V. Strong Coupling (Eliashberg) Equations for HTSC

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- Self-Energy in SC state

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- Equations for $G_N(\mathbf{k},\omega)$ and $\Delta(\mathbf{k},\omega)$

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- s- versus d-wave Pairing in HTSC

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- EPI beyond Migdal Approximation

↓

- EPI vs Spin Fluctuations

↓

- Effects of Impurities

↓

- ARPES spectra (see **M. Kulic**, **O. Dolgov**, cond-mat/0308597 v1 Aug 2003)

SELF-ENERGY IN SC STATE

↓

- Gor'kov-Nambu Formalism
- Electron ($\hat{\psi}_{\uparrow}(\mathbf{r})$) and hole ($\hat{\psi}^{\dagger}_{\downarrow}(\mathbf{r})$) states
- Pseudospin Pauli matrices $\{\tau_i,\ i = 0,1,2,3\}$ in the electron-hole space

↓

$$\Psi(\mathbf{r}) = \left(\begin{array}{c} \psi_{\uparrow}(\mathbf{r}) \\ \psi^{\dagger}_{\downarrow}(\mathbf{r}) \end{array} \right),$$

$$\Psi^{\dagger}(\mathbf{r}) = (\psi^{\dagger}_{\uparrow}(\mathbf{r})\ \psi_{\downarrow}(\mathbf{r})). \qquad (1)$$

↓

$$H = H_e + H_i + H_{el}. \qquad (2)$$

↓

↓

$$H_e = \int d^3r \Psi^{\dagger}(\mathbf{r}) \tau_3 \epsilon_0(\hat{p}) \Psi(\mathbf{r}) +$$

$$+ \frac{1}{2} \int d^3r d^3r' \Psi^{\dagger}(\mathbf{r}) \tau_3 \Psi(\mathbf{r}) V_c(\mathbf{r} - \mathbf{r}') \Psi^{\dagger}(\mathbf{r}') \tau_3 \Psi(\mathbf{r}'),$$

(3)

↓

$$H_{el} = \sum_n \int d^3r V_{el}(\mathbf{r} - \mathbf{R}_n^0)\Psi^\dagger(\mathbf{r})\tau_3\Psi(\mathbf{r})$$

$$+ \int d^3r \Phi(\mathbf{r})\Psi^\dagger(\mathbf{r})\tau_3\Psi(\mathbf{r}). \tag{4}$$

↓

$$\Phi(\mathbf{r}) = -\sum_{n,\alpha} \hat{u}_{\alpha n}\nabla_\alpha V_{e-i}(\mathbf{r} - \mathbf{R}_n^0)$$

$$+ (1/2)\sum_{n,\alpha,\beta} \hat{u}_{\alpha n}\hat{u}_{\beta n}\nabla_\alpha\nabla_\beta V_{e-i}(\mathbf{r} - \mathbf{R}_n^0) + \Phi_{anh}(\mathbf{r}) \tag{5}$$

↓

Green’s functions

↓

$$G(1,2) = -\langle T\Psi(1)\Psi^\dagger(2)\rangle, \tag{6}$$

↓

$$\tilde{D}(1-2) = -\langle T\Phi(1)\Phi(2)\rangle, \tag{7}$$

↓

↓

- LOW-ENERGY PHYSICS (**LEGEP**)

↓

- Migdal theory

↓

- Similar theory as without SC

↓

- Coulomb self-energy

↓

$$\Sigma_c(1,2) = -V_c^{sc}(1,\bar{1})\tau_3 G(1,\bar{2})\Gamma_c(\bar{2},2;\bar{1}), \quad (8)$$

↓

$$V_c^{sc}(1,2) = V_c(1,\bar{2})\varepsilon_e^{-1}(\bar{2},2) \quad (9)$$

↓

- EPI self-energy (see **Fig**. **SC**)

↓

$$\Sigma_{ep}(1,2) = -V_{EP}(\bar{1},\bar{2})\Gamma_c(1,\bar{3};\bar{1})G(\bar{3},\bar{4})\Gamma_c(\bar{4},2;\bar{2}), \quad (10)$$

where

$$V_{ep}(1,2) = \varepsilon_e^{-1}(1,\bar{1})\tilde{D}(\bar{1},\bar{2})\varepsilon_e^{-1}(\bar{2},2) \quad (11)$$

$\downarrow$

$$\Gamma_c(1,2;3) = \tau_3\delta(1-2)\delta(1-3) + \frac{\delta\Sigma_c(1,2)}{\delta U_{eff}(3)} \tag{12}$$

$\downarrow$

- Matrix structure of G, Σ and Γ_c

$\downarrow$

$$G^{-1}(\mathbf{k},\omega_n) = i\omega_n Z(\mathbf{k},\omega_n)\tau_0 - [\epsilon(\mathbf{k}) + X(\mathbf{k},\omega_n)]\tau_3 - Z(\mathbf{k},\omega_n)\Delta(\mathbf{k},\omega_n)\tau_1 \tag{13}$$

$\downarrow$

$$\Sigma = \Sigma_n + \Sigma_s\tau_1 = i\omega_n(1 - Z(\mathbf{k},\omega_n))\tau_0 + X(\mathbf{k},\omega_n)\tau_3 + Z(\mathbf{k},\omega_n)\Delta(\mathbf{k},\omega_n)\tau_1 \tag{14}$$

$\downarrow$

$$\Gamma_c = \Gamma_{nc} + \Gamma_s\tau_1 = \Gamma_{n0}\tau_0 + \Gamma_{n3}\tau_3 + \Gamma_s\tau_1, \tag{15}$$

$\downarrow$

Eliashberg equations (imaginary frequency $i\omega_n$) in SC state

$\downarrow$

- Renormalization of the spectrum

$\Rightarrow Z(\mathbf{k},\omega_n)$ and $X(\mathbf{k},\omega_n)$

$\downarrow$

$$\omega_n[1 - Z(\mathbf{k},\omega_n)] \;=\; -\frac{T}{N}\sum_{\mathbf{k}',n'}\frac{V_Z(\mathbf{k},\mathbf{k}',\omega_n-\omega_{n'})\omega_{n'}Z(\mathbf{k}',\omega_{n'})}{Det(\mathbf{k}',\omega_{n'})}, \tag{16}$$

$\downarrow$

$$X(\mathbf{k},\omega_n) \;=\; -\frac{T}{N}\sum_{\mathbf{k}',n'}\frac{V_Z(\mathbf{k},\mathbf{k}',\omega_n-\omega_{n'})[\epsilon(\mathbf{k}')+X(\mathbf{k}',\omega_{n'})]}{Det(\mathbf{k}',\omega_{n'})}, \tag{17}$$

$\downarrow$

$\downarrow$

- SC pairing function $\Delta(\mathbf{k}, \omega_n)$

$$Z(\mathbf{k}, \omega_n)\Delta(\mathbf{k}, \omega_n) = \frac{T}{N}\sum_{\mathbf{k}', n'} \frac{V_\Delta(\mathbf{k}, \mathbf{k}', \omega_n - \omega_{n'})Z(\mathbf{k}', \omega_{n'})\Delta(\mathbf{k}', \omega_{n'})}{Det(\mathbf{k}', \omega_{n'})}, \tag{18}$$

where,

$$Det(\mathbf{k}, \omega_n) = [\omega_n Z(\mathbf{k}, \omega_n)]^2 + + [\epsilon(\mathbf{k}) + X(\mathbf{k}, \omega_n)]^2 + Z^2(\mathbf{k}, \omega_n)\Delta^2(\mathbf{k}, \omega_n) \tag{19}$$

↓

- Normal-state renormalization potential V_Z

↓

$$V_Z(\mathbf{k}, \mathbf{k}', \omega_n - \omega_{n'}) = \frac{V_c(\mathbf{k}, \mathbf{k}', n - n')\Gamma_{nc}(\mathbf{k}, \mathbf{k}', n, n')}{\varepsilon_e(\mathbf{k}, \mathbf{k}', n - n')} + \sum_{\alpha.\beta} \left| \frac{\Gamma_{nc}(\mathbf{k}, \mathbf{k}', n - n')}{\epsilon_e(\mathbf{k}, \mathbf{k}', n - n')} \right|^2 \times \tilde{g}_\alpha(\mathbf{k}, \mathbf{k}')\tilde{g}^*_\beta(\mathbf{k}, \mathbf{k}')[-D_{\alpha\beta}(\mathbf{k} - \mathbf{k}', n - n')] \tag{20}$$

PAIRING-POTENTIAL V_Δ

$\downarrow$

$$V_\Delta(\mathbf{k},\mathbf{k}',\omega_n-\omega_{n'}) = -\frac{V_c(\mathbf{k},\mathbf{k}',n-n')}{\varepsilon_e(\mathbf{k},\mathbf{k}',n-n')}|\Gamma_{nc}(\mathbf{k},\mathbf{k}',n-n')|^2 + \sum_{\alpha,\beta} \left| \frac{\Gamma_{nc}(\mathbf{k},\mathbf{k}',n-n')}{\varepsilon_e(\mathbf{k},\mathbf{k}',n-n')} \right|^2 \times \tilde{g}_\alpha(\mathbf{k},\mathbf{k}')\tilde{g}^*_\beta(\mathbf{k},\mathbf{k}')[-D_{\alpha\beta}(\mathbf{k}-\mathbf{k}',n-n')]. \quad (21)$$

$\downarrow$

-Vector coupling $\tilde{g}_\alpha(\mathbf{k},\mathbf{k}')$

$\downarrow$

$$\tilde{g}_\alpha(\mathbf{k},\mathbf{k}') = \langle\psi_{\mathbf{k}} \mid \nabla_\alpha V_{ei} \mid \psi_{\mathbf{k}'}\rangle. \quad (22)$$

$\downarrow$

$\psi_{\mathbf{k}}(\mathbf{x}) =< \psi_{\mathbf{k}} \mid \mathbf{x} >$ solution of

$$h_0(\mathbf{x},\bar{\mathbf{y}})\psi_{\mathbf{k}}(\bar{\mathbf{y}}) = \epsilon(\mathbf{k})\psi_{\mathbf{k}}(\mathbf{x}). \quad (23)$$

$$h_0(x,\mathbf{y}) = (-\frac{1}{2m}\nabla^2_{\mathbf{x}} - \mu)\delta(\mathbf{x}-\mathbf{y}) + V_{ren}(\mathbf{x},\mathbf{y}). \quad (24)$$

$\downarrow$

$V_{ren}(\mathbf{x},\mathbf{y})$ - nonlocal **excitation potential**

$\downarrow$

- Note, $\Gamma_{nc}/\varepsilon_e$ vs $|\ \Gamma_{nc}\ |^2\ /\varepsilon_e$ in Z and Δ
- Various definitions

$\downarrow$

$$\tilde{g}_\alpha(\mathbf{k},\mathbf{k}')\tilde{g}^*_\beta(\mathbf{k},\mathbf{k}')[D_{\alpha\beta}(\mathbf{k}-\mathbf{k}',\omega_n-\omega_{n'})]$$
$$= g(\mathbf{k},\mathbf{k}')g^*(\mathbf{k},\mathbf{k}')[D(\mathbf{k}-\mathbf{k}',\omega_n-\omega_{n'})], \tag{25}$$

where

$$g(\mathbf{k},\mathbf{k}') \equiv \sum_\alpha \tilde{g}_\alpha(\mathbf{k},\mathbf{k}')e^\alpha(\mathbf{k}-\mathbf{k}'), \tag{26}$$

and

$$D_{\alpha\beta}(\mathbf{k},\omega_n) \equiv e^\alpha(\mathbf{k})D(\mathbf{k},\omega_n)e^{*\beta}(\mathbf{k}). \tag{27}$$

ELIASHBERG EQUATIONS FOR STRONGLY CORRELATED SYSTEMS

↓

- Replacements

$$\Gamma_{nc} = \gamma_c \tag{28}$$

↓

$$\epsilon(\mathbf{k}) = \delta\epsilon_0(\mathbf{k}) \tag{29}$$

↓

CRITICAL TEMPERATURE T_c

↓

- Linearization of Eqs.(16-18) w.r.t. $\Delta(\mathbf{k}, \omega_n)$

↓

- Spectral-function in different pairing channels

↓

$$\alpha^2 F_i(\tilde{\mathbf{k}}, \tilde{\mathbf{k}}', \omega) = \frac{N_{sc}(0)}{8} \sum_{\nu,j} \mid g_{eff}(\tilde{\mathbf{k}}, \tilde{\mathbf{k}} - T_j \tilde{\mathbf{k}}', \nu) \mid^2$$

$$\times \delta(\omega - \omega_\nu(\tilde{\mathbf{k}} - T_j \tilde{\mathbf{k}}')) \mid \gamma_c(\tilde{\mathbf{k}}, \tilde{\mathbf{k}} - T_j \tilde{\mathbf{k}}') \mid^2 D_i(j). \tag{30}$$

↓

$$\alpha^2 F_i(\omega) = \langle\langle \alpha^2 F_i(\tilde{\mathbf{k}}, \tilde{\mathbf{k}}', \omega)\rangle_{\tilde{\mathbf{k}}}\rangle_{\tilde{\mathbf{k}}'} \quad (31)$$

$\downarrow$

$$\lambda_i = 2\int d\omega \frac{\alpha^2 F_i(\omega)}{\omega} \quad (32)$$

$\downarrow$

$\tilde{\mathbf{k}}$ and $\tilde{\mathbf{k}}'$ on the Fermi line in the irreducible (1/8) Brillouin zone

$\downarrow$

T_j , $j = 1,..8,$ - eight elements of **the point-group of the square lattice**

$\downarrow$

- Five irreducible representations $i = 1,2,...5$

$\downarrow$

- $D_i(j)$ - **i-th representation matrix**

$\downarrow$

- For $i = 1 \Rightarrow s - wave \;\; SC$

$\Downarrow$

$\Delta(\mathbf{k}, \omega_n) \approx \Delta(\omega_n)$

$\downarrow$

$$T_c^{(s)} \sim <\omega_{ph}> e^{-\frac{1+\lambda_S}{\lambda_S - \mu^*_{(s)}}} \quad (33)$$

$\downarrow$

- For $i = 3 \Rightarrow d - wave$ SC

$\Downarrow$

$\Delta(\mathbf{k}, \omega_n) \approx (\cos k_x - \cos k_y)\Delta(\omega_n)$

$\downarrow$

$$T_c^{(d)} \sim < \omega_{ph} > e^{-\frac{1+\lambda_d}{\lambda_s - \mu^*_{(d)}}} \quad (34)$$

$\downarrow$

- Near optimal doping $\delta_{op} \Rightarrow \lambda_s \approx \lambda_d$. Experiments imply (i) $\mu^*_{(s)} >> \mu^*_{(d)}$, or (ii) $\mu^*_{(s)} > 0$ and $\mu^*_{(d)} < 0$

$\Downarrow$

$$T_c^{(d)} > T_c^{(s)} \quad (35)$$

$\downarrow$

CONCLUSION

$\downarrow$

- SC in HTSC is due basically to EPI and triggered to d-wave by (small) residual Coulomb interaction !

$\downarrow$

$\lambda > 1$ and $\lambda_{tr} \sim 0.5$

$\downarrow$

EPI beyond Migdal Approximation

$\downarrow$

- If $(\omega_D/E_F) << 1 \Rightarrow$ Migdal theory
- For $(\omega_D/E_F) \lesssim 1 \Rightarrow$ Non-Migdal correction

$\Downarrow$

- Vertex correction (see **Fig**. **Vertex**)
- Linearized Eqs.(16-21) but

$V_Z(k,k') \rightarrow \tilde{V}_Z(k,k')$

$V_\Delta(k,k') \rightarrow \tilde{V}_\Delta(k,k')$

$\downarrow$

$$\tilde{V}_Z(k,k') = V_{ep}(k-k')[1 + \\ + \sum_q V_{ep}(k-q)G(q-k+k')G(q)] \quad (36)$$

$$\tilde{V}_\Delta(k,k') = V_{EP}(k-k')[1 + \\ \sum_q V_{ep}(q-k)G(-q+k-k')G(-q)] \quad (37)$$

$$+\sum_q V_{ep}(k-q)V_{ep}(q-k')G(q-k+k')G(q).$$

$\downarrow$

- Here, $k = (\mathbf{k}, \omega_n)$, $\sum_q = T\sum_{\mathbf{q},\omega_n}$.

$\downarrow$

- Assume Einstein phonon

$\Rightarrow \omega_{ph}(\mathbf{q}) = \omega_0$

$\downarrow$

- Forward scattering peak in $\gamma_c(\mathbf{k}_F, \mathbf{q})$

$\Rightarrow \gamma_c(\mathbf{k}_F, \mathbf{q}) = 0$ for $q < Q_c << k_F$

$\Downarrow$

- Increase of T_c ! (see Fig.T_c)

$\downarrow$

$$T_c \approx 1.13\tilde{\omega}_0(m)e^{-\frac{1+\lambda_Z/(1+m)}{\lambda_\Delta}} \tag{38}$$

$$\tilde{\omega}_0(m) = \omega_0 \frac{e^{\frac{m}{2(1+m)}}}{(1+m)\sqrt{e}}$$

$\downarrow$

$$\lambda_\Delta = \lambda[1 + 2\lambda P_V(\omega_n, \omega_m; Q_c) + \lambda P_c(\omega_n, \omega_m; Q_c)], \tag{39}$$

↓

$$\lambda_Z = \lambda[1 + \lambda P_V(\omega_n, \omega_m; Q_c)]. \quad (40)$$

↓

- For $Q_c << k_F$ and finite $m = \omega_D/E_F$

$\Rightarrow P_V(\omega_n, \omega_m; Q_c) > 0$

$P_c(\omega_n, \omega_m; Q_c) > 0$

↓

- Vertex correction might be important in fullerenes A_3C_{60}

↓

- Comparison of A_3C_{60} and intercalated KC_8

$\Rightarrow$ (ω_D/E_F) ~ 1 in A_3C_{60} !

↓

	KC_8	A_3C_{60}
T_c	$0.1 - 0.2\ K$	$20 - 35\ K$
ω_D	$2000\ K$	$2000\ K$
λ	~ 0.25	~ 0.5
μ^*	~ 0.1	~ 0.4
E_F	$10\ eV$	$0.2\ eV$

↓

EPI vs Spin Fluctuations (SF)

↓

- For EPI well defined theory !
- For SF not well-defined theory (RPA) !

↓

- SF self-energy

↓

$$\Sigma_{sf}(\mathbf{k},\omega_n) = -\frac{T}{N}\sum_{\mathbf{k}',m} g_{sf}^2 P(\mathbf{k}-\mathbf{k}',n-m) \times \tau_0 G(\mathbf{k}',\omega_m)\tau_0 \qquad (41)$$

↓

$P(\mathbf{k}-\mathbf{k}',n-m)$ - propagator of spin fluctuations

↓

$$P(\mathbf{q},n-m) \approx \chi(\mathbf{q},n-m). \qquad (42)$$

↓

$\chi(\mathbf{q},n-m)$ - dynamical spin susceptibility

↓

- **Phenomenology** for $\chi(\mathbf{q},\omega)$

↓

- Pines et al. model

↓

$$\mathrm{Im}\,\chi_P(\mathbf{q},\omega+i0^+)$$

$$=\frac{\omega}{\omega_{sf}}\frac{\chi_Q}{(1+\xi_M^2\,|\,\mathbf{q}-\mathbf{Q}\,|^2\,)^2}\Theta(\omega_c-|\,\omega\,|),$$

(43)

↓

- Fit by $\omega_c = 400\ meV$
- From $NMR \Rightarrow \chi_Q \approx (30-40)\chi_0 \sim 100\ eV^{-1}$

↓

- R(ULN) model from neutron scattering for underdoped HTSC !

$$\mathrm{Im}\,\chi_R(\mathbf{q},\omega+i0^+) = [\frac{\sqrt{C}}{1+J_0[\cos q_x+\cos q_y]}]^2$$

$$\times\frac{3(T+5)\omega}{1.5\omega^2-60\,|\,\omega\,|\,+900+3(T+5)^2}\Theta(\tilde{\omega}_c-|\,\omega\,|$$

(4

where

$\tilde{\omega}_c = 100\ meV$, $J_0 = 0.3\ eV$, $C = 0.19\ eV^{-1}$

↓↓

- **Repulsive pairing potential** $V_{SF}(\mathbf{q},\omega)$ (see **Fig.SF**)

$\downarrow$

$$V_{SF}(\mathbf{q},\omega+i0^+) = g_{SF}^2 \int_{-\infty}^{\infty} \frac{d\Omega}{\pi} \frac{\mathrm{Im}\,\chi(\mathbf{q},\Omega+i0^+)}{\Omega-\omega}, \tag{45}$$

$\downarrow$

- Pairing spectral function in $d-wave$ channel

$\Rightarrow Y_d(\mathbf{k}) \sim \cos k_x - \cos k_y$

$$\alpha_d^2 F(\omega) = -\frac{\langle\langle Y_d(\mathbf{k}) Y_d(\mathbf{k}') V_{sf}(\mathbf{k}-\mathbf{k}',\omega+i0^+)\rangle\rangle}{\langle Y_d^2(\mathbf{k})\rangle}. \tag{46}$$

$\downarrow$

- In order to get $T_c \sim 100\ K$ Pines et al. **assume to large coupling**

$\Rightarrow g_{SF} \sim 0.64\ eV$! $\Rightarrow \lambda_{sf} \gtrsim 2$!

$\downarrow$

- Even for such large $g_{SF} \sim 0.64\ eV \Rightarrow$ R(ULN) model gets much smaller T_c

$\downarrow$

- For optimal doping (highest T$_c$) $\Rightarrow$neutron scattering shows

$\Downarrow$

$$\mathrm{Im}\,\chi_{\exp}(\mathbf{Q},\Omega) << \mathrm{Im}\,\chi_{P}(\mathbf{Q},\Omega) \qquad (47)$$

$\downarrow$

- Other experiments (ARPES, $\rho(T)$)

$\Rightarrow g_{SF} \sim 0.1\ eV\,! \Rightarrow \lambda_{sf} < 0.2\,!$

$\Downarrow$

- SF pairing-mechanism ineffective in HTSC !

$\downarrow$

- Numerical calculations (see Fig.MC)

$\Rightarrow$ **No HTSC in Hubbard and t-J models** !

Effects of Impurities

$\downarrow$

- In HTSC **d-wave pairing** is realized

- **Isotropic impurity** scattering - with $u^2(\mathbf{q}) = u_0^2 = const$

suppress d-wave pairing strongly!

$\downarrow$

- **Experiments**

(a) $T_c(\rho_i)$ for substitutional defects $Cu \rightarrow Zn$, $Y \rightarrow \mathrm{Pr}$

(b) $T_c(\rho_i)$ for ion (Ne^+) bombardment of $HTSC$

ρ_i - residual resistivity

$\Downarrow$

T_c in HTSC is **robusst** against impurities !!

- How to reconcile T_c robustness and d-wave pairing

- **Answer**:

Forward scattering peak in impurity scattering potential

$\Downarrow$

$u_{i,0}^2 \to u_i^2(\mathbf{q}) = u_{i,0}^2\gamma_c^2(\mathbf{q})$

$\downarrow$

- **Self-energy**

$$\Sigma = \Sigma_{ep} + \Sigma_i \qquad (48a)$$

$\downarrow$

- Impurity self-energy Σ_i

$$\Sigma_i(\mathbf{p}, i\omega_n) = n_i \int \frac{d^2p'}{(2\pi)^2} u_i^2(\mathbf{p} - \mathbf{p}')G(\mathbf{p}', i\omega_n). \qquad (48b)$$

$\omega_n = \pi T(2n+1)$ - Matsubara frequencies

n_i - concentration of impurities frequencies

$\downarrow$

- Eq.(48) gives renormalized frequency $\tilde{\omega}_n(\theta)$ and gap $\tilde{\Delta}_n(\theta)$

$\downarrow$

$$\tilde{\omega}_n(\theta) = \omega_n + \frac{1}{2}\langle\Gamma(\theta, \theta')\frac{\tilde{\omega}_n(\theta')}{\sqrt{\tilde{\omega}_n^2(\theta') + \tilde{\Delta}_n^2(\theta')}}\rangle_{\theta'} \qquad (49)$$

$\downarrow$

$$\tilde{\Delta}_n(\theta) = \Delta(\theta) + \frac{1}{2}\langle\Gamma(\theta,\theta')\frac{\tilde{\Delta}_n(\theta')}{\sqrt{\tilde{\omega}_n^2(\theta') + \tilde{\Delta}_n^2(\theta')}}\rangle_{\theta'}. \quad (50)$$

↓

$$\Gamma(\theta,\theta') = u_i^2(\theta,\theta')N(\theta) \quad (51a)$$

where

$$\Gamma(\theta,\theta') = \Gamma_s(\theta,\theta') + $$
$$+ \Gamma_d Y_d(\theta) Y_d(\theta') + \Gamma_p Y_p(\theta) Y_p(\theta') + ... \quad (51b)$$

and

$$\int_0^{2\pi}\frac{d\theta'}{2\pi}(...) \equiv \langle(...)\rangle_{\theta'} \quad (52)$$

↓

- Self-consistent gap equation

↓

$$\Delta(\theta) = T_c\sum_n\langle\lambda(\theta,\theta')\frac{\tilde{\Delta}_n(\theta')}{\sqrt{\tilde{\omega}_n^2(\theta') + \tilde{\Delta}_n^2(\theta')}}\rangle_{\theta'}. \quad (53)$$

↓

- For d-wave pairing ⇒pairing-potential $\lambda(\theta,\theta')$

$$\lambda(\theta,\theta') = \lambda Y_d(\theta)Y_d(\theta') \qquad (54)$$

↓

- SC order parameter

$$\Delta(\theta) = \Delta_0 Y_d(\theta) = -\Delta(\theta + \pi/2) \qquad (55)$$

$$\tilde{\Delta}(\theta) = \tilde{\Delta}_0 Y_d(\theta)$$

↓

- Note, $\langle Y_d(\theta)\rangle_\theta = 0$ and $\langle Y_d^2(\theta)\rangle_\theta = 1$.
- Near T_c linearization of $Eqs.(49, 50, 53)$

↓

- Critical temperature T_c (see **Fig**.$T_c(\rho_i)$)

↓

$$ln\frac{T_c}{T_c^0} = \Psi(\frac{1}{2}) - \Psi[\frac{1}{2} + (1-\beta)x], \qquad (56)$$

where

$$\beta = \frac{\Gamma_d}{\Gamma_s} < 1; \quad x = \frac{\Gamma_s}{4\pi T_c} \qquad (57)$$

- For isotropic scattering $\beta = 0 \Rightarrow$ strong depairing

- In HTSC $\beta \approx 0.8$ (**t-J model**) (see **Fig**.β)

$\Rightarrow$ robustness of d-wave pairing

$\downarrow$

Experiments on HTSC

$\Rightarrow T_c(\rho_i)$ for substitutional defects $Cu \rightarrow Zn$, $Y \rightarrow \mathrm{Pr}$

$\Rightarrow T_c(\rho_i)$ for ion (Ne^+) bombardment

$\downarrow$

- Resistivity $\rho(T)$

$$\rho(T) = \rho_i + \alpha T \quad (58)$$

- Residual resistivity ρ_i

$$\rho_i = \frac{4\pi\Gamma_{tr}}{\omega_p^2} \quad (59)$$

$\downarrow$

$\Gamma_s = p \bullet \Gamma_{tr} \Rightarrow$ from ρ_i

$$\Gamma_{tr} = 2\pi\lambda_{tr}\frac{\rho_i}{\alpha} \quad (60)$$

$\downarrow$

- From $\rho(T)$ **experiments** $\Rightarrow \lambda_{tr} \gtrsim 0.4$
- From $t - J$ model $\Rightarrow p \gtrsim 2.5$ for $\delta \leq 0.2$