Two-Dimensional t-J Model in a Staggered Field

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   - ⋆ Our motivation
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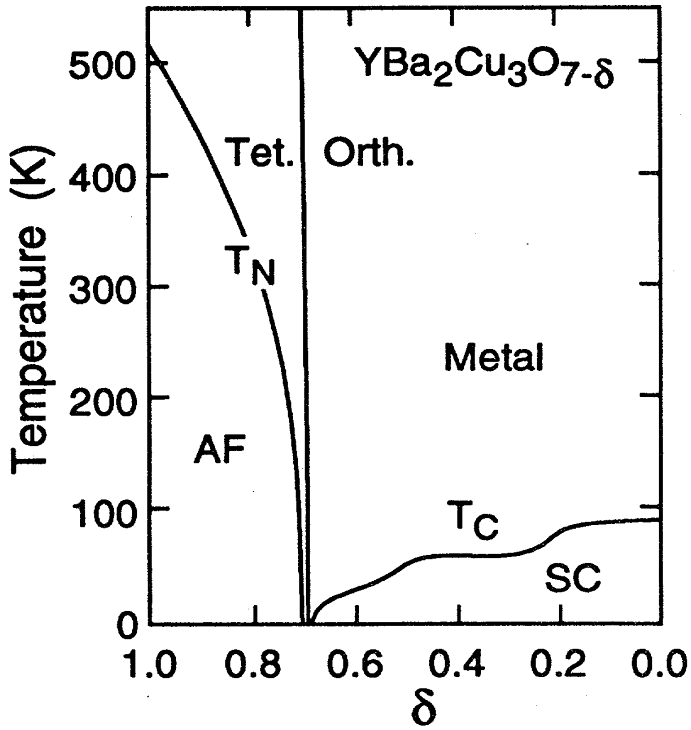
   - ⋆ Pair spectral function

3. Discussion and Summary

Previous phase diagram of YBCO



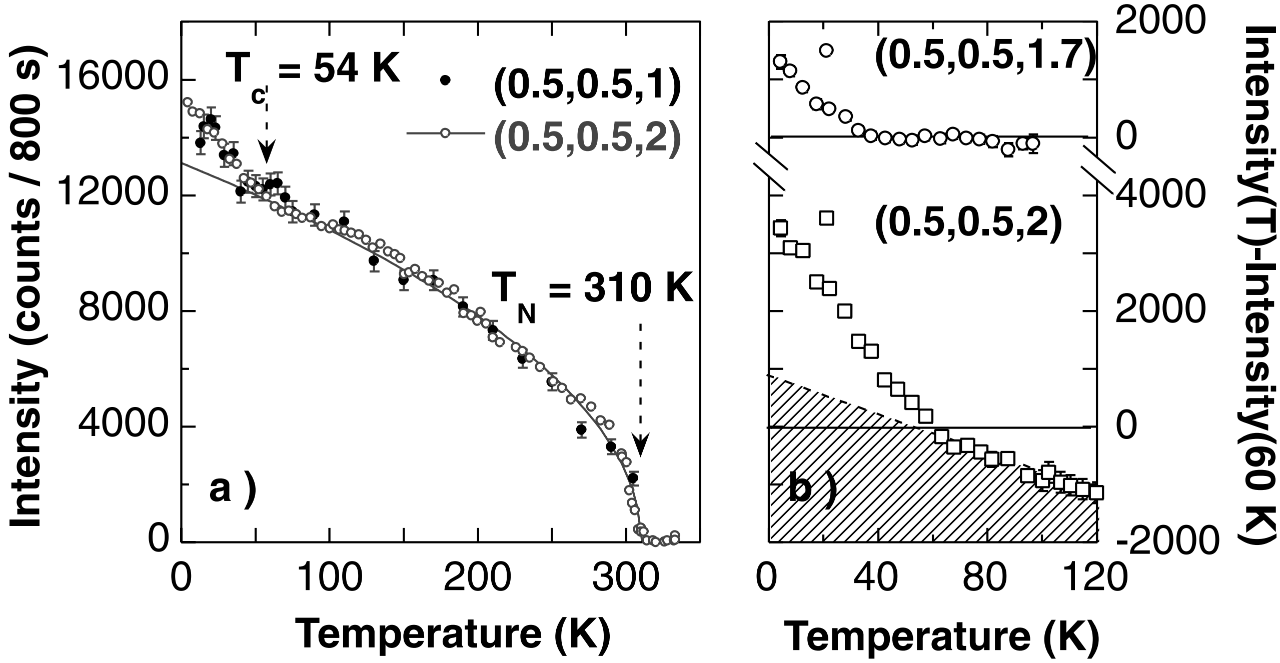
⋆ The AF and SC phases are well separated (also for other high-T_c cuprates).

$$\Downarrow$$

competition?

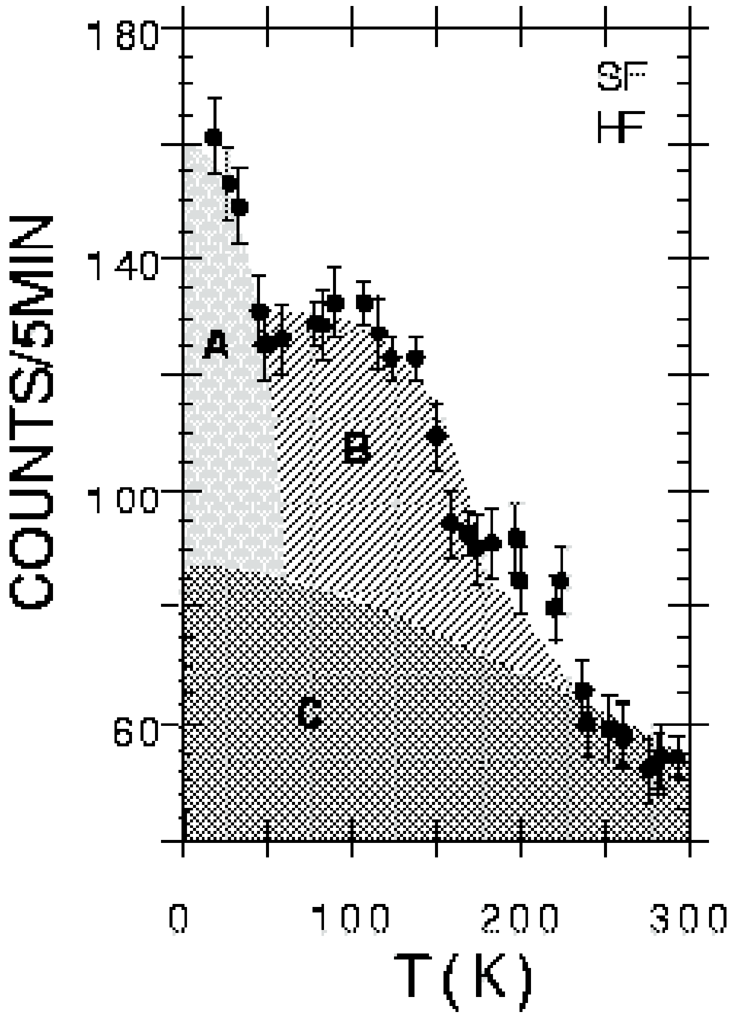
Coexistence of superconductivity and commensurate AF order in YBCO

◇ elastic neutron scattering in $YBa_2Cu_3O_{6.5}$
[Sidis et al. '01]



⋆ The magnetic peak intensity exhibits a marked enhancement at T_c.

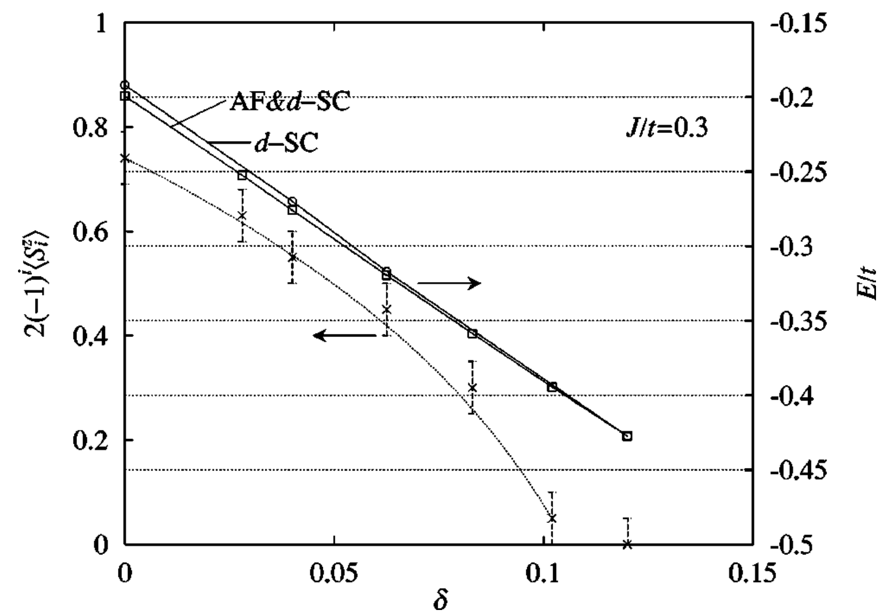
◇ elastic neutron scattering in $YBa_2Cu_3O_{6.6}$
[Mook et al. '01, '02]



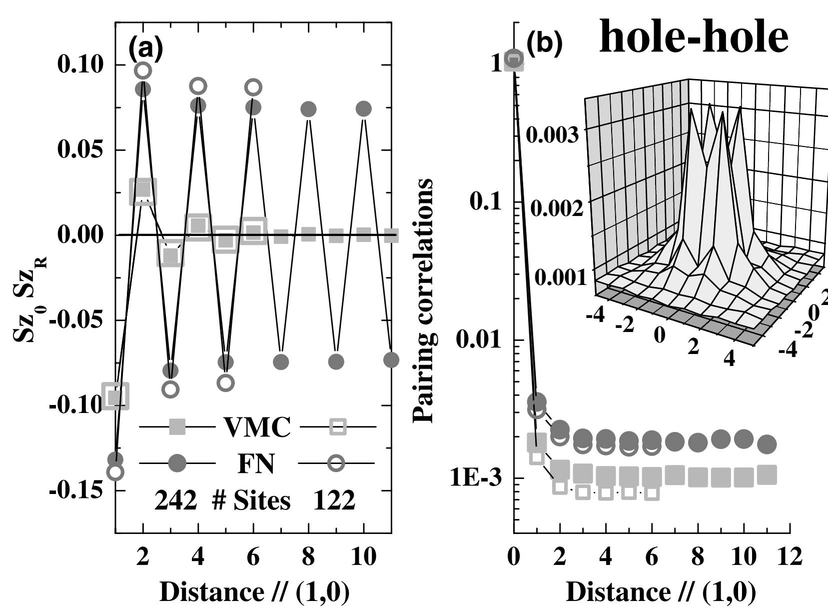
⋆ Searches for magnetic order in $YBa_2Cu_3O_7$ show no signal while a small magnetic intensity is found in $YBa_2Cu_3O_{6.45}$.

Possible coexistence of dSC and AF in the (ordinary) 2D t-J model

(1) variational Monte Carlo method
[Himeda-Ogata '99]



(2) quantum Monte Carlo method [Sorella et al. '02]



Our motivation

$\boxed{\text{Does antiferromagnetism compete or coexist with superconductivity on a square lattice?}}$

$\diamondsuit$ The presence of staggered field is expected to produce (or strengthened) the AF order irrespective of the hole density.

$$\Downarrow$$

How does the superconducting correlation change?

t-J model in a staggered field

$$\begin{aligned}\mathcal{H} = & -t \sum_{\langle ij \rangle \sigma} \left(\tilde{c}^\dagger_{i\sigma} \tilde{c}_{j\sigma} + \mathrm{H.c.} \right) \\ & + J \sum_{\langle ij \rangle} \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j - \frac{1}{4} n_i n_j \right) \\ & - h \sum_{i \in \mathrm{A}} S^z_i + h \sum_{j \in \mathrm{B}} S^z_j\end{aligned}$$

Previous study in the 1D case

[Bonča et al. '92, Prelovšek et al. '93]

(1) binding energy

$$E_{\mathrm{B}} = E_0(N_{\mathrm{h}} = 2) + E_0(N_{\mathrm{h}} = 0) - 2E_0(N_{\mathrm{h}} = 1)$$

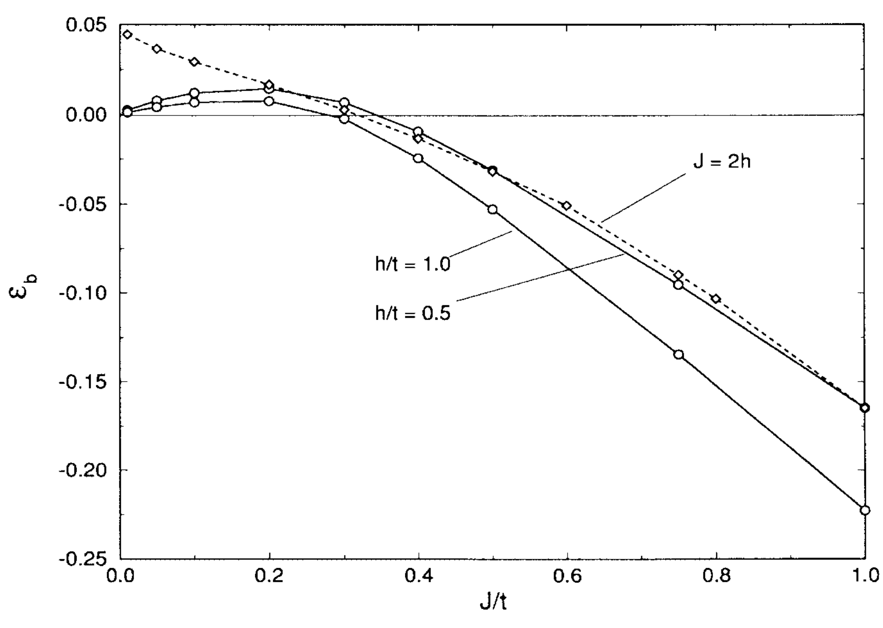
If $E_{\mathrm{B}} < 0$ for $J > J_{\mathrm{c}}$, a bound hole-pair is formed.

♣ t-J_z-h model (analytic), $J = J_z$

$$h \ll t: \; E_{\mathrm{B}} = 0.909(th^2)^{1/3} - J$$
$$J_{\mathrm{c}} = 0.909(th^2)^{1/3}$$
$$h \gg t: \; E_{\mathrm{B}} = -\frac{J}{2} + \frac{2(2h-J)t^4}{(h+J/2)^3(2h+J/2)}$$
$$J_{\mathrm{c}}/t \sim 4(t/h)^3 \ll 1$$

cf. $h = 0$: Binding occurs only for $J > J_{\mathrm{c}} = 4t$.

♣ t-J-h model (numerical)



(2) correlation exponent at finite hole doping

charge-density-wave correlations:

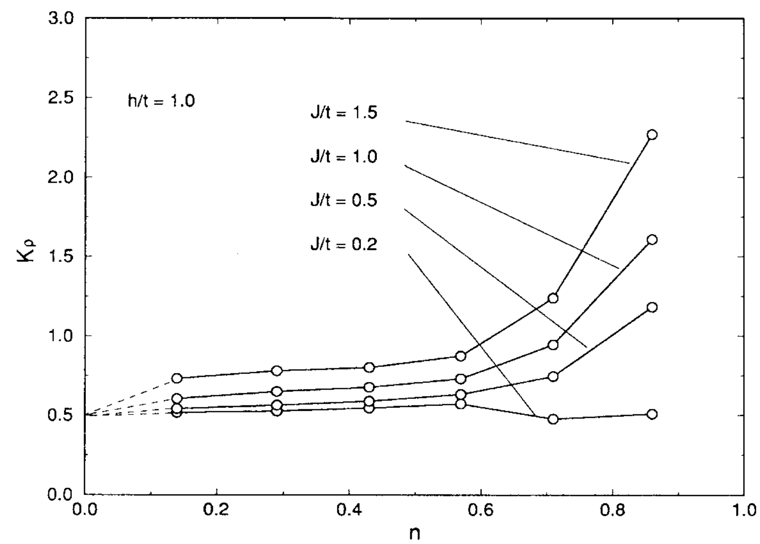
$$C_{\rm CDW}(r) \sim r^{-K_\rho} \text{ for } r \gg 1$$

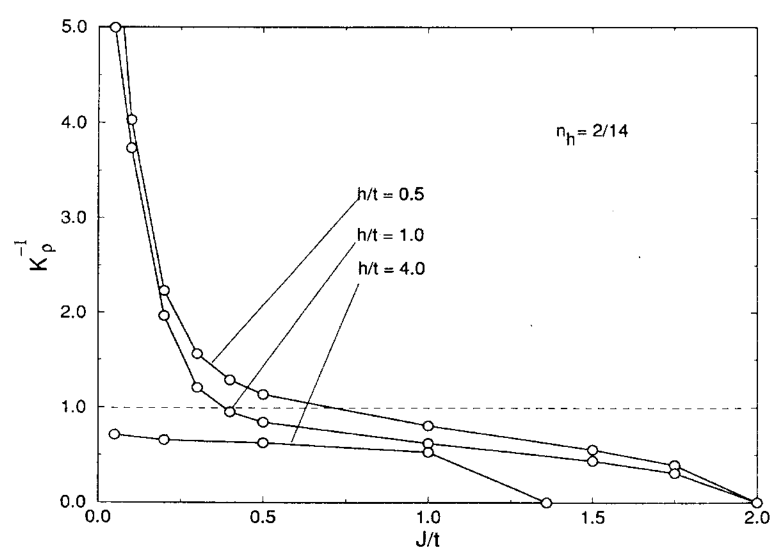
superconducting correlations:

$$C_{\rm SC}(r) \sim r^{-1/K_\rho} \text{ for } r \gg 1$$

⋆ For $K_\rho < 1$, CDW is dominant.

⋆ For $K_\rho > 1$, SC is dominant.





Extension to the 2D case

$\star$ We use exact-diagonalization technique
to investigate the following quantities at $T = 0$.
[system size: $N = 4 \times 4, \sqrt{18} \times \sqrt{18}, \sqrt{20} \times \sqrt{20}$]

(1) Binding energy

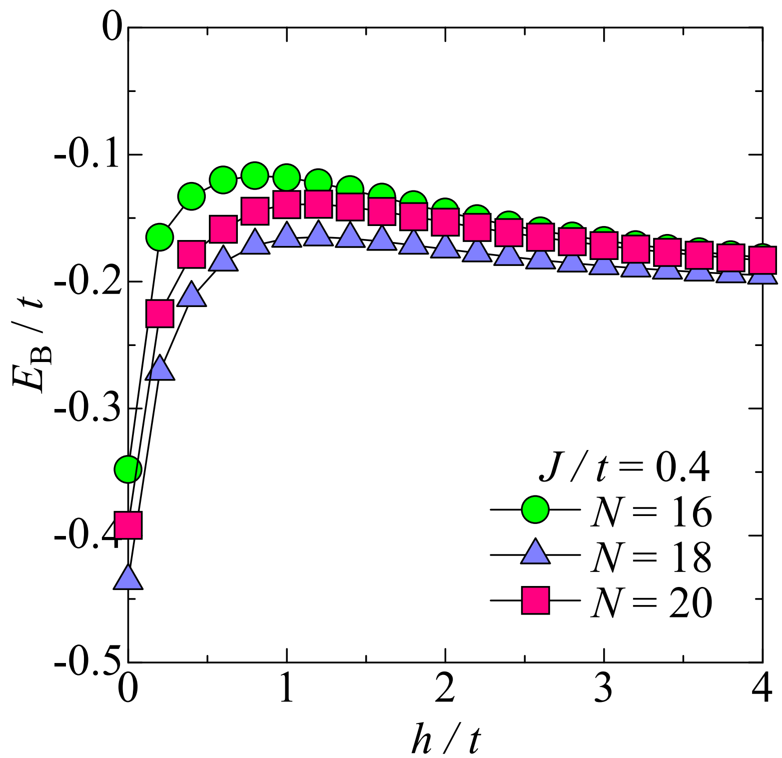
(2) Hole correlations

(3) Staggered-spin correlations

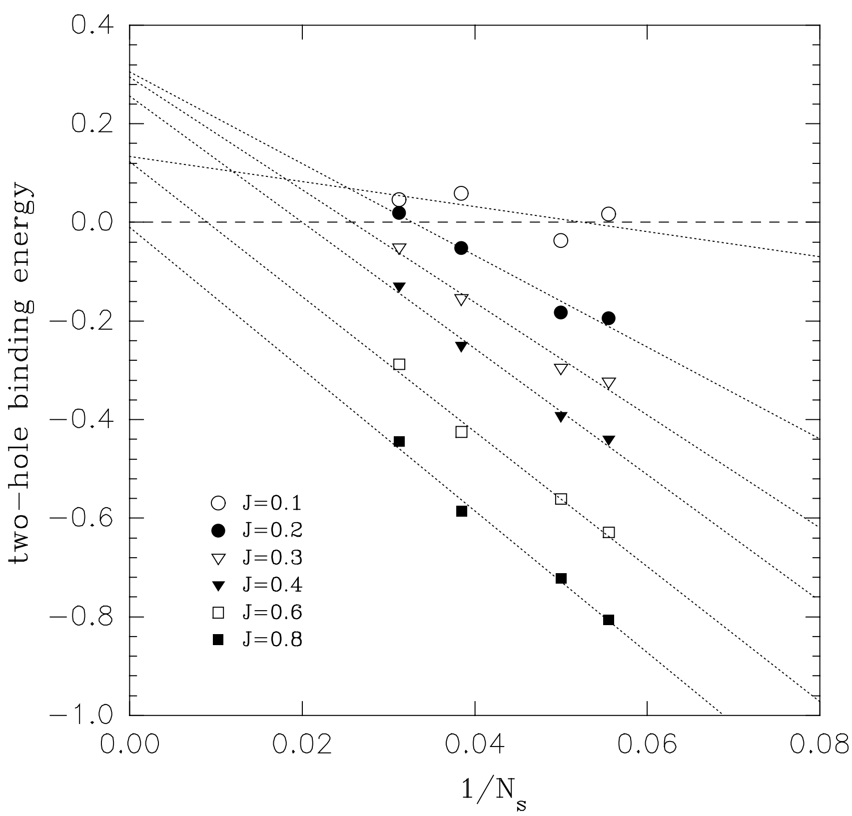
(4) Superconducting correlations

(5) Pair spectral function

Binding energy

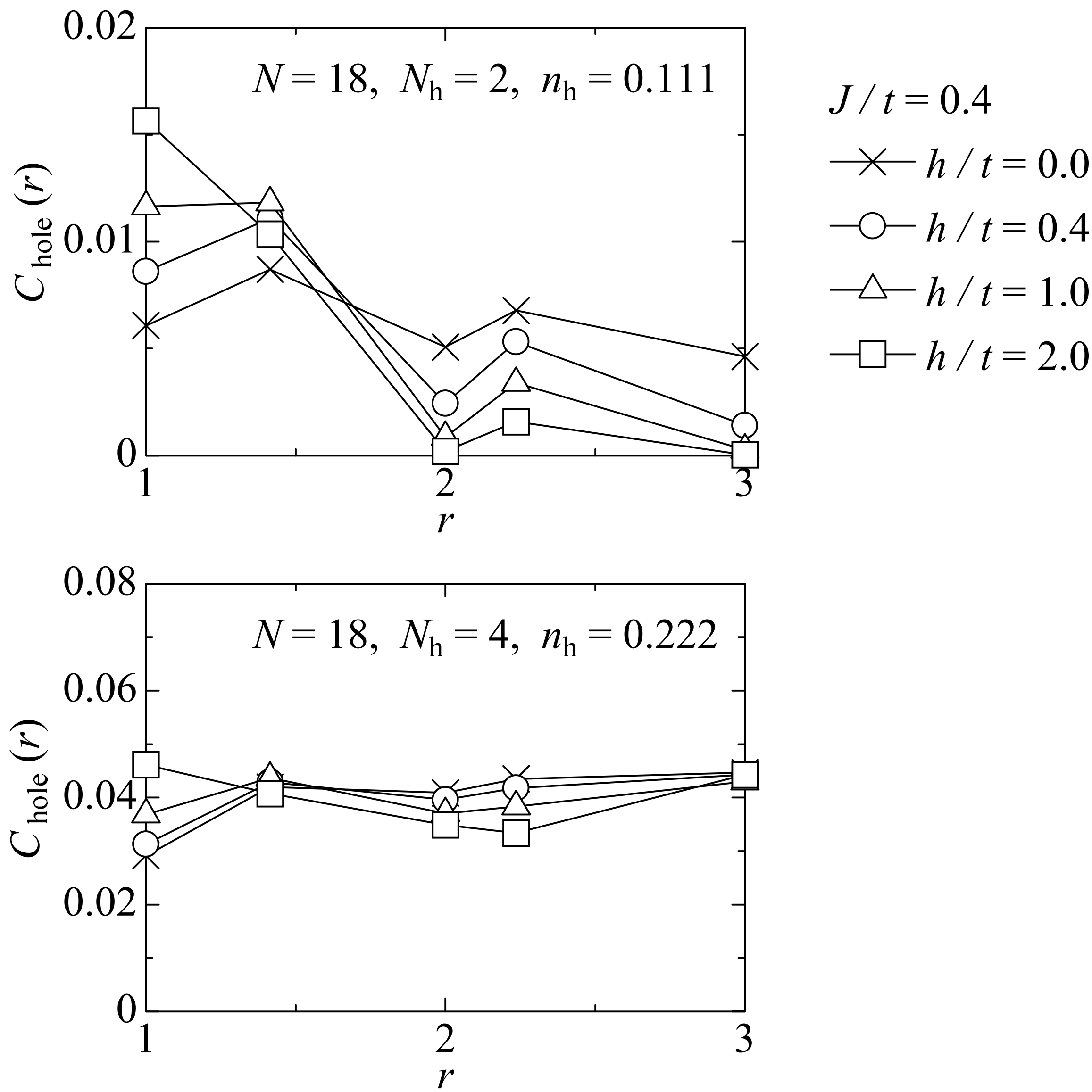


◇ size dependence for $h/t = 0$ [Shih et al. '98]



Hole correlations $(J/t = 0.4)$

$$C_{\mathrm{hole}}(\vec{r}) = \frac{1}{N} \sum \langle n_{\mathrm{h}}(\vec{i}) n_{\mathrm{h}}(\vec{i} + \vec{r}) \rangle$$

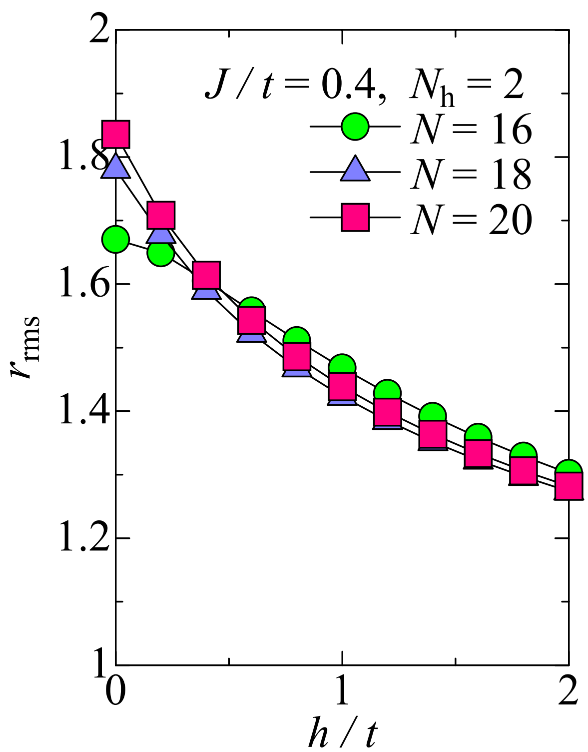


$\diamondsuit$ root-mean-square separation of hole pair

$$r_{\rm rms} \equiv \sqrt{\langle r^2 \rangle}$$

$$\langle r^2 \rangle = \sum_{\vec{r}(\neq \vec{0})} |\vec{r'}|^2 C_{\rm hole}(\vec{r}) \Bigg/ \sum_{\vec{r}(\neq \vec{0})} C_{\rm hole}(\vec{r})$$

$|\vec{r'}|$: substantial distance between two sites under p.b.c.



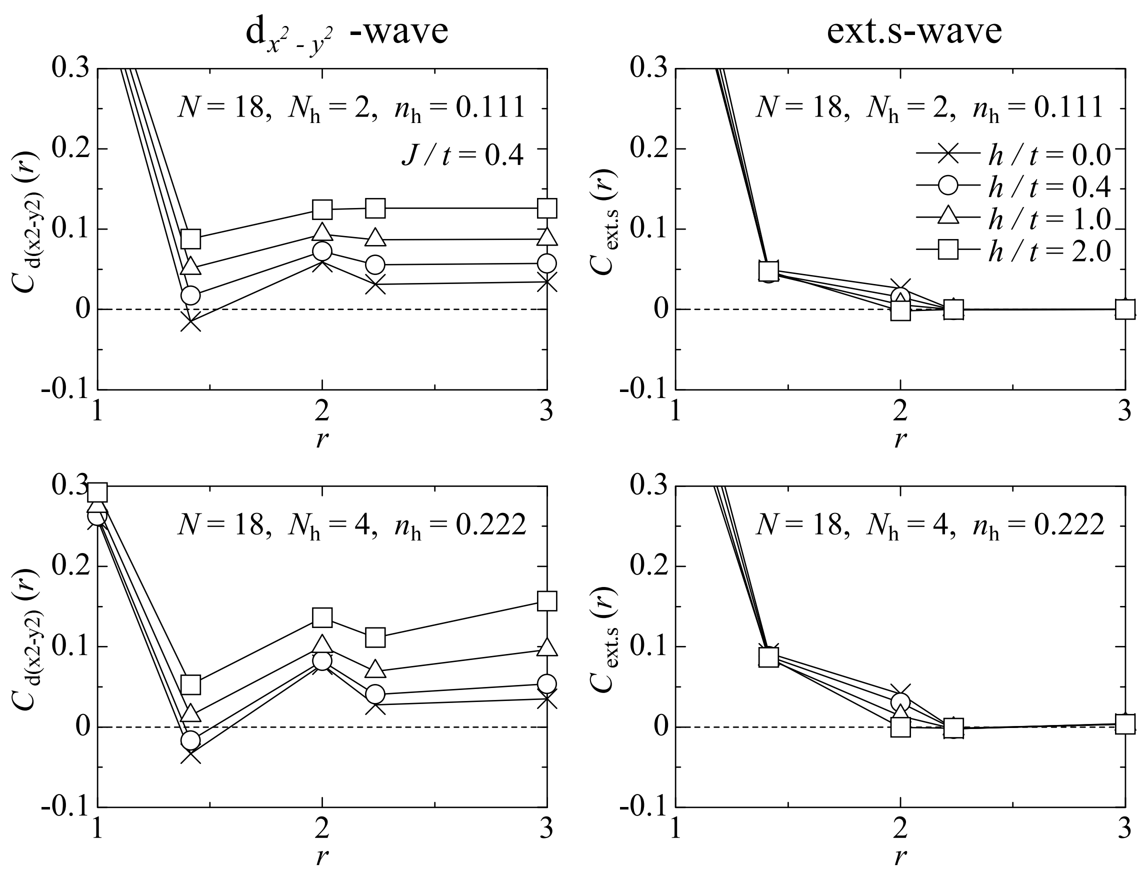
$\star$ The separation of hole pair becomes smaller with increasing staggered field.

Superconducting correlations

$$C_\alpha(\vec{r}) = \frac{1}{N}\sum_{\vec{i}}\langle\Delta_\alpha^\dagger(\vec{i})\Delta_\alpha(\vec{i}+\vec{r})\rangle, \quad \alpha = \mathrm{ext.s}, \mathrm{d}_{x^2-y^2}$$

$$\Delta_\alpha(\vec{i}) = \frac{1}{\sqrt{2}}\sum_{\vec{\epsilon}} f_\alpha(\vec{\epsilon})\left(c_{\vec{i}\uparrow}c_{\vec{i}+\vec{\epsilon},\downarrow} - c_{\vec{i}\downarrow}c_{\vec{i}+\vec{\epsilon},\uparrow}\right)$$

where $\vec{\epsilon}$ is $(\pm1, 0)$ and $(0, \pm1)$.



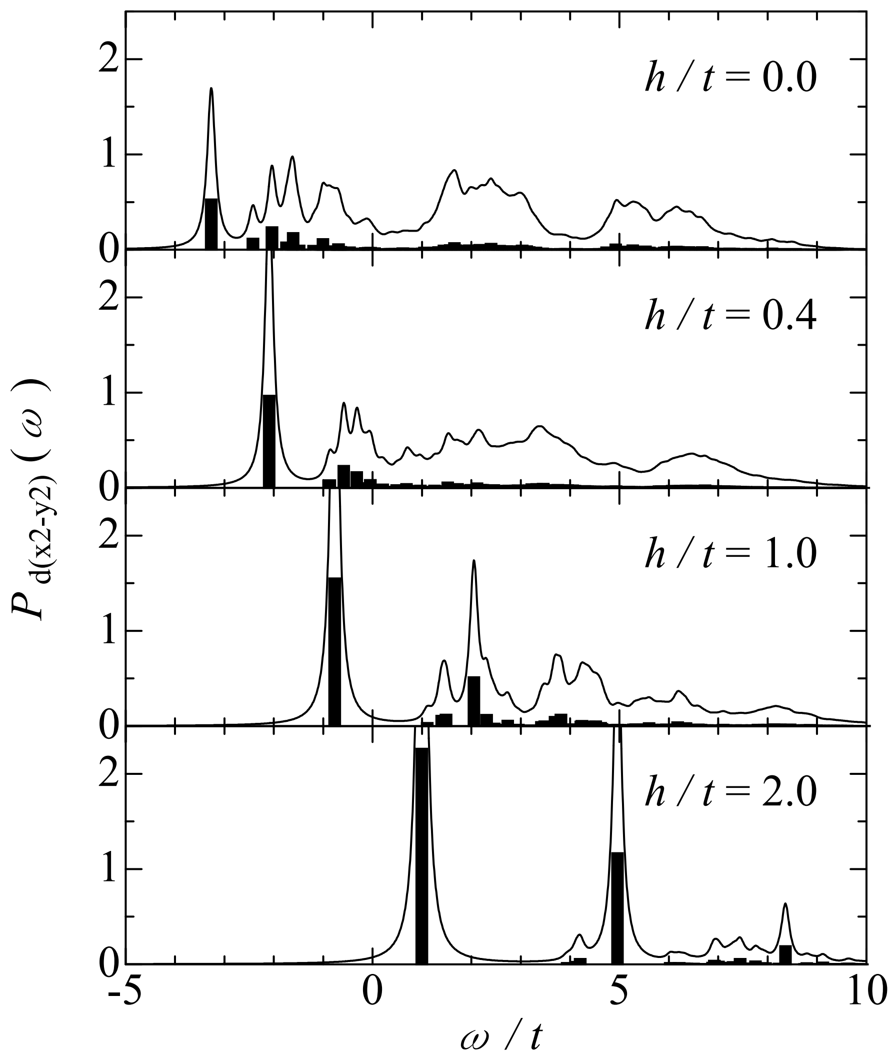
$\star$ For $n_h \lesssim 0.2$, the $\mathrm{d}_{x^2-y^2}$-wave correlations are enhanced by a staggered field.

$\star$ The extended-s-wave ones hardly change at long distances.

Pair spectral function

$$P_\alpha(\omega) = \sum_n |\langle \Psi_n(N_\mathrm{h}=2)|\Delta|\Psi_0(N_\mathrm{h}=0)\rangle|^2$$
$$\times \delta\left(\omega - E_n(N_\mathrm{h}=2) + E_0(N_\mathrm{h}=0)\right)$$

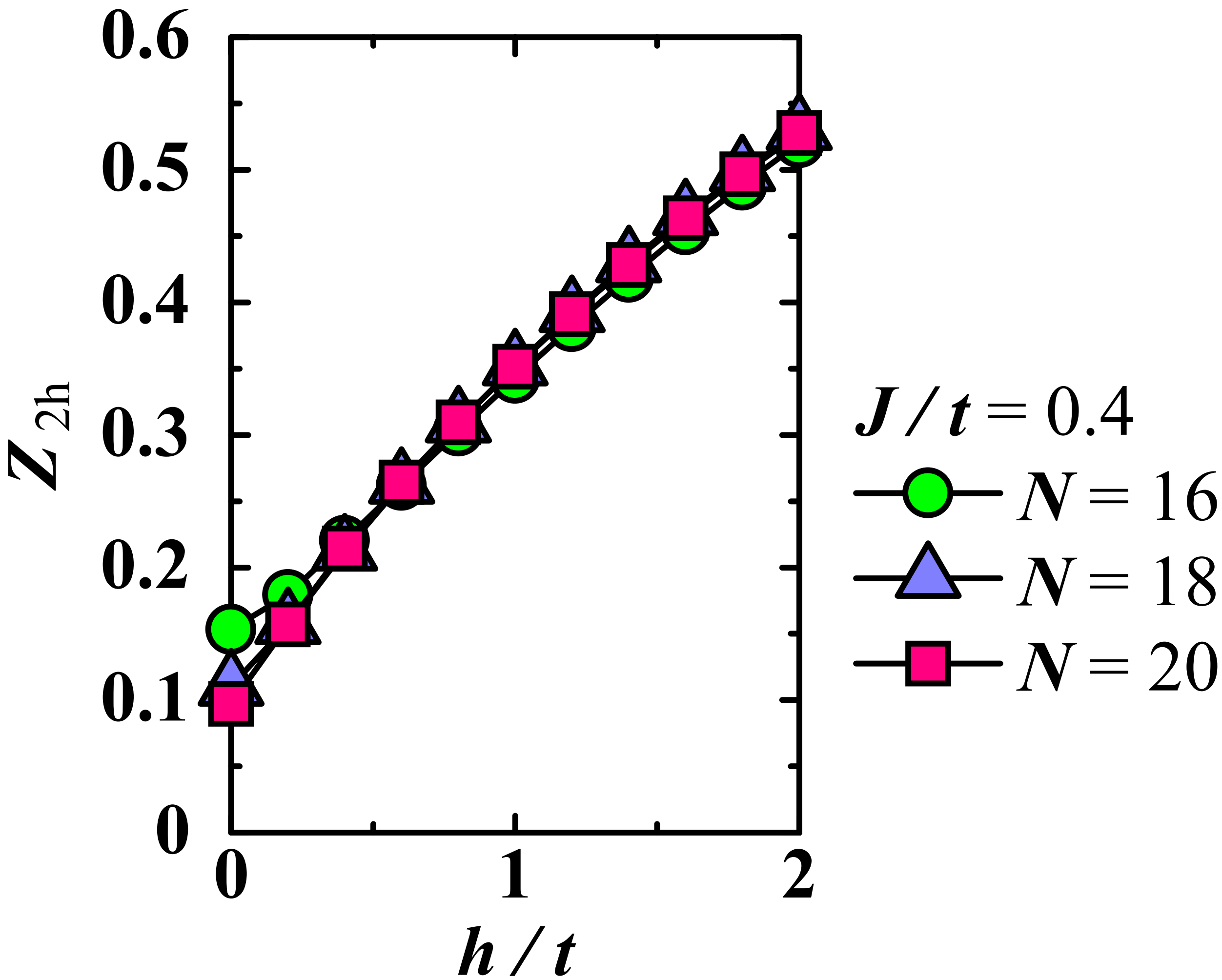
$$\Delta = \frac{1}{\sqrt{N}} \sum_{\vec{i},\vec{\epsilon}} f_\alpha(\vec{\epsilon}) \left(\tilde{c}_{\vec{i}\uparrow} \tilde{c}_{\vec{i}+\vec{\epsilon},\downarrow} - \tilde{c}_{\vec{i}\downarrow} \tilde{c}_{\vec{i}+\vec{\epsilon},\uparrow} \right)$$



$N = 18$, $J/t = 0.4$
$N_\mathrm{it} = 500$, Lorentzian width = 0.1

Spectral weight of "quasi-particle"

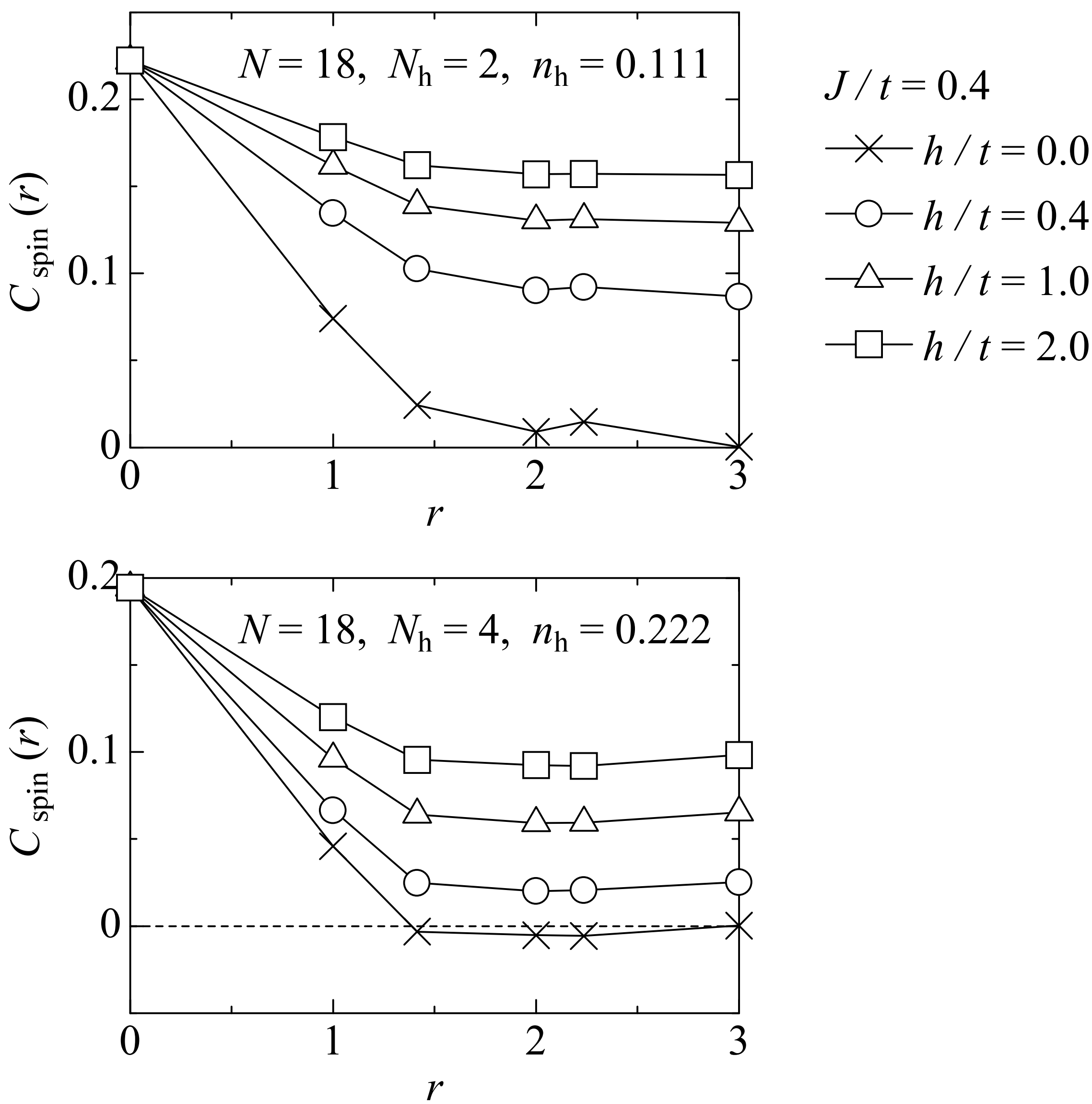
$$Z_{2\mathrm{h}} = \frac{|\langle \Psi_0(N_\mathrm{h}=2)|\Delta|\Psi_0(N_\mathrm{h}=0)\rangle|^2}{\langle \Psi_0(N_\mathrm{h}=0)|\Delta^\dagger\Delta|\Psi_0(N_\mathrm{h}=0)\rangle}$$



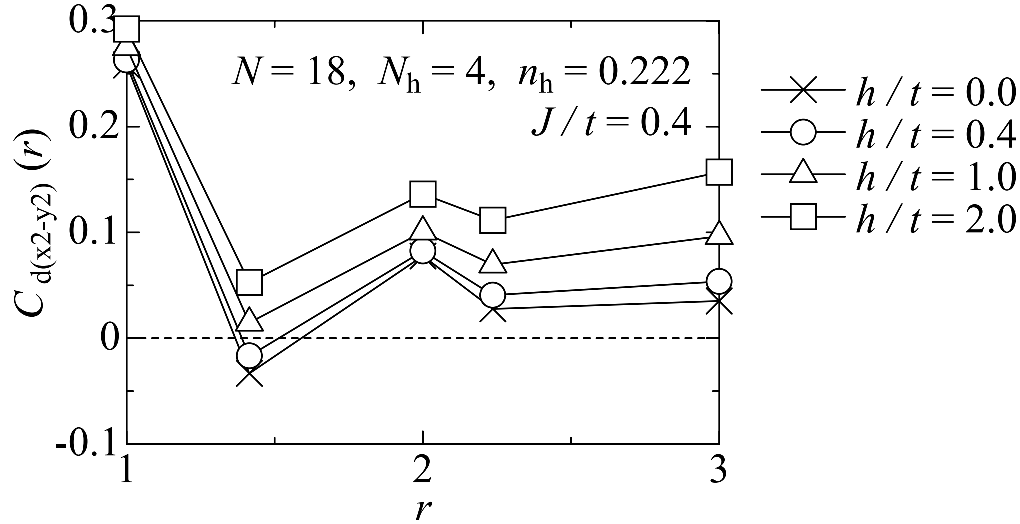
⋆ The "quasi-particle" peak seems to survive in the thermodynamic limit, especially for large h/t.

Staggered-spin correlations $(J/t = 0.4)$

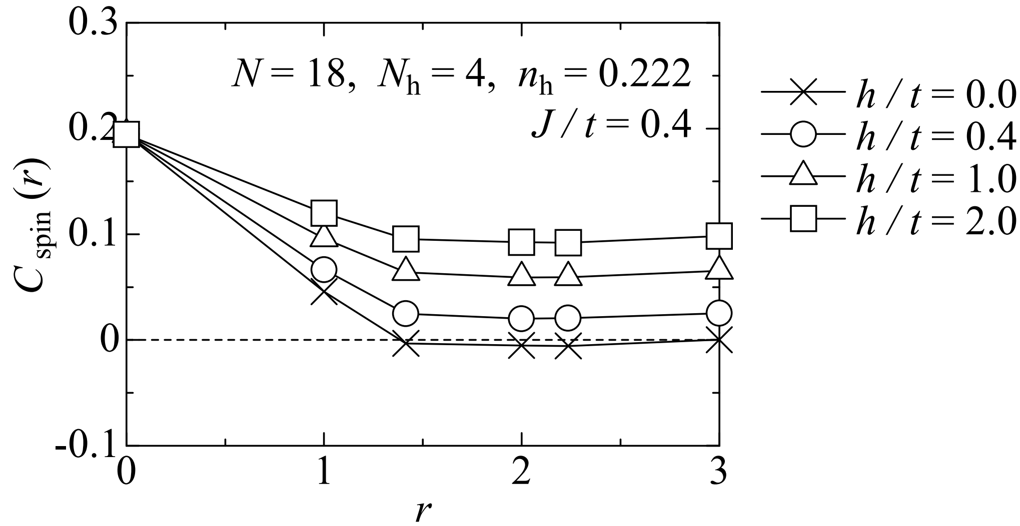
$$C_{\rm spin}(\vec{r}) = \frac{1}{N}\sum_{\vec{i}} (-1)^{r_x + r_y} \langle S^z_{\vec{i}} S^z_{\vec{i}+\vec{r}} \rangle$$



d$_{x^2-y^2}$-wave superconducting correlations



staggered-spin correlations



$\star$ Possibility of coexistence of the d$_{x^2-y^2}$ superconducting order and the commensurate AF order induced by a staggered field.

Summary

For the low-hole-density region ($n_{\mathrm{h}} \lesssim 0.2$),

(1) The presence of staggered field

– **strengthens** the attraction between two holes.

– **helps** the $\mathrm{d}_{x^2-y^2}$-wave superconductivity.

(2) Coexistence of the $\mathrm{d}_{x^2-y^2}$-wave superconducting order and the commensurate AF order is likely to occur in a staggered field.