II. Hubbard model and HTSC

$\downarrow$

- Tight-binding model Hamiltonian
- Coulomb interaction and Hubbard model
- Hubbard model in terms of Hubbard operators $X^{\alpha\beta}$
- strongly correlated system $\Rightarrow U \gg W$

$\downarrow$

Tight-binding Hamiltonian

$\downarrow$

- Hamiltonian for solid with N electrons

$H_e = T + H_{el} + H_c$

$\downarrow$

$$H_e = \int d^3 r \hat{\psi}^\dagger(\mathbf{r}) \epsilon_0(\hat{p}) \hat{\psi}(\mathbf{r}) +$$

$$+ \frac{1}{2} \int d^3 r d^3 r' \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}) V_c(\mathbf{r} - \mathbf{r}') \hat{\psi}^\dagger(\mathbf{r}') \hat{\psi}(\mathbf{r}') \quad (1)$$

$\downarrow$

$$\{\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')\} = \delta(\mathbf{r} - \mathbf{r}') \qquad (2)$$

$\downarrow$

$\epsilon_0(\hat{p}) = \hat{p}^2/2m$ - kinetic energy of electron

$V_c(\mathbf{r} - \mathbf{r}') = \frac{e^2}{|\mathbf{r}-\mathbf{r}'|}$ - Coulomb **e**-**e** interaction

$\downarrow$

$$\hat{\psi}(\mathbf{r}) = \sum_i \phi_i(\mathbf{r})\hat{f}_i \qquad (3)$$

$\downarrow$

- complete basis $\{\phi_j(\mathbf{r})\}$

$$\int d^3r \phi_j^*(\mathbf{r})\phi_i(\mathbf{r}) = \delta_{ji} \qquad (4)$$

$\downarrow$

$$\{\hat{f}_i, \hat{f}_j^\dagger\} = \delta_{ij}$$

$\downarrow$

- in solids two set of basis are usually used: plane waves and tight-binding

Plane waves basis

↓

$$\hat{\psi}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}\sigma} \hat{f}_{\mathbf{k}\sigma} e^{i\mathbf{kr}} \chi_\sigma \qquad (5)$$

↓

$$\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}) = \frac{1}{V} \sum_{\mathbf{k}\sigma} \hat{\rho}_{\mathbf{k}} e^{-i\mathbf{kr}} \qquad (6)$$

$$\hat{\rho}_{\mathbf{k}} = \sum_{\mathbf{q}\sigma} \hat{f}^\dagger_{\mathbf{k}+\mathbf{q}\sigma} \hat{f}_{\mathbf{q}\sigma}$$

↓

- kinetic energy T

$$T = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \hat{f}^\dagger_{\mathbf{k}\sigma} \hat{f}_{\mathbf{k}\sigma} \qquad (7)$$

↓

- Electron-lattice interaction (**G**-reciprocal lattice vectors)

↓

$$H_{el} = \sum_{\mathbf{G}} V_{el,\mathbf{G}} \hat{\rho}_{\mathbf{G}} \qquad (8)$$

↓

- e-e Coulomb interaction

↓

$$H_c = \sum_{\mathbf{k}} V_{c,\mathbf{k}} \hat{\rho}_{\mathbf{k}}^{\dagger} \hat{\rho}_{\mathbf{k}} \qquad (9a)$$

↓

- Coulomb interaction $V_{c,\mathbf{k}}$,

$$V_{c,\mathbf{k}} = \frac{4\pi e^2}{k^2 \varepsilon_\infty \Omega} \qquad (9b)$$

↓

ε_∞ - dielectric function (high energy screening)

↓

$\Omega = V/N$

Tight-binding basis

↓

- Wannier orbitals $\phi(\mathbf{r} - \mathbf{R}_m)$

↓

$$T = \sum_{m\sigma} T_{0,m} \hat{f}^{\dagger}_{m\sigma} \hat{f}_{m\sigma} + \sum_{m \neq n\sigma} T_{mn} \hat{f}^{\dagger}_{m\sigma} \hat{f}_{n\sigma} \quad (10)$$

↓

$$T_{mn} = \frac{\hbar^2}{2m} \int d^3 r \nabla \phi^* (\mathbf{r} - \mathbf{R}_m) \nabla \phi (\mathbf{r} - \mathbf{R}_n) \quad (11)$$

↓

- electron-lattice interaction

↓

$$H_{el} = \sum_{m\sigma} V_{el,0,m} \hat{f}^{\dagger}_{m\sigma} \hat{f}_{m\sigma} + \sum_{m \neq n\sigma} V_{el,mn} \hat{f}^{\dagger}_{m\sigma} \hat{f}_{n\sigma} \quad (12)$$

↓

$$V_{el,mn} = \int d^3r\phi^*(\mathbf{r} - \mathbf{R}_m)V_{el}(\mathbf{r})\phi(\mathbf{r} - \mathbf{R}_n) \quad (13)$$

$\downarrow$

- e-e Coulomb interaction (only two-center terms included)

$$H_{el} = \frac{U}{2}\sum_{m\sigma_1\sigma_2}\hat{n}_{m\sigma_1}\hat{n}_{m\sigma_2} + \frac{1}{2}\sum_{m\neq n\sigma\sigma'}V_{c,mn}\hat{n}_{m\sigma}\hat{n}_{n\sigma'} \quad (14)$$

$\downarrow$

$\hat{n}_{m\sigma} = \hat{f}^{\dagger}_{m\sigma}\hat{f}_{m\sigma}$

$\downarrow$

$$V_{c,mn} = e^2\int\int\frac{d^3rd^3r'}{|\ \mathbf{r} - \mathbf{r}'\ |} \quad (15)$$

$$\times |\ \phi^*(\mathbf{r} - \mathbf{R}_m)\ |^2\ \ |\ \phi(\mathbf{r} - \mathbf{R}_n)\ |^2$$

$\downarrow$

- for well localized $\phi(\mathbf{r} - \mathbf{R}_n) \Rightarrow V_{c,mn} \approx \frac{e^2}{R_{mn}}$

- note $\hat{n}^2_{m\sigma} = \hat{n}_{m\sigma}$

- "atomic level" $\epsilon_{a,m} = T_{0,m} + V_{el,0,m} < 0$

↓

EXTENDED HUBBARD MODEL

↓

$$H_H^e = \sum_{m\sigma} \epsilon_{a,m} \hat{n}_{m\sigma} - \sum_{m \neq n\sigma} t_{mn} \hat{f}_{m\sigma}^\dagger \hat{f}_{n\sigma}$$

$$+ U \sum_m \hat{n}_{m\uparrow} \hat{n}_{m\downarrow} + \frac{1}{2} \sum_{m \neq n\sigma\sigma'} V_{c,mn} \hat{n}_{m\sigma} \hat{n}_{n\sigma'} \quad (16)$$

↓

- Note, $t_{mn} = -(T_{mn} + V_{el,mn})$

↓

- Brillouin zone wave vector basis

↓

$$\hat{f}_{n\sigma} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} \hat{f}_{\mathbf{k}\sigma} e^{i\mathbf{k}\mathbf{R}_n} \quad (17)$$

↓

$$\sum_n \hat{f}_{n\sigma}^\dagger \hat{f}_{n\sigma} = \sum_{\mathbf{k}} \hat{f}_{\mathbf{k}\sigma}^\dagger \hat{f}_{\mathbf{k}\sigma} \quad (18)$$

↓

$$H_H^e = \epsilon_a \sum_{\mathbf{k}\sigma} \hat{n}_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} t_{\mathbf{k}} \hat{f}^{\dagger}_{\mathbf{k}\sigma} \hat{f}_{\mathbf{k}\sigma}$$

$$+ \frac{U}{N} \sum_{\mathbf{k}} \rho^{\dagger}_{k\uparrow} \rho_{k\downarrow} + \frac{1}{2N} \sum_{\mathbf{k}\sigma\sigma'} V_{c,\mathbf{k}} \rho^{\dagger}_{k\sigma} \rho_{k\sigma'}$$

(19a)

↓

$$t_{\mathbf{k}} = \sum_{\mathbf{R}_n \neq 0} t_n e^{-i\mathbf{k}\mathbf{R}_n}$$

$$V_{c,\mathbf{k}} = \sum_{\mathbf{R}_n \neq 0} (e^2/R_n) e^{i\mathbf{k}\mathbf{R}_n} \qquad (19b)$$

- note and

- usually is V_c neglected and assumed $t_n \neq 0$ for n.n.

↓

Small U Hubbard Model -metals

↓

- small $U \;\; \Rightarrow \;\; U \ll W$ (band width)

↓

- charge susceptibility (in imaginary frequency $i\omega_n$)

↓

$$\chi_c(\mathbf{k}, i\omega_n) = -\frac{1}{N}\int_0^\beta d\tau e^{i\omega_n\tau} < T_\tau \rho_{\mathbf{k}}(\tau)\rho^\dagger_{\mathbf{k}}(0) > \quad (20)$$

$$= 2(\chi_{\uparrow\uparrow} + \chi_{\uparrow\downarrow})$$

↓

$$\rho_{\mathbf{k}} = \rho_{\mathbf{k}\uparrow} + \rho_{\mathbf{k}\downarrow} \quad (21)$$

↓

$$\chi_c(\mathbf{k}, i\omega_n) = \frac{2P(\mathbf{k}, i\omega_n)}{1 - (U + 2V_{c,\mathbf{k}})P(\mathbf{k}, i\omega_n)} \quad (22)$$

↓

- spin susceptibility $\chi_s(\mathbf{k}, i\omega_n)$

↓

$$\chi_s(\mathbf{k}, i\omega_n) = -\frac{1}{N}\int_0^\beta d\tau e^{i\omega_n\tau} < T_\tau s^z_{\mathbf{k}}(\tau)s^{z\dagger}_{\mathbf{k}}(0) > \quad (23)$$

$$= 2(\chi_{\uparrow\uparrow} - \chi_{\uparrow\downarrow})$$

↓

$$s^z_{\mathbf{k}}(\tau) = \rho_{\mathbf{k}\uparrow} - \rho_{\mathbf{k}\downarrow} \tag{24}$$

$\downarrow$

$$\chi_s(\mathbf{k}, i\omega_n) = \frac{2P(\mathbf{k}, i\omega_n)}{1 + UP(\mathbf{k}, i\omega_n)} \tag{25}$$

$\downarrow$

$$P^{RPA}_{\sigma}(\mathbf{k}, i\omega_n) = -\frac{1}{N}\sum_q G(q)G(k+q)$$

$$= \frac{1}{N}\sum_{\mathbf{q}} \frac{n_F(\xi_{\mathbf{q}}) - n_F(\xi_{\mathbf{k}+\mathbf{q}})}{i\omega_n + \xi_{\mathbf{q}} - \xi_{\mathbf{k}+\mathbf{q}}} \tag{26}$$

$\downarrow$

$$\chi_c(\mathbf{k}) = -\frac{N(0)}{1 + N(0)U + \frac{k_{sc}^2}{k^2}} \tag{27}$$

$\downarrow$

- in metals $\chi_c(\mathbf{k} \to \mathbf{0}) \to 0$

Charge collective modes for U<<W

↓

- define $x = \frac{\omega}{k v_F}$
- for $x \to 0$

$$P_\sigma^{RPA}(0) = -N(0) \tag{28}$$

- for $x \to \infty$ (in d-dimensions)

↓

$$P_\sigma^{RPA}(k) = \frac{k^2 a^2 E}{\omega^2 d} \tag{29}$$

↓

- average "kinetic" energy E

↓

$$E = \frac{1}{N} \sum_{\mathbf{k}} t_{\mathbf{k}} n_F(\xi_{\mathbf{k}})$$

↓

$\xi_{\mathbf{k}} = -t_{\mathbf{k}} - \mu$

↓

- in metals

$$\chi_c(\mathbf{k} \to \mathbf{0}, \omega) = -\frac{2n_0 a^2 k^2 E}{\omega^2 - \omega_{pl}^2} \qquad (30)$$

↓

$$\omega_{pl}^2 = \frac{4\pi^2 n_0 e^2}{\varepsilon_\infty m} \qquad (31)$$

↓

- in metals plasma collective mode with ω_{pl}

↓

- neutral systems $\Rightarrow V_c = 0 \Rightarrow$ pure Hubbard model

↓

$$\chi_c(\mathbf{k} \to \mathbf{0}, \omega) = -\frac{2n_0 a^2 k^2 E}{\omega^2 - \omega_k^2} \qquad (32a)$$

- sound-like mode ($v_c^2 = a^2 UE/d$)

$$\omega_k^2 = v_c^2 k^2 \qquad (32b)$$

↓

a - nearest neighbour distance

↓

n_0 - density of electrons

↓

Antiferromagnetism and spin collective modes

↓

- limit $x \to \infty$

↓

$$\chi_s(\mathbf{k} \to \mathbf{0}, \omega) = \frac{2v_c^2 k^2/U}{\omega^2 + v_c^2 k^2} \qquad (33)$$

↓

- there is relaxation mode $\omega_{rel} = iv_c k$

↓

AF instability - **2D system with**

↓

- Case: half-filling $n_0 = 1$ and $\mu = 0$)
- for 2D n.n. $\epsilon_{\mathbf{k}} = -2t(\cos k_x a + \cos k_x a)$

↓

- density of states

$$N(\omega) = \frac{1}{2\pi^2 t} \ln \frac{16t}{\omega} \qquad (34)$$

- static $\chi_s(\mathbf{k})$

↓

$$\chi_s(\mathbf{k}) = \frac{2P(\mathbf{k})}{1 + UP(\mathbf{k})} \qquad (35)$$

- at $T > 0$

↓

- at $\mathbf{k} = 0$

$$P^{RPA}(\mathbf{0}) \sim -\frac{1}{t}\ln\frac{t}{T} \qquad (36)$$

- at $\mathbf{Q} = (\boldsymbol{\pi}, \boldsymbol{\pi})$

$$P(\mathbf{k} = \mathbf{Q}) \sim -\frac{1}{t}\ln^2\frac{t}{T} \qquad (37)$$

↓

- AF (SDW) instability at T_{SDW}

↓

$$1 + UP(\mathbf{Q}) = 0 \qquad (38)$$

↓

$$T_{SDW} \sim te^{-2\pi(t/U)^2} \qquad (39)$$

↓

- for $n \lesssim 1$ AF fluctuations are inherent (**Fig.AF**)

↓

Hubbard model with large U>>W

↓

$$H = -t \sum_{m \neq n\sigma} \hat{f}^{\dagger}_{m\sigma} \hat{f}_{n\sigma} + U \sum_{m} \hat{n}_{m\uparrow} \hat{n}_{m\downarrow} \quad (40)$$

↓

- doubly occupation (”doublons”) is suppressed for $U >> W$

↓

- novel types of screening is expected

↓

- Hilbert space $\{|\ \alpha > \Rightarrow |\ 0 >, |\ 2 >, |\uparrow>, |\downarrow>\}$
- Hubbard projection operators $X^{\alpha\beta}$;
$\alpha, \beta = 0, 2, \sigma = \uparrow (+), \sigma = \downarrow (-)$

↓

$$X^{\alpha\beta} = |\ \alpha >< \beta\ | \quad (41)$$

↓

$$X^{\alpha\beta} X^{\gamma\delta} = \delta_{\beta\gamma} X^{\alpha\delta} \quad (42)$$

↓

- ”ugly” algebra

↓

$$X_i^{\alpha\beta} X_j^{\gamma\delta} \pm X_j^{\gamma\delta} X_i^{\alpha\beta} = \delta_{ij}(\delta_{\beta\gamma} X_i^{\alpha\delta} \pm \delta_{\delta\alpha} X^{\gamma\beta} \quad (43)$$

↓

- completeness relation

↓

$$X_i^{00} + X_i^{22} + \sum_{\sigma} X_i^{\sigma\sigma} = 1 \quad (44)$$

↓

- $\hat{f}_{i\sigma}$ versus $X^{\alpha\beta}$

(if $\sigma =\uparrow \Rightarrow \bar{\sigma} =\downarrow$)

↓

$$\hat{f}_{i\sigma} = X_i^{0\sigma} + \sigma X_i^{\bar{\sigma}2}$$

$$\hat{f}_{i\sigma}^{\dagger} = X_i^{\sigma 0} + \sigma X_i^{2\bar{\sigma}} \quad (45)$$

$$n_i = 1 - X_i^{00} + X_i^{22} \quad (46)$$

$$S_i^{+} = \hat{f}_{i\uparrow}^{\dagger} \hat{f}_{i\downarrow} = X_i^{+-} = (S_i^{-})^{\dagger} = (X_i^{-+})^{\dagger}$$

$$S_i^z = \frac{1}{2}(\hat{f}_{i\uparrow}^\dagger \hat{f}_{i\uparrow} - \hat{f}_{i\downarrow}^\dagger \hat{f}_{i\downarrow}) = \frac{1}{2}(X_i^{++} - X_i^{--})$$

(47)

↓

$X^{\alpha\beta}$ **VERSUS** $\hat{f}_{i\sigma}$

↓

$$X^{\sigma 0} = \hat{f}_\sigma^\dagger (1 - \hat{n}_{\bar{\sigma}}); \quad X^{\sigma\bar{\sigma}} = \hat{f}_\sigma^\dagger \hat{f}_{\bar{\sigma}} \qquad (48)$$

$$X^{\sigma\sigma} = \hat{n}_\sigma (1 - \hat{n}_{\bar{\sigma}}) \qquad (49)$$

$$X^{00} = (1 - \hat{n}_\uparrow)(1 - \hat{n}_\downarrow) \qquad (50)$$

$$X^{2\sigma} = \sigma \hat{f}_{\bar{\sigma}}^\dagger \hat{n}_\sigma \ ; \quad X^{20} = \sigma \hat{f}_{\bar{\sigma}}^\dagger \hat{f}_\sigma \qquad (51)$$

$$X^{22} = n_\uparrow n_\downarrow \qquad (52)$$

↓

- Hamiltonian in terms of $X^{\alpha\beta} \Rightarrow$ correlated motion of holes (electrons)

↓

$$H = -t \sum_{ij\sigma} (X_i^{\sigma 0} X_j^{0\sigma} + X_i^{2\sigma} X_j^{\sigma 2})$$

$$- t \sum_{ij\sigma} \sigma (X_i^{\sigma 0} X_j^{\bar{\sigma} 2} + X_i^{2 \bar{\sigma}} X_j^{0 \sigma}) + U \sum_i X_i^{22}$$

$$= H_1 + H_{12} + H_2 \qquad (53)$$

$H_1 \Rightarrow$ single hole motion $\Rightarrow$ lower Hubbard band

$H_2 \Rightarrow$ two holes $\Rightarrow$ upper Hubbard band

$H_{12} \Rightarrow$ connect two bands

$\downarrow$

Effective Hamiltonian for $U >> t$

$\downarrow$

VARIOUS METHODS

$\downarrow$

- perturbation over U-term

$\downarrow$

- canonical transformation $S \Rightarrow$ mixes lower and upper band

$\downarrow$

$$H_{eff} = e^S H e^{-S}$$

$$= H + [S, H] + \frac{1}{2} [S, [S, H]] + .. \qquad (54)$$

↓

$$S = \varkappa \sum_{ij\sigma} (X_i^{\sigma 0} X_j^{\bar{\sigma} 2} - X_i^{2\bar{\sigma}} X_j^{0\sigma}) \qquad (55)$$

↓

$\varkappa \;\Rightarrow$ disappear all L-U processes $\sim t$

$$\varkappa = -\frac{t}{U}$$

$$H_{12} + [S, H_2] = 0 \qquad (56)$$

↓

$$H_{eff} = -t \sum_{ij\sigma} X_i^{\sigma 0} X_j^{0\sigma} + H_{3s}$$

$$+ J \sum_{ij\sigma} (\mathbf{S}_i \mathbf{S}_j - \frac{1}{4} \hat{n}_i \hat{n}_j) + H_2 \qquad (57)$$

↓

- exchange energy $J = 2t^2/U$
- $H_2 \;\Rightarrow$ motion of ”doublons”

↓

$$H_2 = U \sum_i X_i^{22} - t \sum_{ij\sigma} X_i^{2\sigma} X_j^{\sigma 2} \qquad (58)$$

$\downarrow$

- three sites term H_{3s} (usually neglected in t-J model)

$\downarrow$

$$H_{3s} = \frac{J}{2} \sum_{ijl\sigma} (X_i^{\bar{\sigma}0} X_l^{\sigma\bar{\sigma}} X_j^{0\sigma} - X_i^{\sigma 0} X_l^{\bar{\sigma}\bar{\sigma}} X_j^{0\sigma}) \tag{59}$$

$\downarrow$

- projection on the lower band

$\downarrow$

$$PH_{eff}P = H_{tJ}$$

$\downarrow$

$$H_{tJ} = -t \sum_{ij\sigma} X_i^{\sigma 0} X_j^{0\sigma} + +J \sum_{ij\sigma} (\mathbf{S}_i \mathbf{S}_j - \frac{1}{4} \hat{n}_i \hat{n}_j)$$

$$= -t \sum_{ij\sigma} X_i^{\sigma 0} X_j^{0\sigma} + \frac{J}{2} \sum_{ij\sigma} (X_i^{\sigma\bar{\sigma}} X_j^{\bar{\sigma}\sigma} - X_i^{\sigma\sigma} X_j^{\bar{\sigma}\bar{\sigma}}) \tag{60}$$

$\downarrow$

- Spin operators $S^{\pm}, S^z$ do not describe correctly the electron spin!

$S = 0, 1/2$

$\downarrow$

$$[S_i^+, S_j^-] = 2\delta_{ij} S_i^z$$

$$[S_i^z, S_j^\pm] = \pm\delta_{ij} S_i^\pm \qquad (61)$$

$$\mathbf{S}_i^2 = \frac{3}{4}\hat{n}_i \neq \frac{3}{4} \qquad (62)$$

$\downarrow$

Ugly algebra of $X^{\alpha\beta}$ $\Rightarrow$ How to treat H_{tJ} ?

$\downarrow$

Various representations of $X^{\alpha\beta}$

$\downarrow$

SLAVE BOSON METHOD

$\downarrow$

$F_{i\sigma}$ - fermion (spinon); B_i - boson (holon)

$\downarrow$

$$X^{0\sigma} = F_\sigma B^\dagger \qquad (63)$$

$\downarrow$

- constraint on Hilbert space (completeness)

$\downarrow$

$$B^\dagger B + \sum_\sigma F_\sigma^\dagger F_\sigma = 1 \qquad (64)$$

$$H_{tJ} = -t \sum_{ij\sigma} F_{i\sigma}^\dagger F_{j\sigma} B_i B_j^\dagger + \frac{J}{2} \sum_{ij\sigma\sigma'} F_{i\sigma}^\dagger F_{j\sigma} F_{j\sigma'}^\dagger F_{i\sigma'}$$

(65)

$\downarrow$

- partition function ($F_{i\sigma}$ - Grassman variable)

$\downarrow$

$$Z = \int D\lambda_i DB_i DB_i^* DF_{i\sigma} DF_{i\sigma}^* e^{-\int_0^\beta (L+H_{tJ})d\tau}$$

(66)

$\downarrow$

$$L == \sum_{i\sigma} F_{i\sigma}^\dagger \left(\frac{\partial}{\partial\tau} - \mu\right) F_{j\sigma} + \sum_i B_i^\dagger \frac{\partial}{\partial\tau} B_i$$

$$+ \sum_i \lambda_i \left(B_i^\dagger B_i + \sum_\sigma F_{i\sigma}^\dagger F_{i\sigma} - 1\right) \qquad (67)$$

$\downarrow$

- 1/N expansion as a controllable methods

$\downarrow$

SLAVE FERMION METHOD

$\downarrow$

$$X^{0\sigma} = B^{\dagger}_{\sigma} F \quad (68)$$

$\downarrow$

- constraint on Hilbert space

$\downarrow$

$$F^{\dagger} F + \sum_{\sigma} B^{\dagger}_{\sigma} B_{\sigma} = 1 \quad (69)$$

$\downarrow$

SPIN FERMION METHOD

$\downarrow$

$$\hat{f}^{\dagger}_{\uparrow} \hat{f}_{\uparrow} + \hat{f}^{\dagger}_{\downarrow} \hat{f}_{\downarrow} = 1 - F^{\dagger} F \quad (70)$$

$$\mathbf{S} = \mathbf{s}(1 - F^{\dagger} F) \quad (71)$$

$\downarrow$

$$H_{tJ} = 2t \sum_{ij} F^{\dagger}_{i} F_{j} (\mathbf{s}_{i} \mathbf{s}_{j} + \frac{1}{4})$$

$$+ J \sum_{ij} (1 - F_i^\dagger F_i)(\mathbf{s}_i \mathbf{s}_j - \frac{1}{4})(1 - F_j^\dagger F_j) \tag{72}$$

↓

PROPERTIES OF REPRESENTATIONS

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- nonuniqueness (ambiguity)
- Fermi-Boson diagram technique possible
- constraint gives rise to singular kinematical interaction
- difficult to find controllable approximation