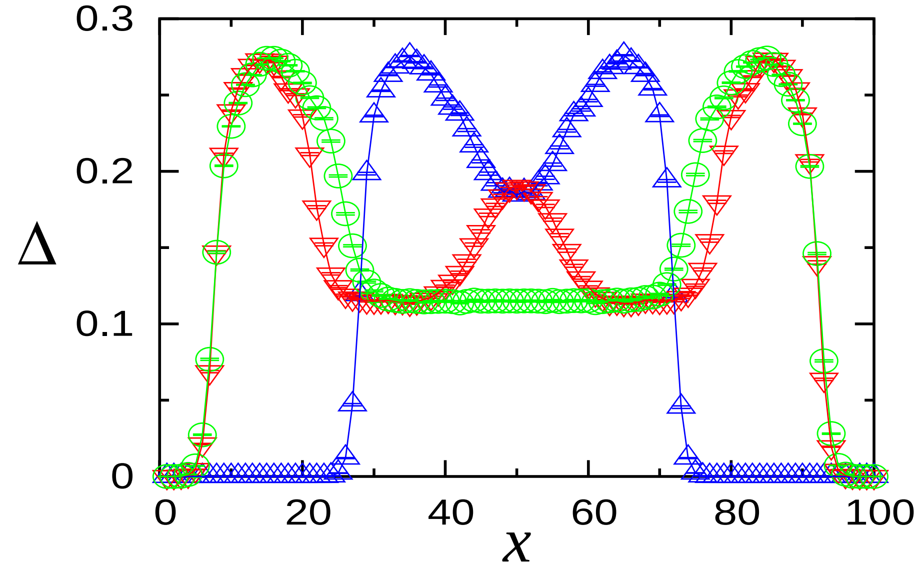


Density, Variance and Compressibility profiles

Density



Variance

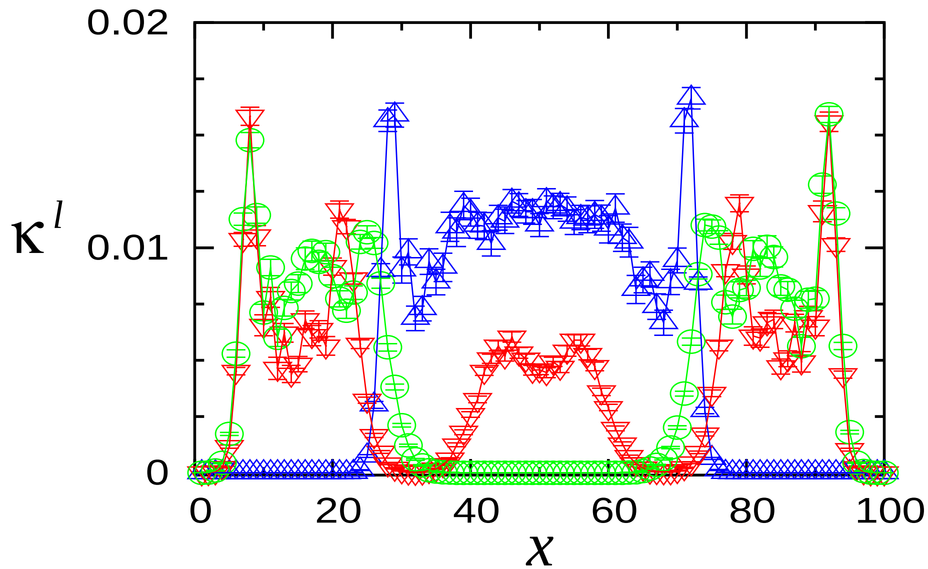


Local Order Parameter: Local Compressibility

$$\kappa_i^l = \sum_{|j| \leq l(U)} \chi_{i,i+j}$$

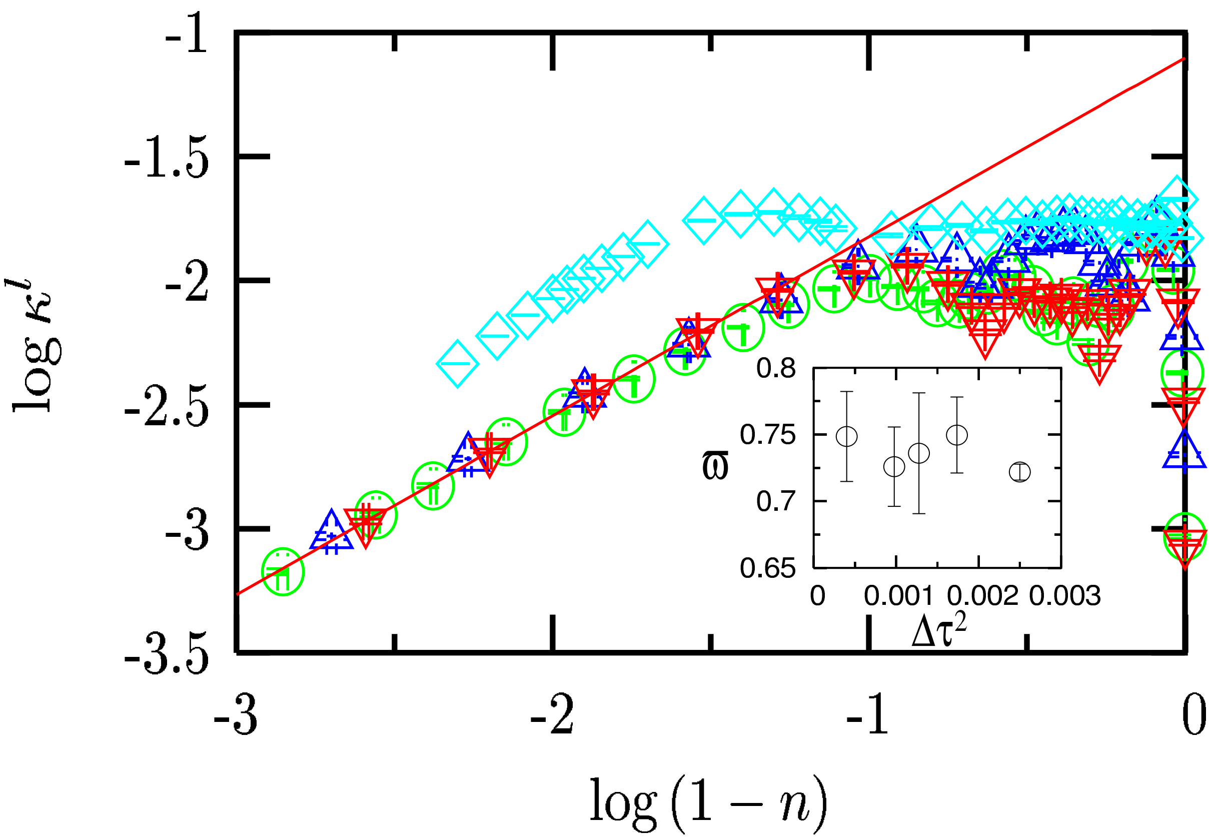
* $\chi_{i,j} = \langle \hat{n}_i \hat{n}_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$

* $l(U) \sim 10\, \xi(U)$
  $\xi(U)$ correlation length of $\chi_{i,j}$ in the homogeneous system with $n = 1$.



Criticality and Universality of the Local Compressibility

Local Compressibility vs Density



Parameters

$\bigtriangledown$: $U = 6t$ + **harmonic**

$\bigtriangleup$: $U = 8t$ + **harmonic**

$\bigcirc$: $U = 6t$ + **quartic**

$\diamond$: $U = 6t$ + **periodic lattice**

Compressibility

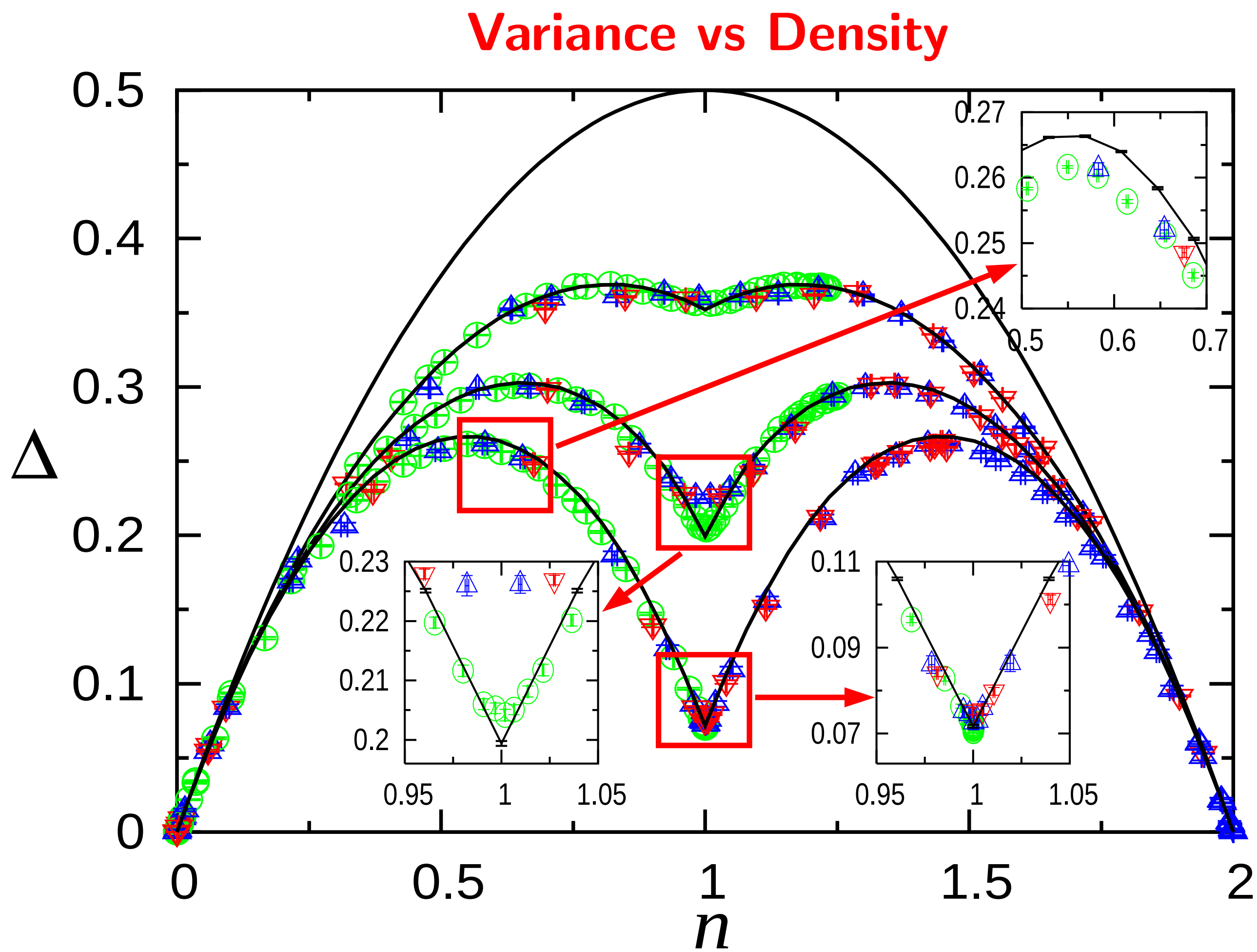
$$\kappa^l \sim (1-n)^{\varpi}$$

Critical exponent ϖ

$$\varpi \sim 0.68 - 0.78$$

Universality: No dependence on confining potential and on-site repulsion for $n \to 1$

Universality of the Variance



Confining potentials

Line: periodic lattice

○: harmonic

△: quartic

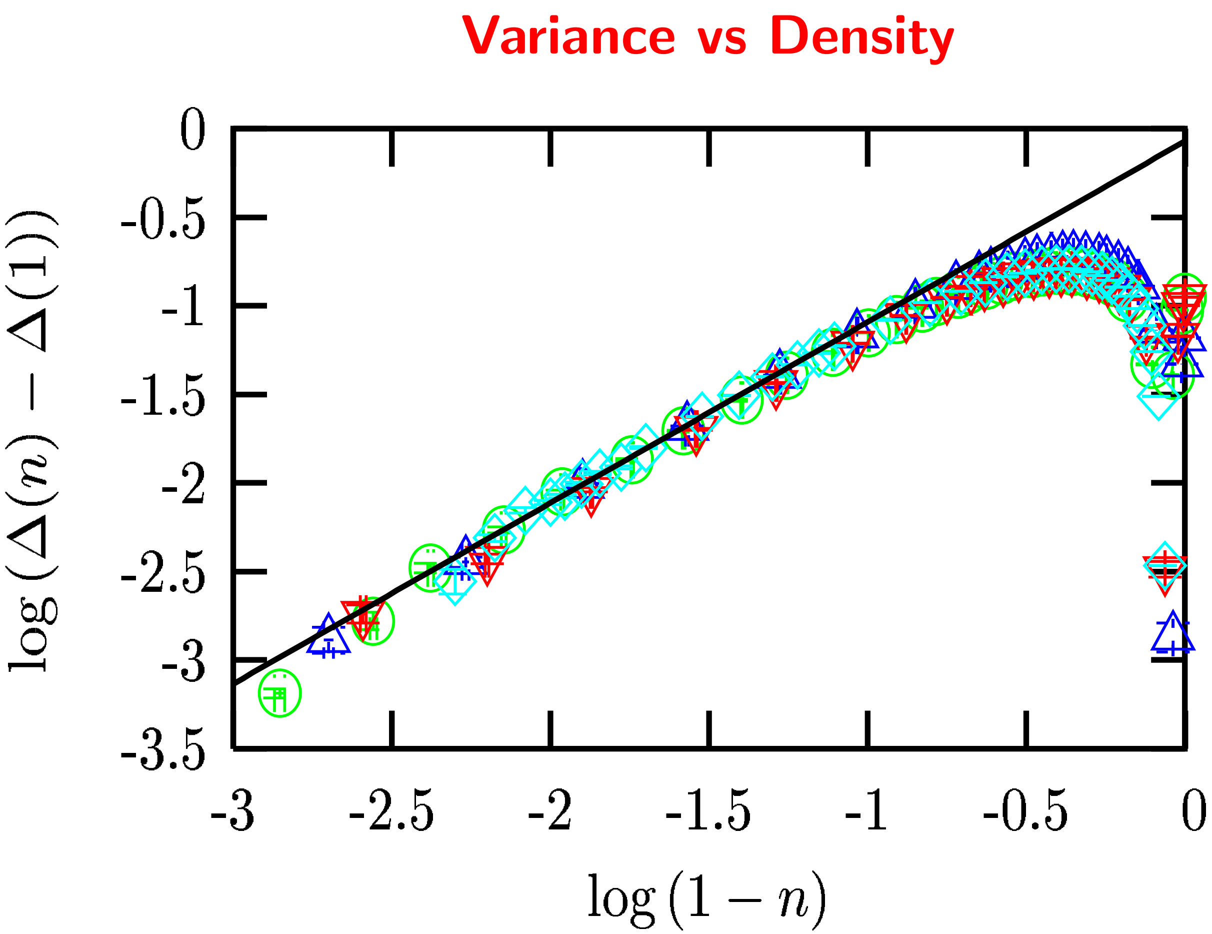
▽: harmonic+cubic+quartic

On site repulsions: (top to bottom)

$U/t = 0.0,\ 2.0,\ 4.0,\ 8.0$

Universality: No dependence on confining potential for $n \to 1$

Universality of the Variance



Parameters

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$\bigcirc$: $U = 6t$ + **quartic**

$\diamond$: $U = 6t$ + **periodic lattice**

Variance

$$\Delta \sim (1-n)^{\varrho}$$

Exponent ϱ

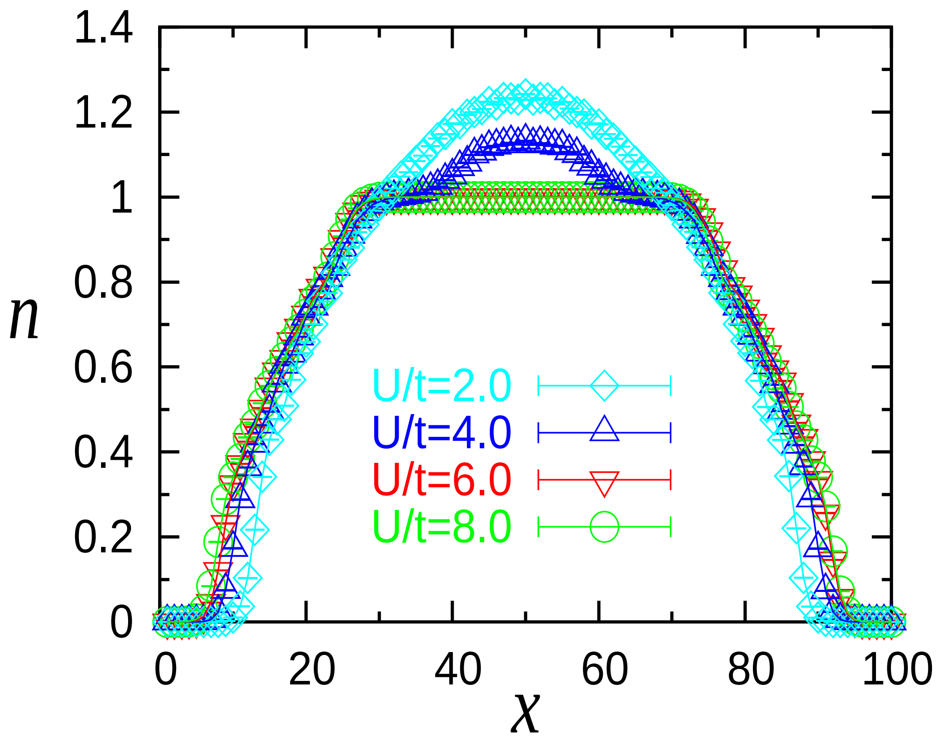
$$\varrho = 1$$

Universality: No dependence on confining potential and on-site repulsion for $n \to 1$

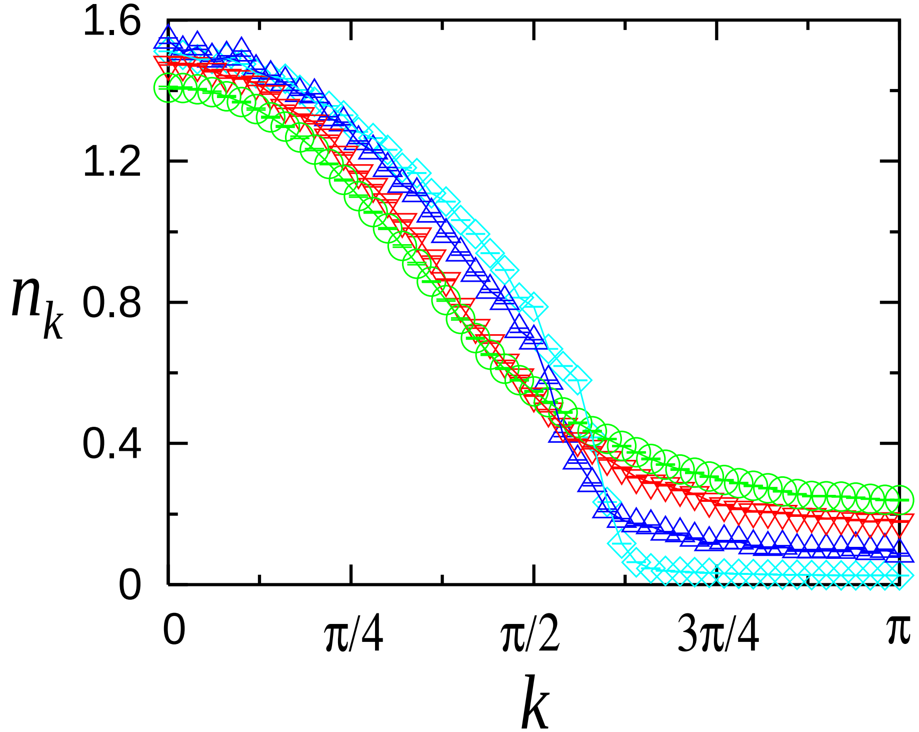
Evolution of the Density and Momentum profiles with U

M. Rigol and A. Muramatsu, cond-mat/0309670

Density



Momentum

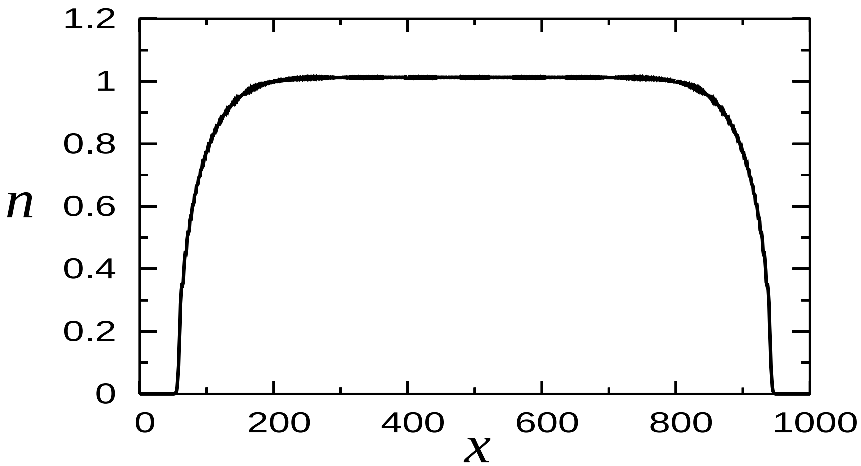


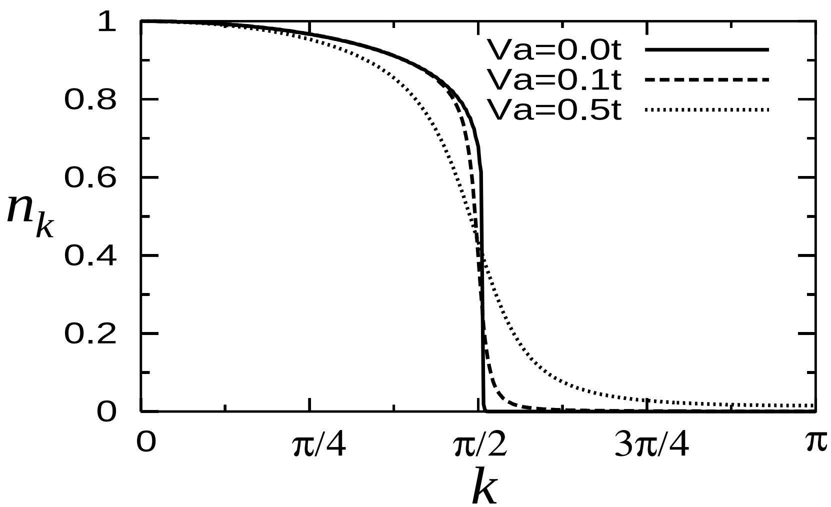
Non-interacting case

Hamiltonian

$$H = -t\sum_{i,\sigma}\left(c^{\dagger}_{i\sigma}c_{i+1\sigma} + h.c.\right) + V_{10}\sum_{i\sigma}\left(i - \frac{N}{2}\right)^{10} n_{i\sigma} + V_a\sum_{i\sigma}(-1)^i n_{i\sigma}$$

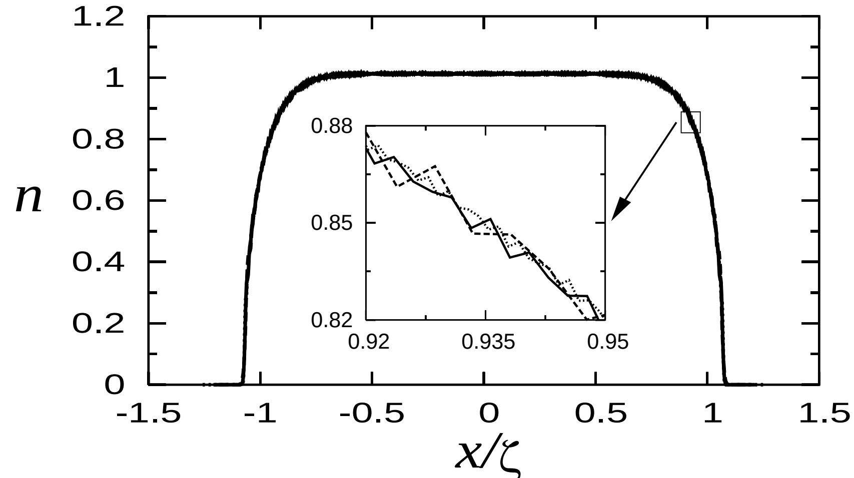
Local Insulator

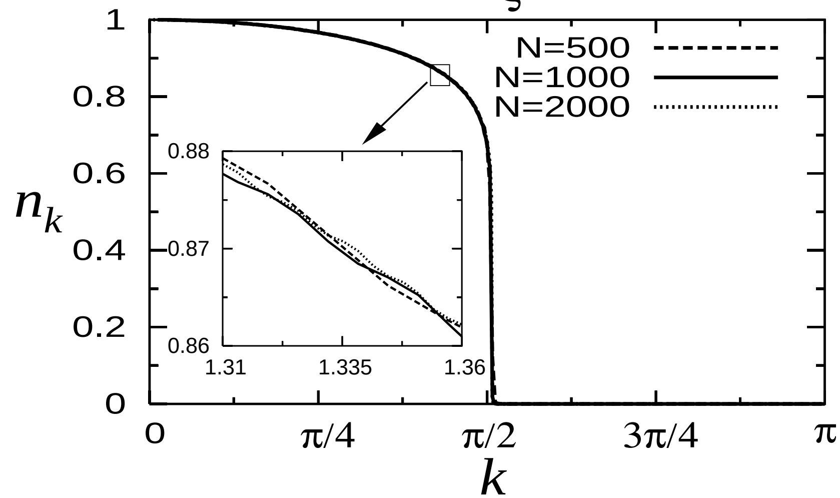




Scaling

$\zeta = (V_\alpha/t)^{-1/\alpha}$ and $\tilde{\rho} = N_f/\zeta$ constant.





Conclusions

i) Local compressibility shows quantum critical behavior for $n \to 1$.

- Critical exponent: $\varpi \sim 0.68 - 0.78$.

- Independent of spatial correlations.

- To be expected in higher dimensions.

ii) Local compressibility and variance show universal behavior for $n \to 1$.

iii) Generic phase diagram.

- Scaling form: $N_f\sqrt{V/t}$ vs U/t.

iv) Momentum distribution function, not appropriate to characterize the MMIT.