Doppler Detection of Planets:
Precise Radial Velocity Measurements

***Period04*: multi-sine fitting with Fourier analysis. Tutorials available plus versions in Mac OS, Windows, and Linux**

http://www.univie.ac.at/tops/Period04/

Generalized Lomb-Scargle Periodogram:

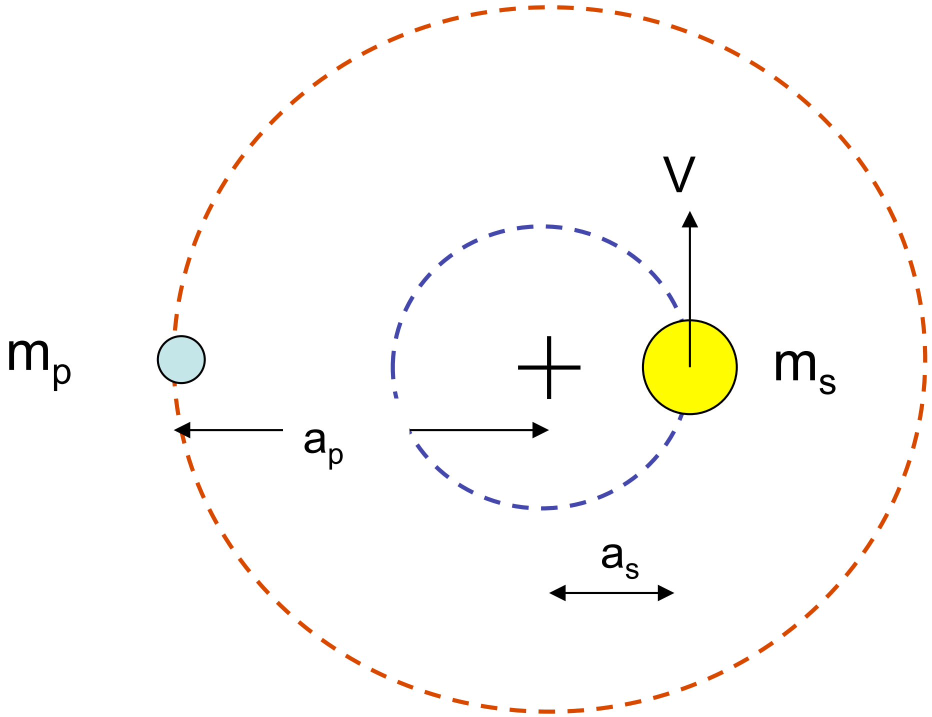
http://www.astro.physik.uni-goettingen.de/~zechmeister/

Orbit fitting with a GUI interface includes periodograms

http://www.stefanom.org/systemic/

Web-based version but only for known systems

Newton's form of Kepler's Law



$$P^2 = \frac{4\pi \quad (a_s + a_p)^3}{G(m_s + m_p)}$$

$$P^2 = \frac{4\pi^2 \quad (a_s + a_p)^3}{G(m_s + m_p)}$$

Approximations: a_p » a_s m_s » m_p

Circular orbits: $$V = \frac{2\pi a_s}{P}$$

Conservation of momentum: $m_s \times a_s = m_p \times a_p$

$$\boxed{V_{obs} = \frac{28.4 \quad m_p \sin i}{P^{1/3} m_s^{2/3}}}$$

m_p in Jupiter masses

m_s in solar masses

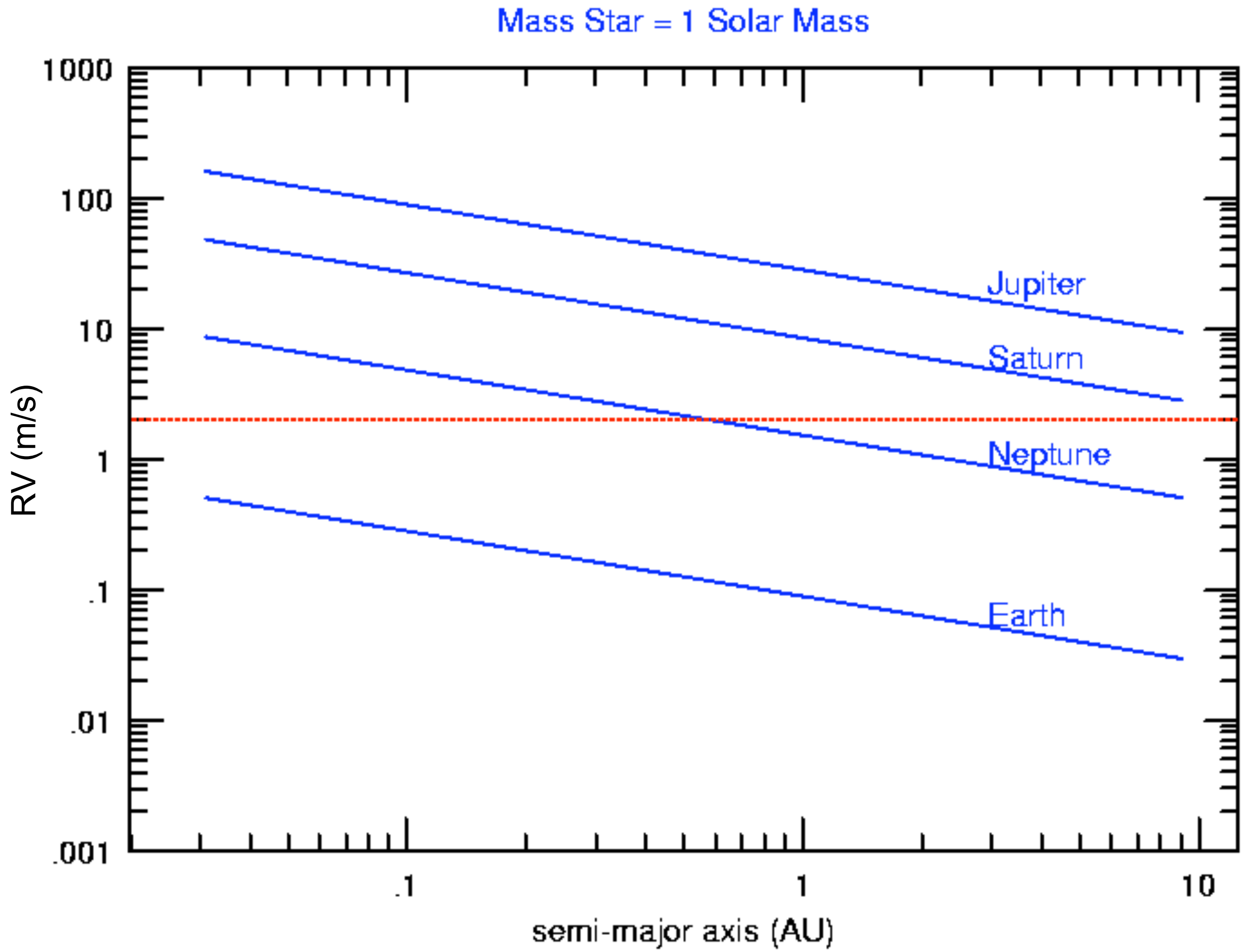
P in years

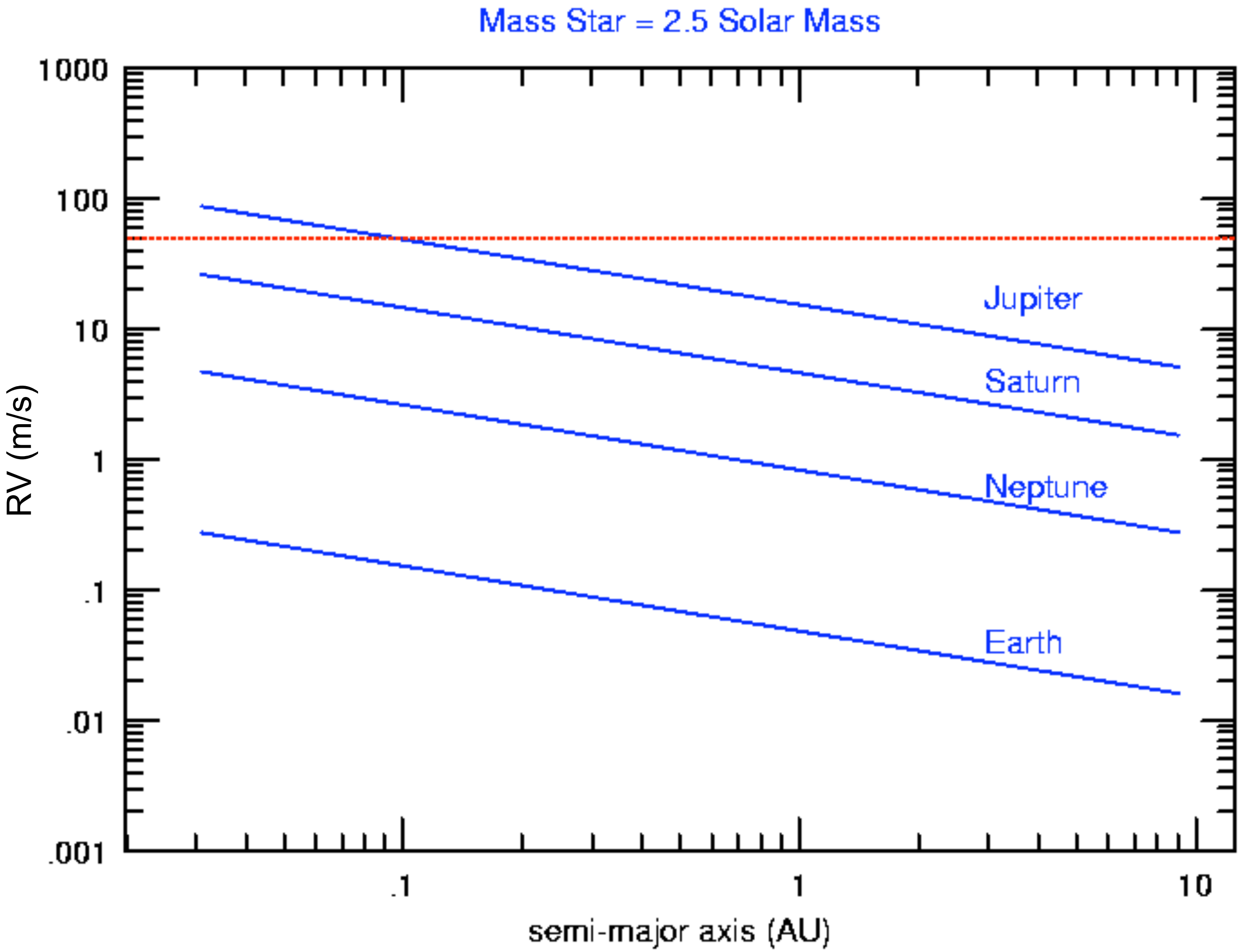
V in m/s

Radial Velocity Amplitude of Planets in the Solar System

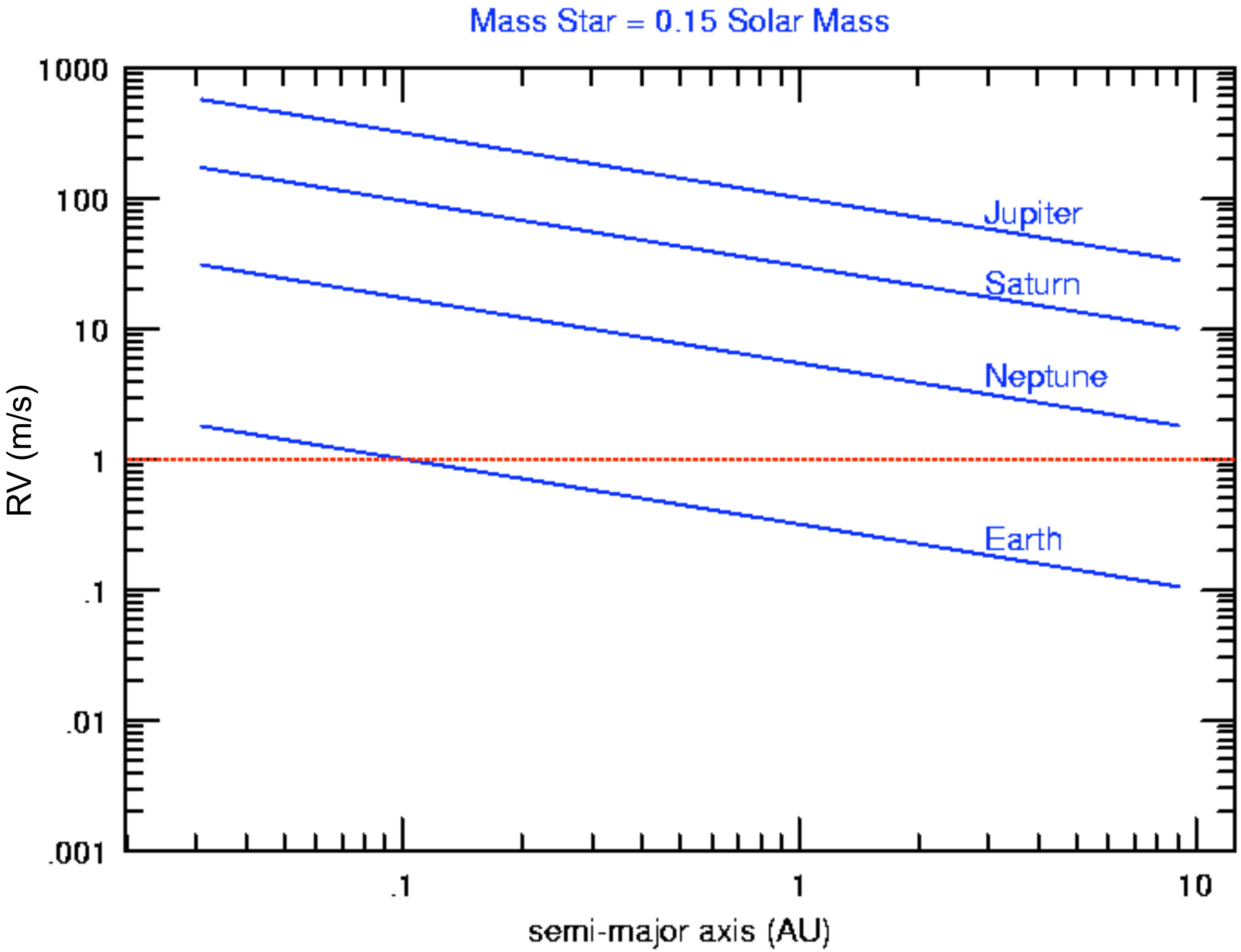
Planet	Mass (M_J)	V(m s^{-1})
Mercury	1.74×10^{-4}	0.008
Venus	2.56×10^{-3}	0.086
Earth	3.15×10^{-3}	0.089
Mars	3.38×10^{-4}	0.008
Jupiter	1.0	12.4
Saturn	0.299	2.75
Uranus	0.046	0.297
Neptune	0.054	0.281
Pluto	1.74×10^{-4}	3×10^{-5}

Radial Velocity Amplitude of Planets at Different *a*





Mass Star = 2.5 Solar Mass
1000
100
10
1
.1
.01
.001
RV (m/s)
Jupiter
Saturn
Neptune
Earth
.1
1
10
semi-major axis (AU)

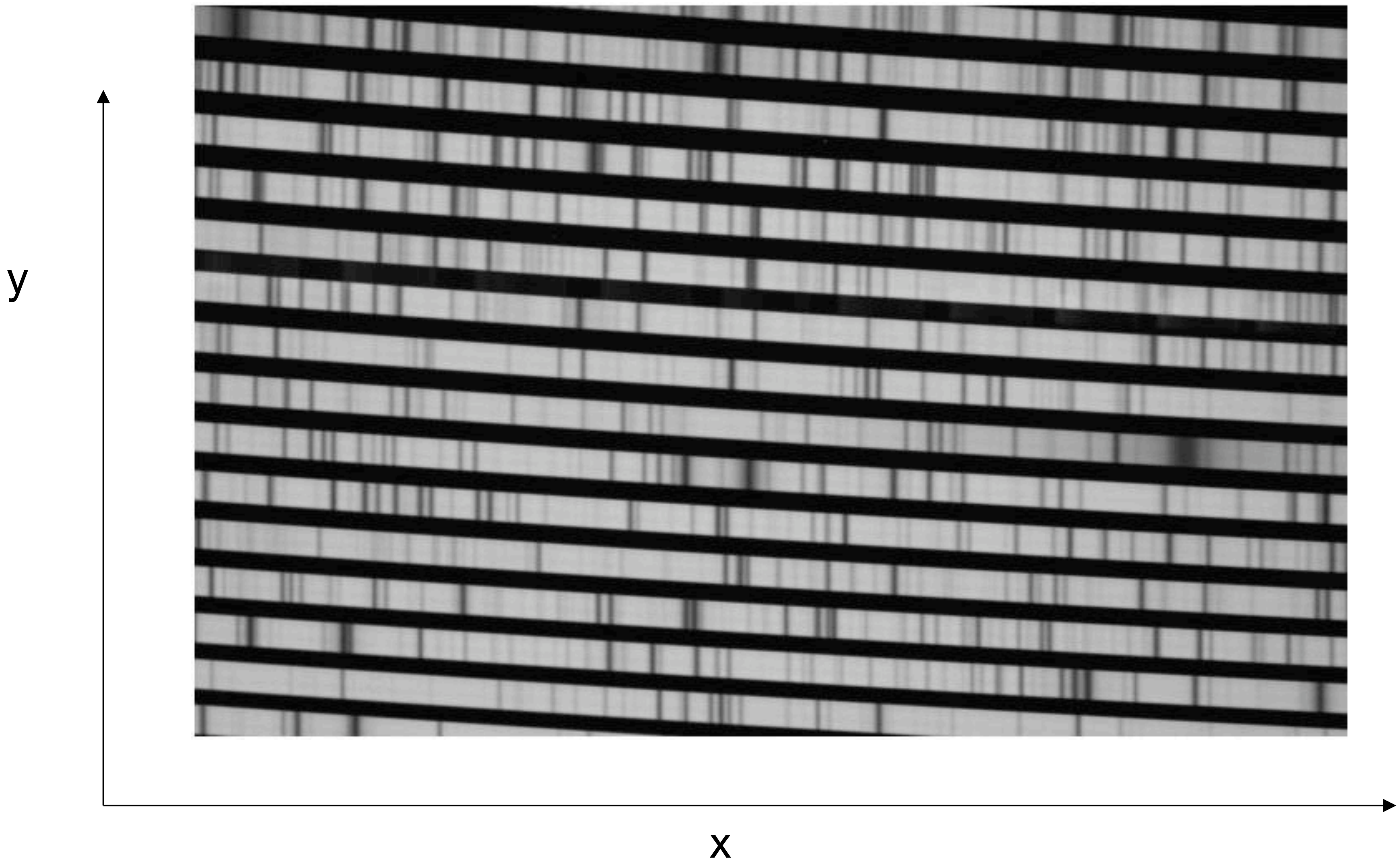

Mass Star = 0.15 Solar Mass
1000
100
10
1
.1
.01
.001
RV (m/s)
.1
1
10
semi-major axis (AU)
Jupiter
Saturn
Neptune
Earth

Measurement of Doppler Shifts

In the non-relativistic case:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta v}{c}$$

We measure Δv by measuring $\Delta\lambda$



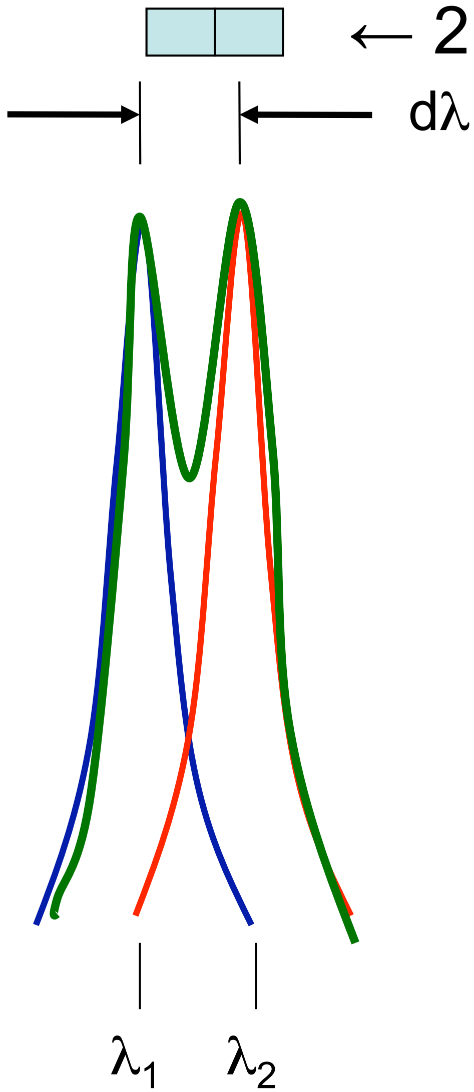
On a detector we only measure x- and y- positions, there is no information about wavelength. For this we need a calibration source

CCD detectors only give you x- and y- position. A doppler shift of spectral lines will appear as Δx

$$\Delta x \rightarrow \Delta\lambda \rightarrow \Delta v$$

How large is Δx ?

Spectral Resolution



← 2 detector pixels

Consider two monochromatic beams

They will just be resolved when they have a wavelength separation of $d\lambda$

Resolving power:

$$\boxed{R = \frac{\lambda}{d\lambda}}$$

$d\lambda$ = full width of half maximum of calibration lamp emission lines

$R = 50.000 \rightarrow \Delta\lambda = 0.11$ Angstroms

$\rightarrow$ 0.055 Angstroms / pixel (2 pixel sampling) @ 5500 Ang.

1 pixel typically 15 μm

$$v = \frac{\Delta\lambda\, c}{\lambda}$$

1 pixel = 0.055 Ang $\rightarrow$ 0.055 x $(3 \cdot 10^8$ m/s)/5500 Ang $\rightarrow$

= 3000 m/s per pixel

Δv = 10 m/s = 1/300 pixel = 0.05 μm = 5 x 10^{-6} cm

Δv = 1 m/s = 1/1000 pixel $\rightarrow$ 5 x 10^{-7} cm = 50 Å

			For Δv = 20 m/s	
R	Ang/pixel	Velocity per pixel (m/s)	Δpixel	Shift in mm
500 000	0.005	300	0.06	0.001
200 000	0.125	750	0.027	4×10^{-4}
100 000	0.025	1500	0.0133	2×10^{-4}
50 000	0.050	3000	0.0067	10^{-4}
25 000	0.10	6000	0.033	5×10^{-5}
10 000	0.25	15000	0.00133	2×10^{-5}
5 000	0.5	30000	6.6×10^{-4}	10^{-5}
1 000	2.5	150000	1.3×10^{-4}	2×10^{-6}

So, one should use high resolution spectrographs….up to a point

How does the RV precision depend on the properties of your spectrograph?

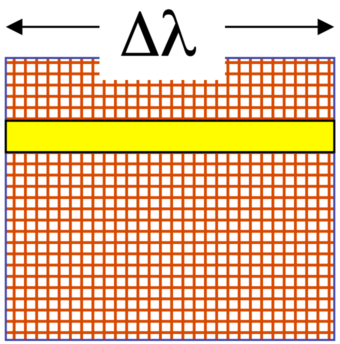
Wavelength coverage:

- Each spectral line gives a measurement of the Doppler shift
- The more lines, the more accurate the measurement:

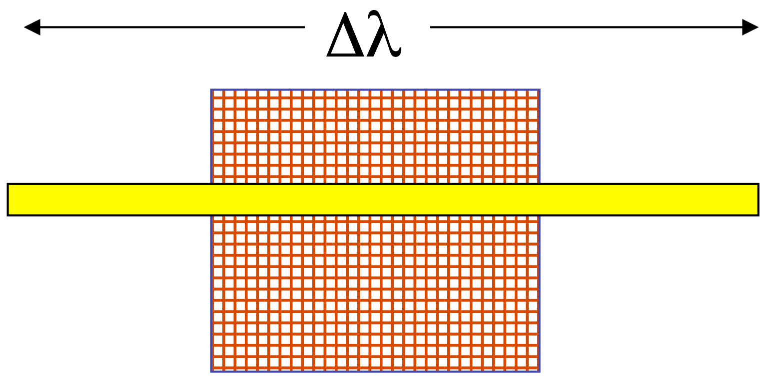
$\sigma_{Nlines} = \sigma_{1line}/\sqrt{N_{lines}}$ → Need broad wavelength coverage

Wavelength coverage is inversely proportional to R:

Low resolution

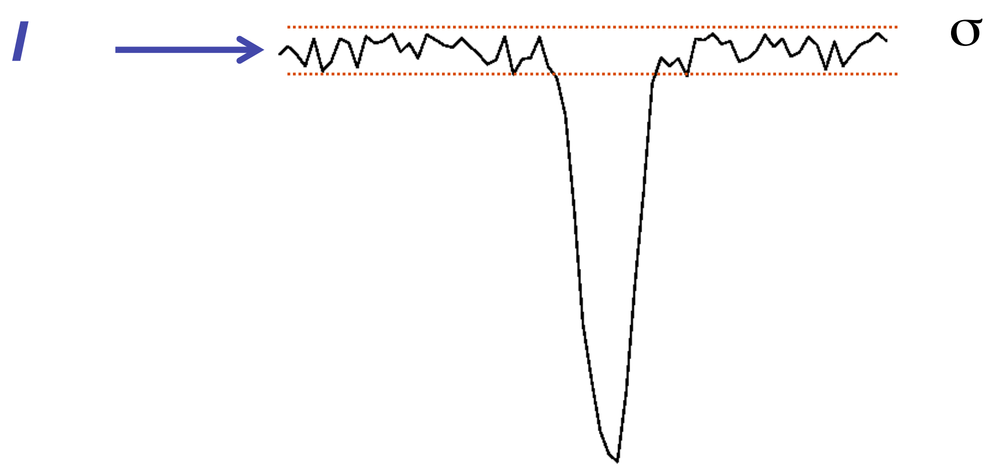


High resolution



detector

Noise:

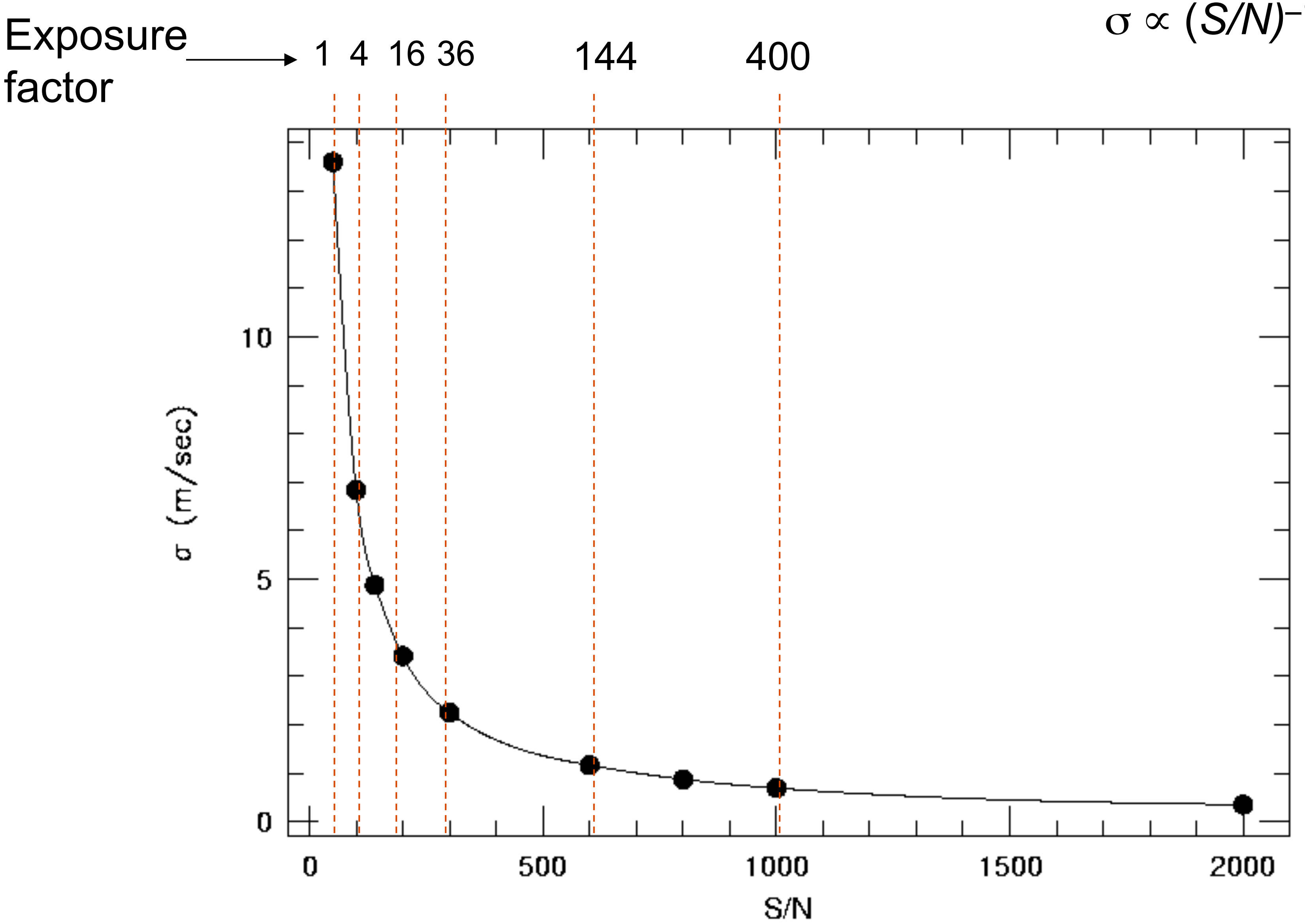


I = detected photons

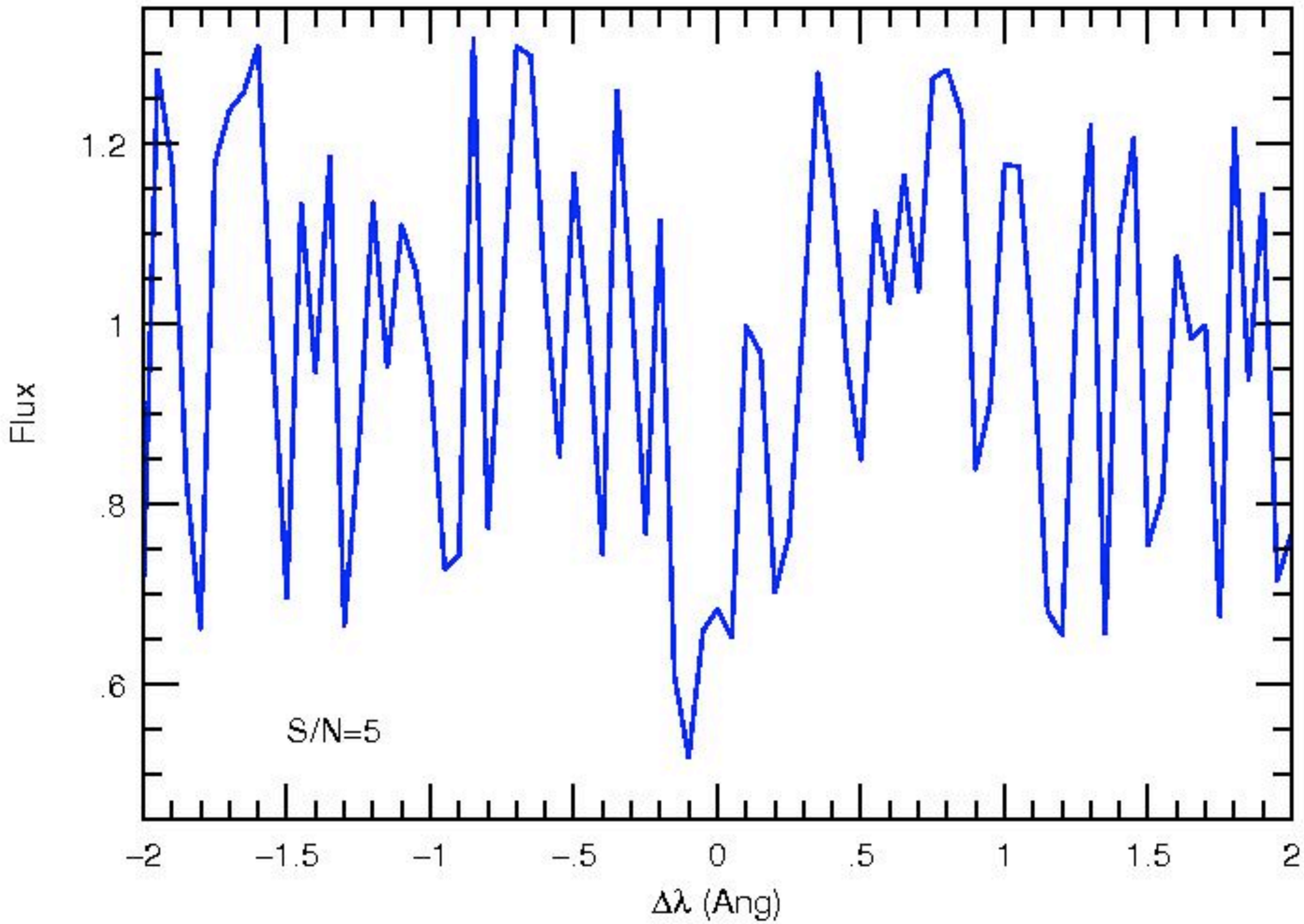
Signal to noise ratio S/N = I/σ

For photon statistics: $\sigma = \sqrt{I} \rightarrow S/N = \sqrt{I}$

$\sigma \propto (S/N)^{-1}$



Price: $S/N \propto t^2_{exposure}$



Note: Photographic plates: S/N = 10-50
CCD Detectors: S/N = 100-500

How does the radial velocity precision depend on all parameters?

$$\sigma \text{ (m/s)} = \text{Constant} \times (S/N)^{-1} R^{-3/2} (\Delta\lambda)^{-1/2}$$

σ: error

R: spectral resolving power

S/N: signal to noise ratio

$\Delta\lambda$: wavelength coverage of spectrograph in Angstroms

For R=110.000, *S/N*=150, $\Delta\lambda$=2000 Å, σ = 2 m/s

$C \approx 2.4 \times 10^{11}$

Predicted radial velocity performance of spectrographs

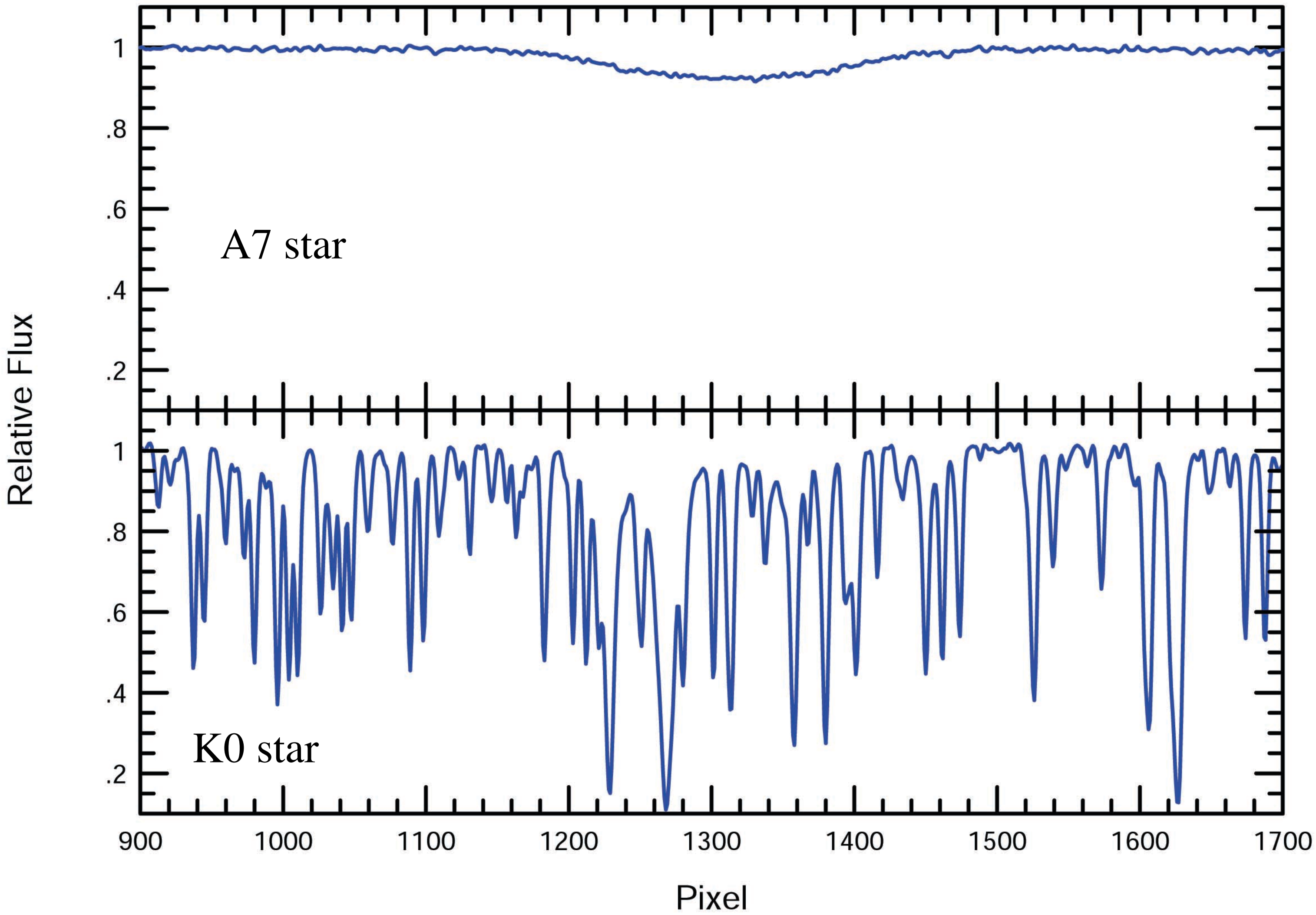
Spectrograph	$\Delta\lambda$ (Ang.)	R	σ_{pred} (m/s)	σ_{actual} (m/s)
McDonald 6ft	9	200 000	8	10
McDonald cs23	1000	60 000	3	3
McDonald cs21	800	180 000	2	4
McDonald CE	800	50 000	5	10
TLS coude	1000	67 000	3	3
ESO CES	43	100 000	8	10
Keck HiRes	1000	80 000	3	3
HARPS	2000	100 000	1	2

The Radial Velocity precision depends not only on the properties of the spectrograph but also on the properties of the star.

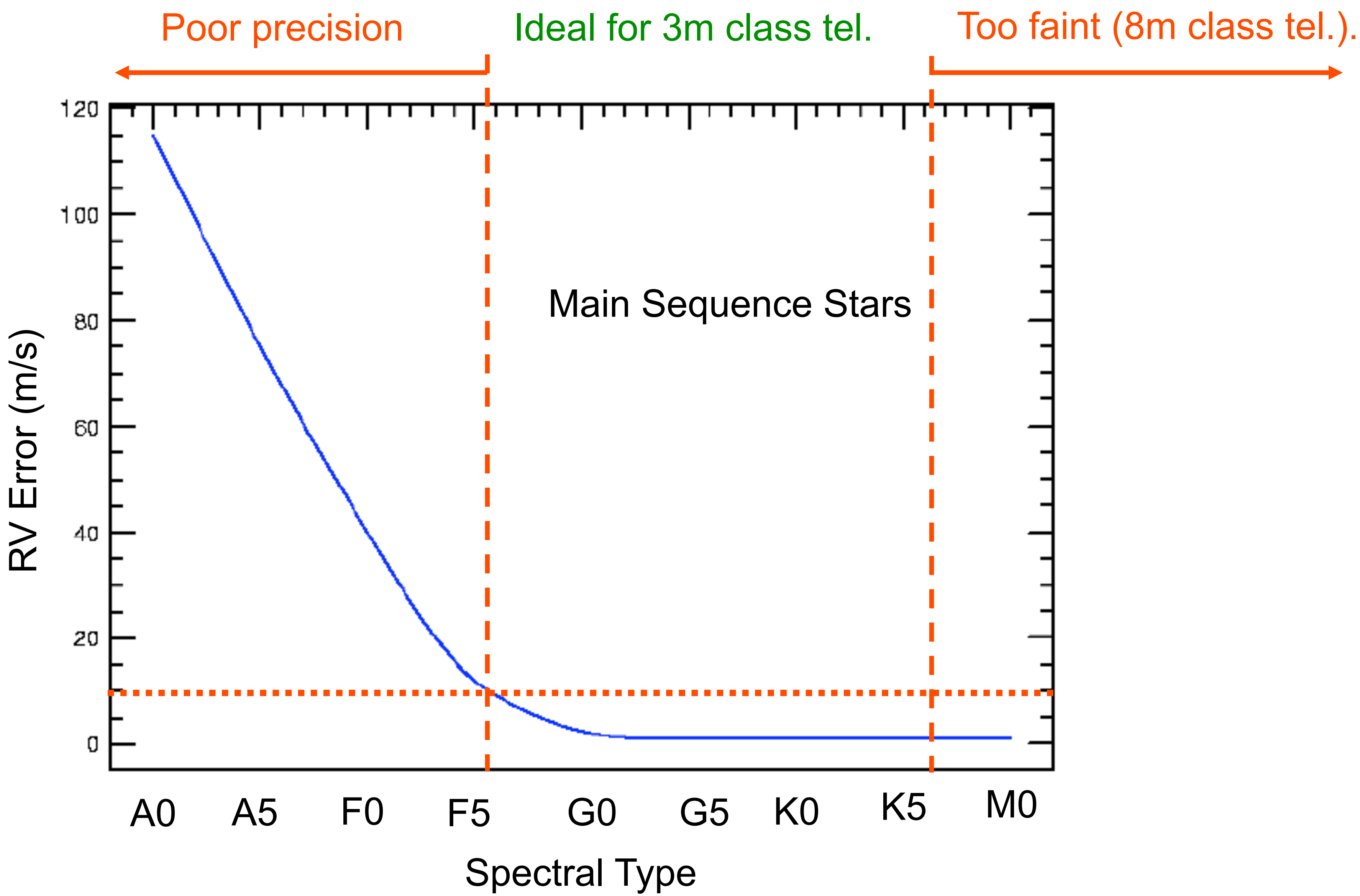
Good RV precision → cool stars of spectral type later than F6

Poor RV precision → cool stars of spectral type earlier than F6

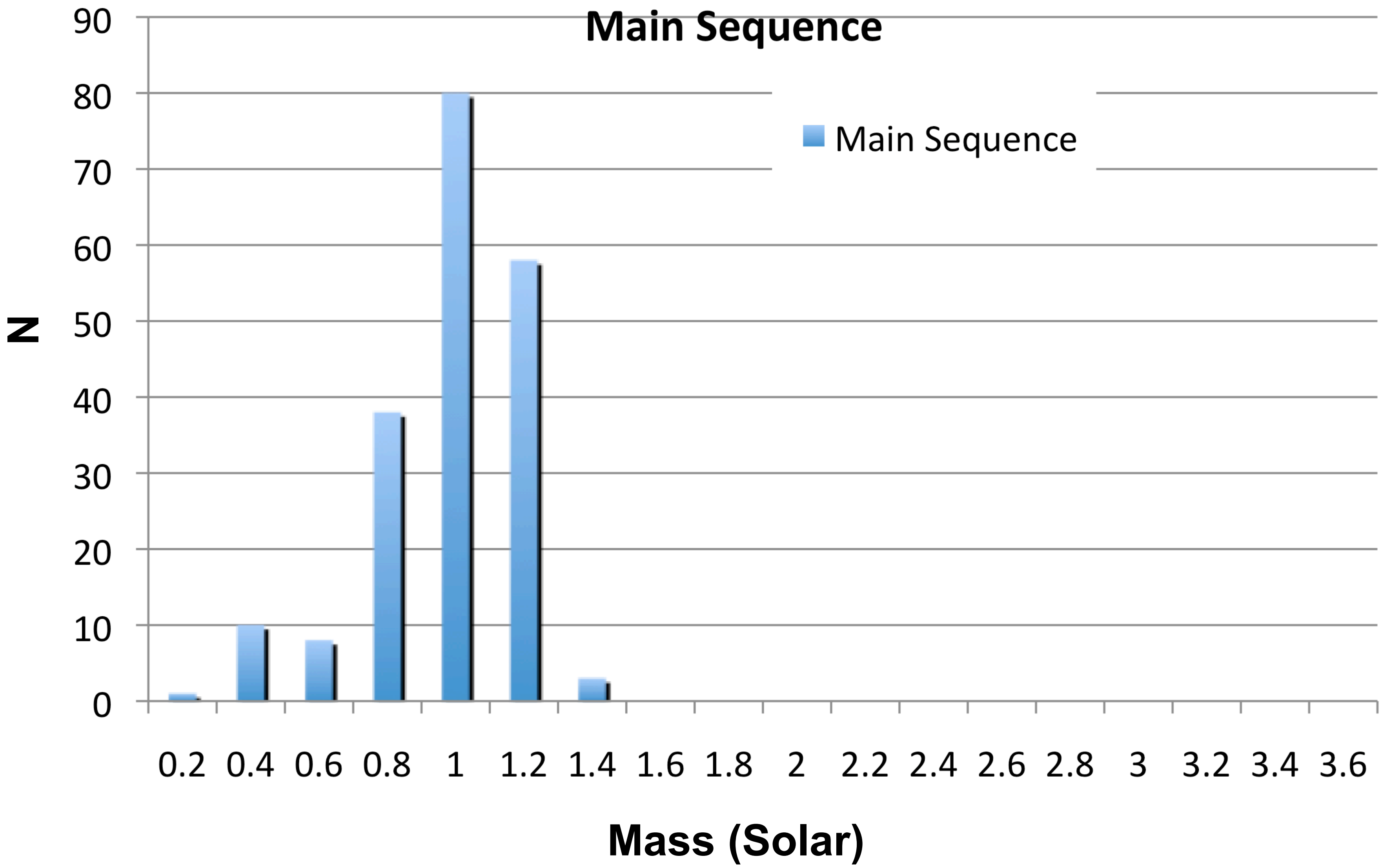
Why?



Early-type stars have few spectral lines (high effective temperatures) and high rotation rates.



98% of known exoplanets are found around stars with spectral types later than F6



Main Sequence
Main Sequence
N
Mass (Solar)
0
10
20
30
40
50
60
70
80
90
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6

Including dependence on stellar parameters

$$\boxed{\sigma \text{ (m/s)} \approx \text{Constant} \times (S/N)^{-1}\, R^{-3/2}\, (\Delta\lambda)^{-1/2}\, (v \sin i)\quad f(T_{\mathrm{eff}})}$$

$v \sin i$: projected rotational velocity of star in km/s

$f(T_{\mathrm{eff}})$ = factor taking into account line density

$f(T_{\mathrm{eff}}) \approx 1$ for solar type star

$f(T_{\mathrm{eff}}) \approx 3$ for A-type star

$f(T_{\mathrm{eff}}) \approx 0.5$ for M-type star

Instrumental Shifts

Recall that on a spectrograph we only measure a Doppler shift in Δx (pixels).

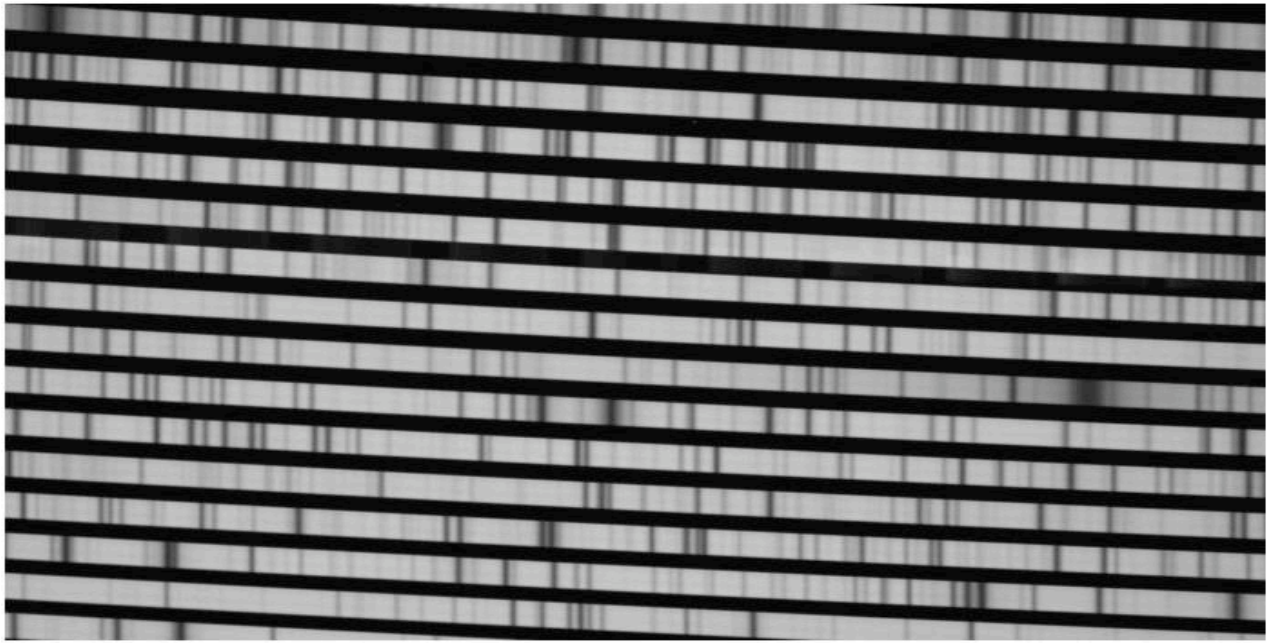
This has to be converted into a wavelength to get the radial velocity shift.

Instrumental shifts (shifts of the detector and/or optics) can introduce „Doppler shifts" larger than the ones due to the stellar motion

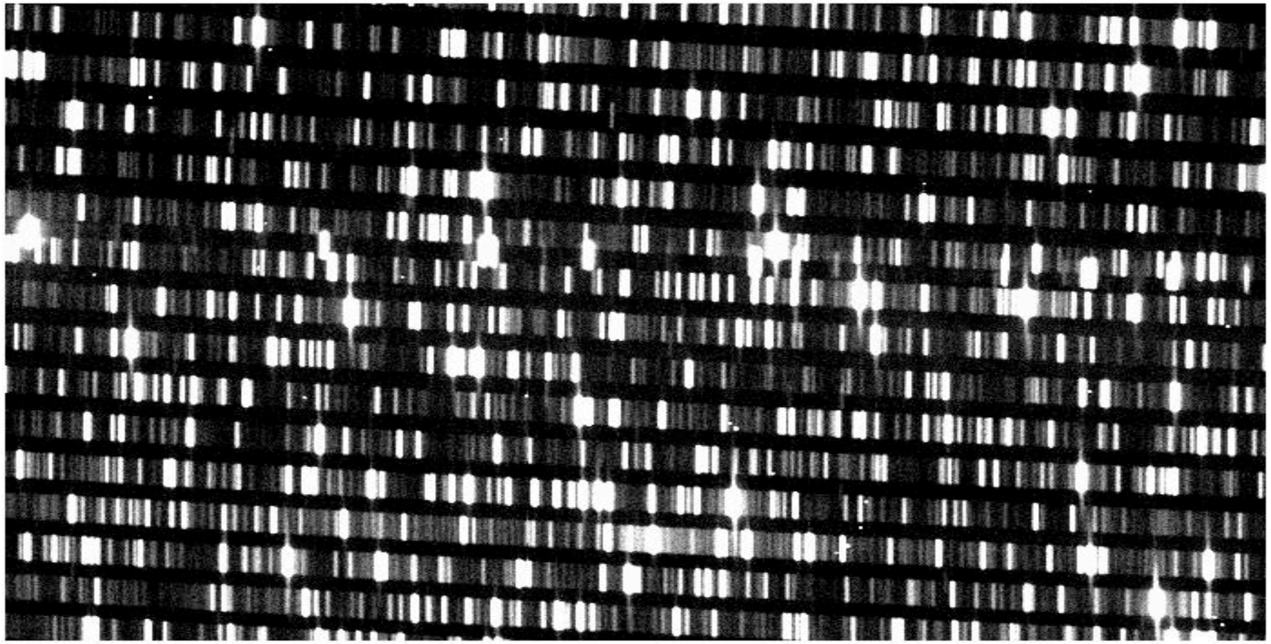
E.g.. for TLS spectrograph with R=67.000 our best RV precision is 1.8 m/s → 1.2 x 10^{-6} cm →120 Å

Traditional method:

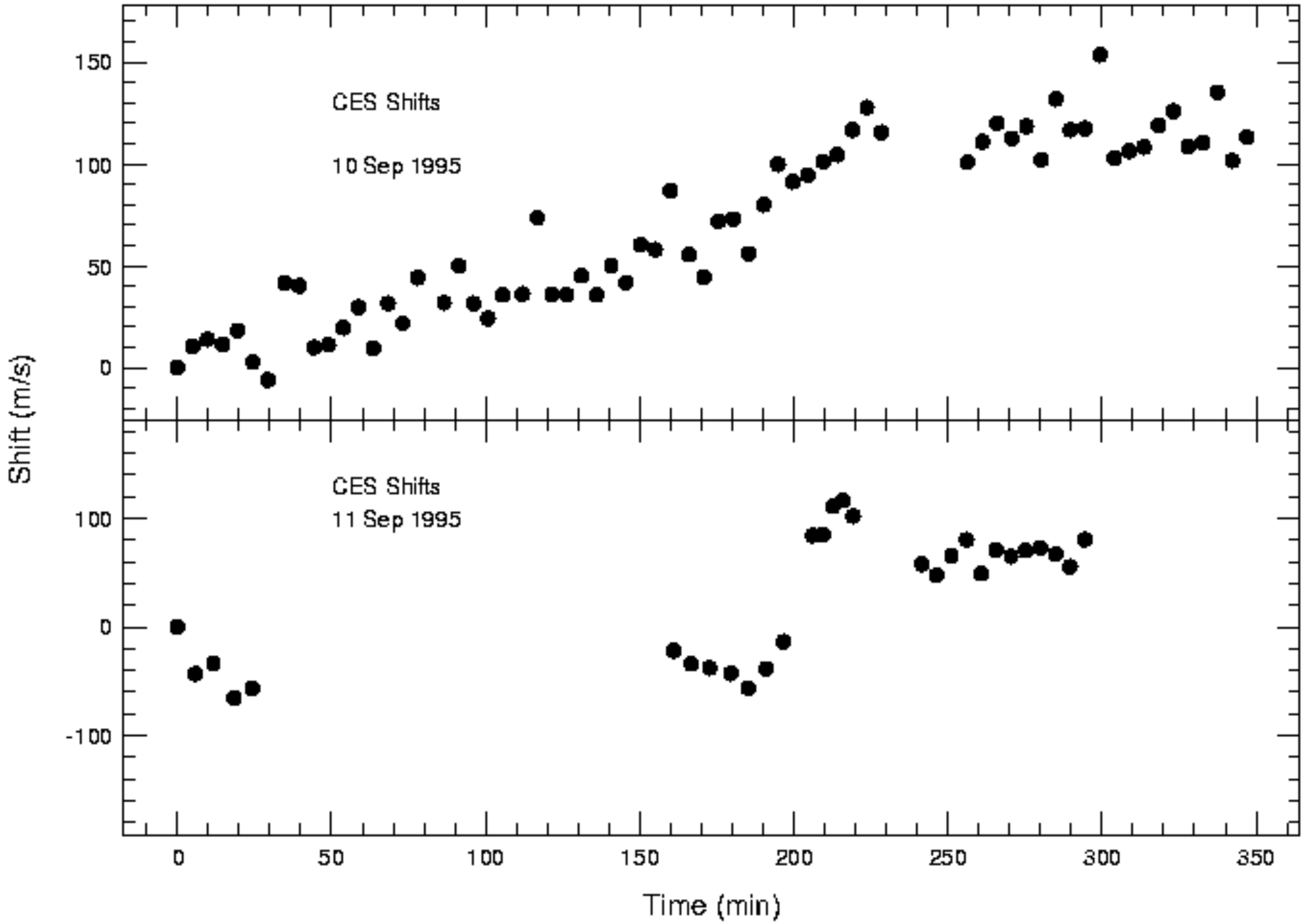
Observe your star→



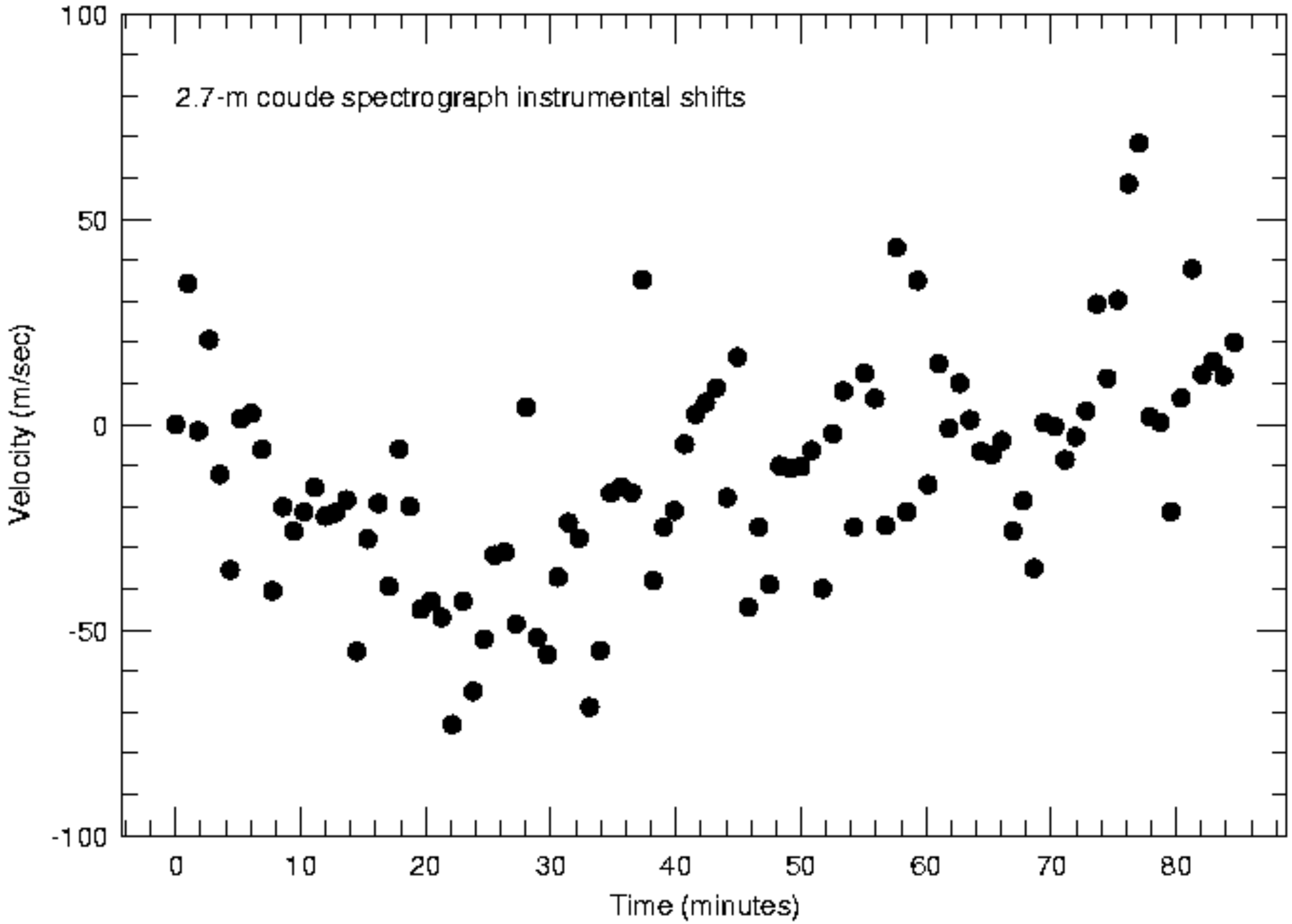
Then your calibration source→



<u>Problem:</u> these are not taken at the same time…



... Short term shifts of the spectrograph can limit precision to several hunrdreds of m/s

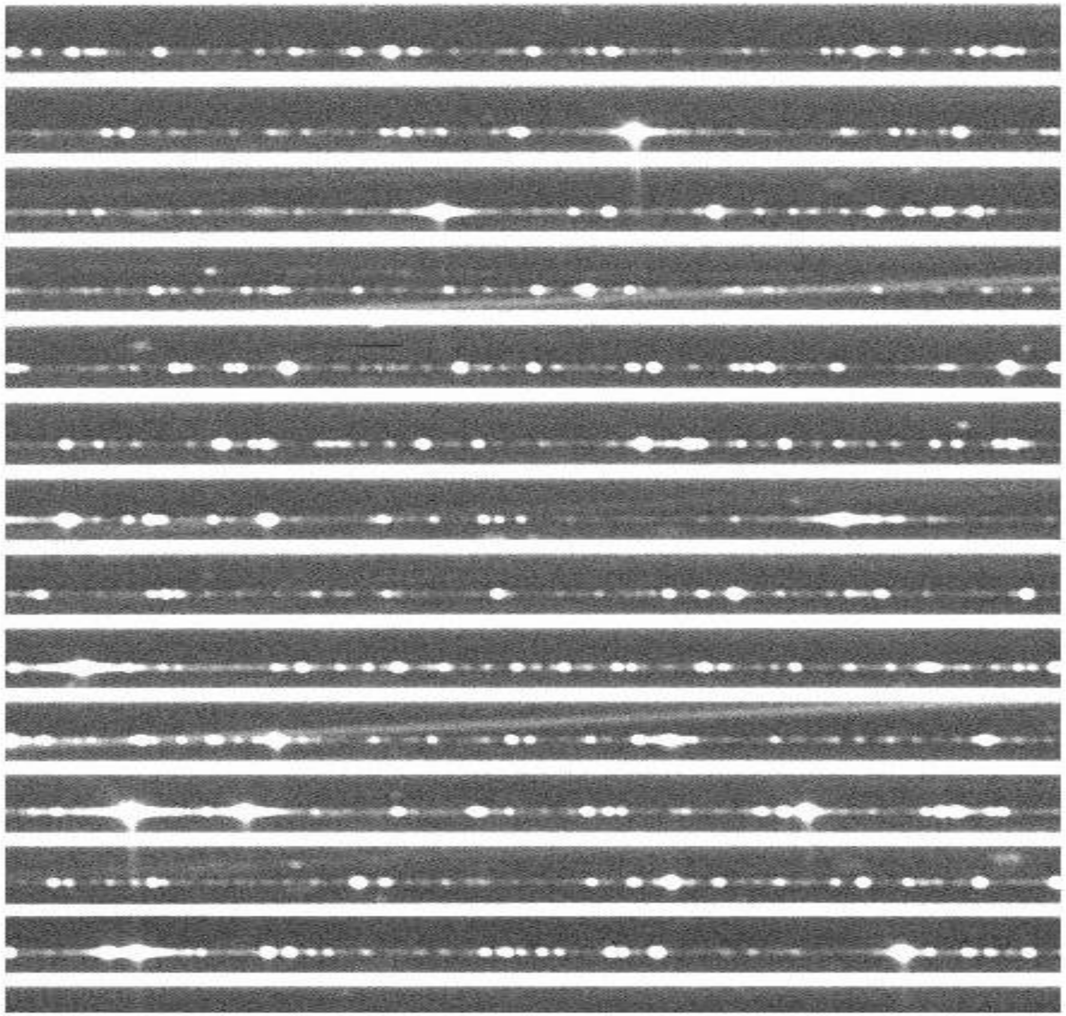


2.7-m coude spectrograph instrumental shifts
Velocity (m/sec)
100
50
0
-50
-100
0
10
20
30
40
50
60
70
80
Time (minutes)

Solution 1: Observe your calibration source (Th-Ar) simultaneously to your data:

Stellar spectrum

Thorium-Argon calibration



Spectrographs: CORALIE, ELODIE, HARPS

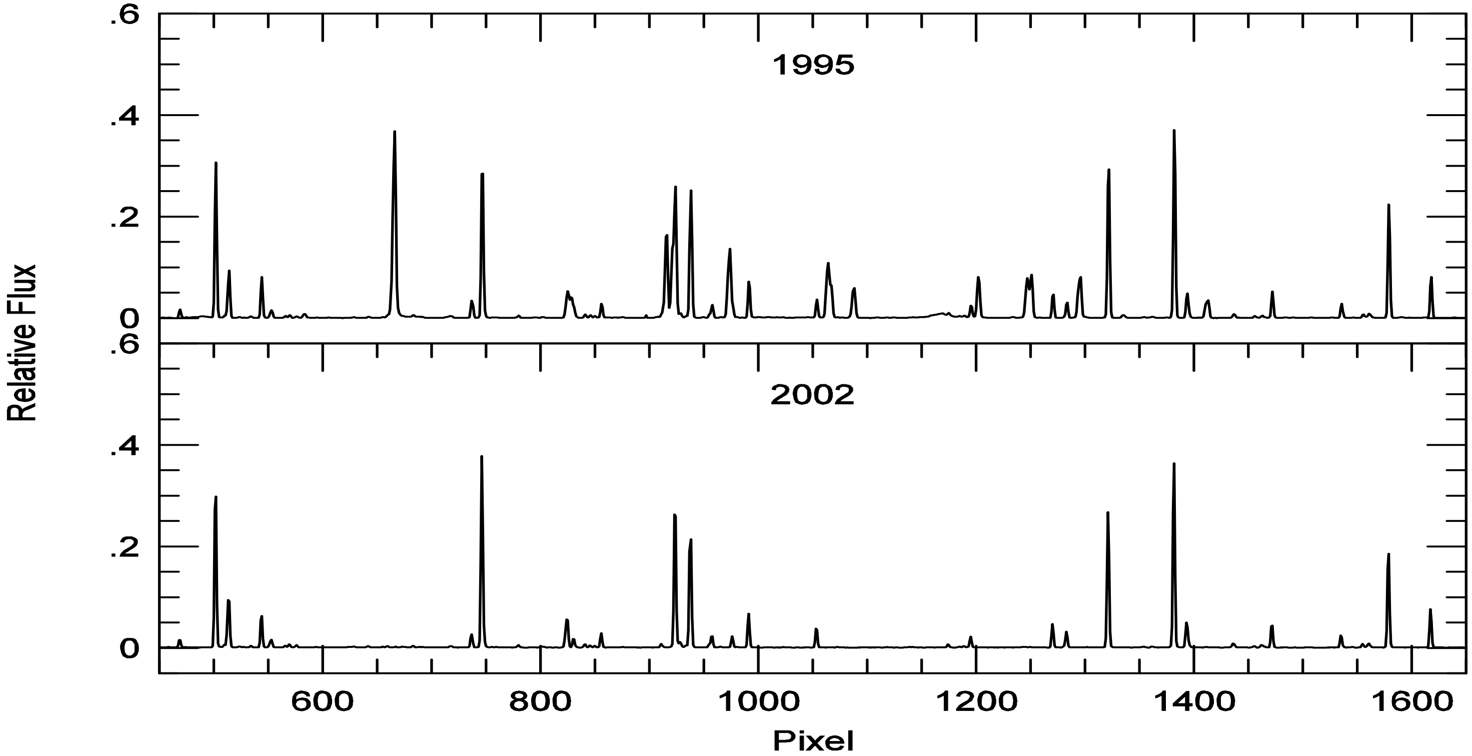
Advantages of simultaneous Th-Ar calibration:

- Large wavelength coverage (2000 – 3000 Å)
- Computationally simple

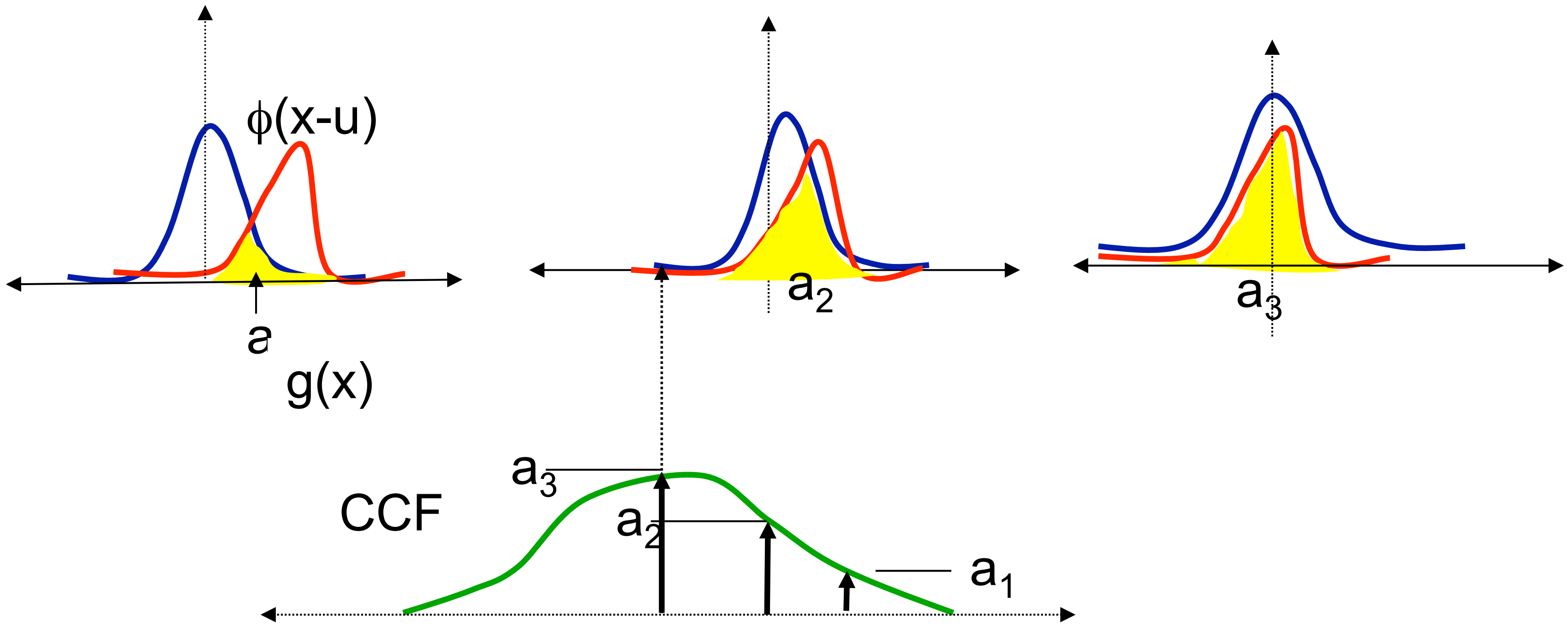
Disadvantages of simultaneous Th-Ar calibration:

- Th-Ar are active devices (need to apply a voltage)
- Lamps change with time
- Th-Ar calibration not on the same region of the detector as the stellar spectrum
- Some contamination that is difficult to model
- Cannot model the instrumental profile, therefore you have to stablize the spectrograph

Th-Ar lamps change with time!

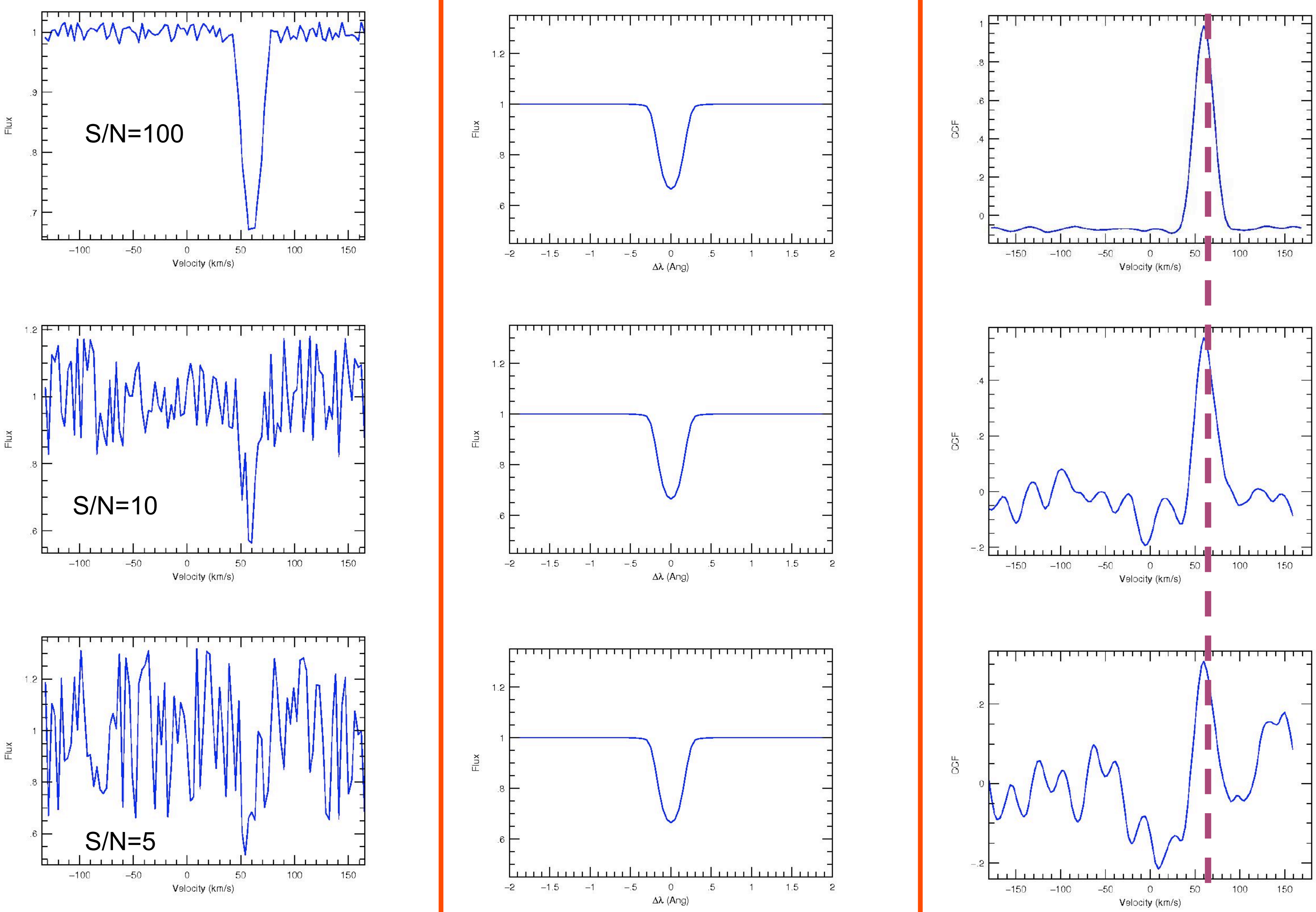


Detecting Doppler Shifts with Cross Correlation



A standard method to computer the Doppler shift is to take a spectrum of a standard star, compute the cross correlation function (CCF) with your data and look for the shift in the peak

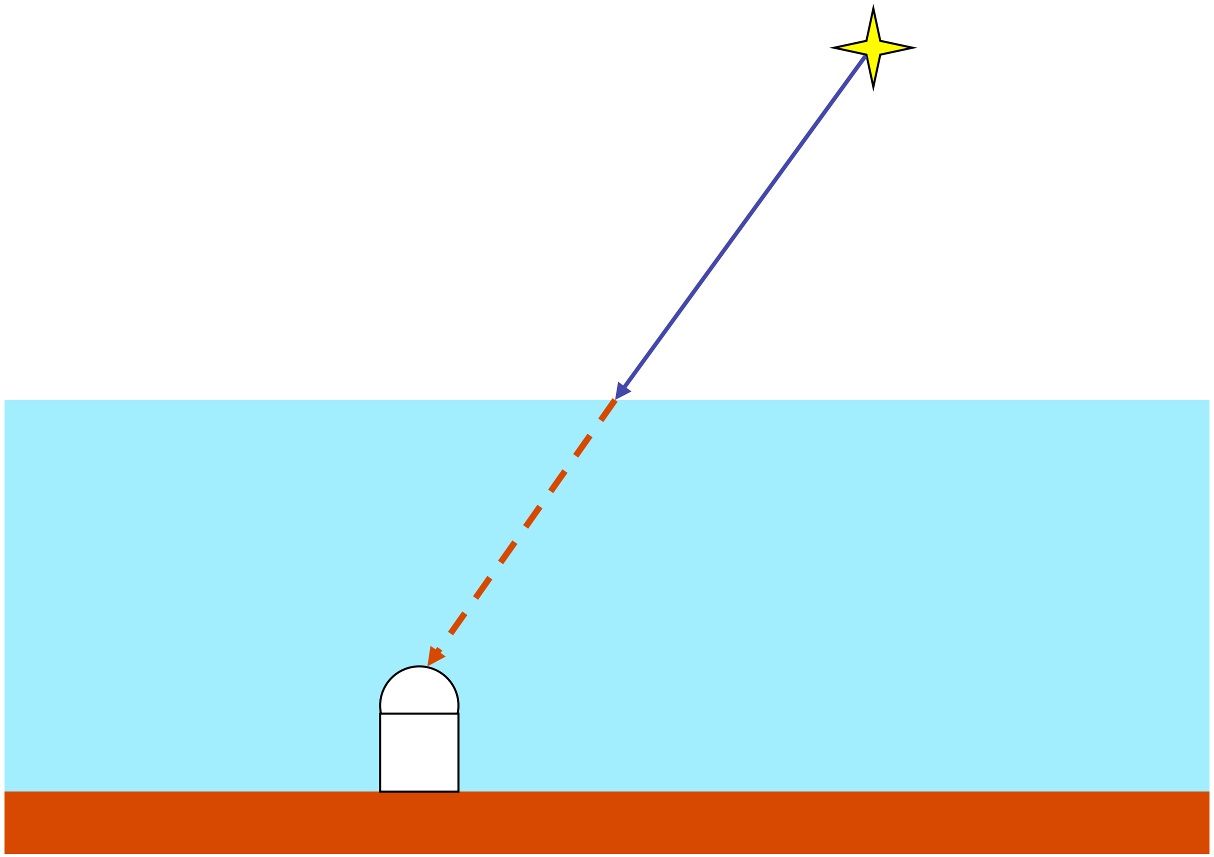
Observation | Standard | CCF

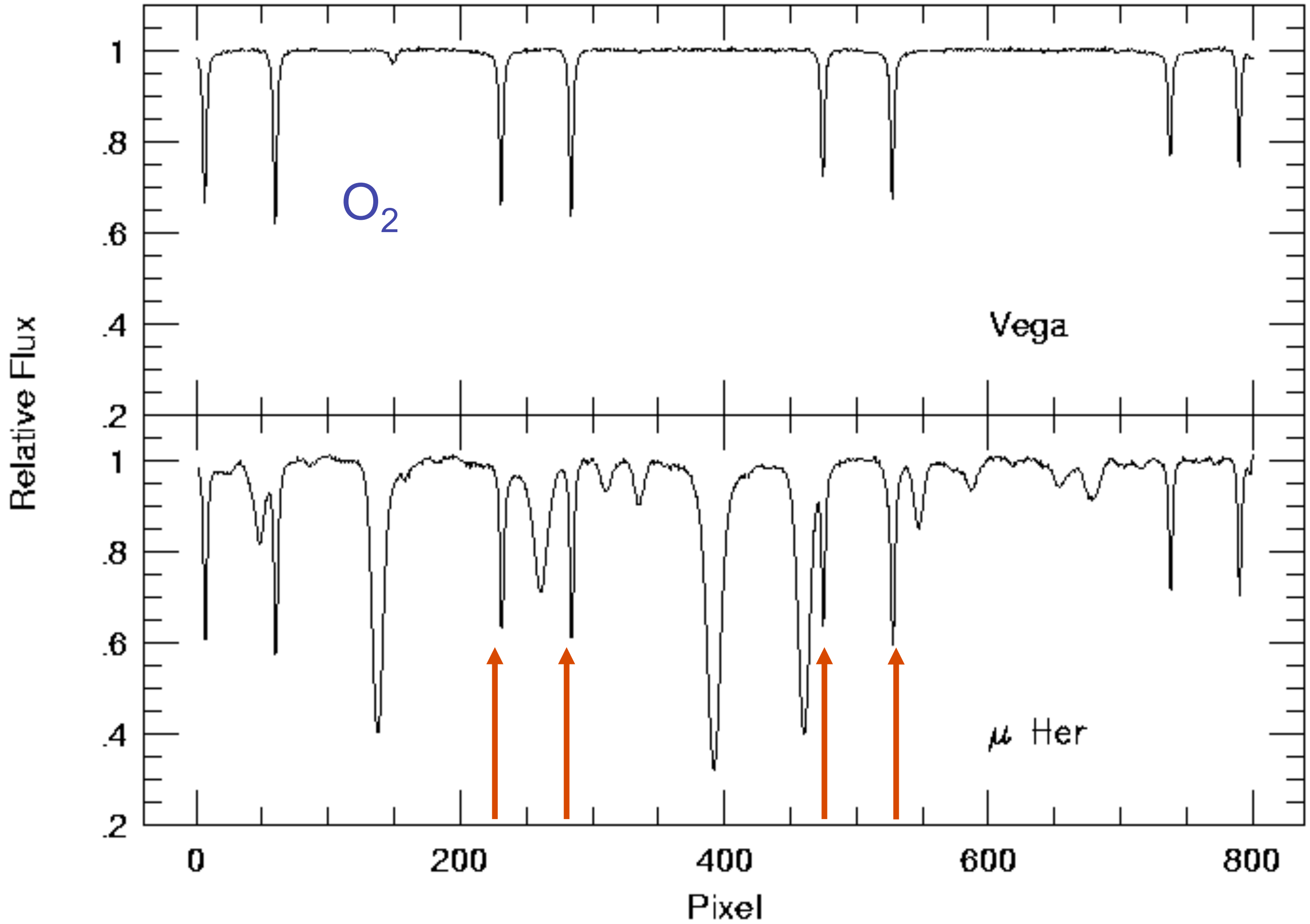


IRAF package: fxcorr

Solution 2: Absorption cell

a) Griffin and Griffin: Use the Earth's atmosphere:





6300 Angstroms

Example: The companion to HD 114762 using the telluric method.
Best precision is 15–30 m/s

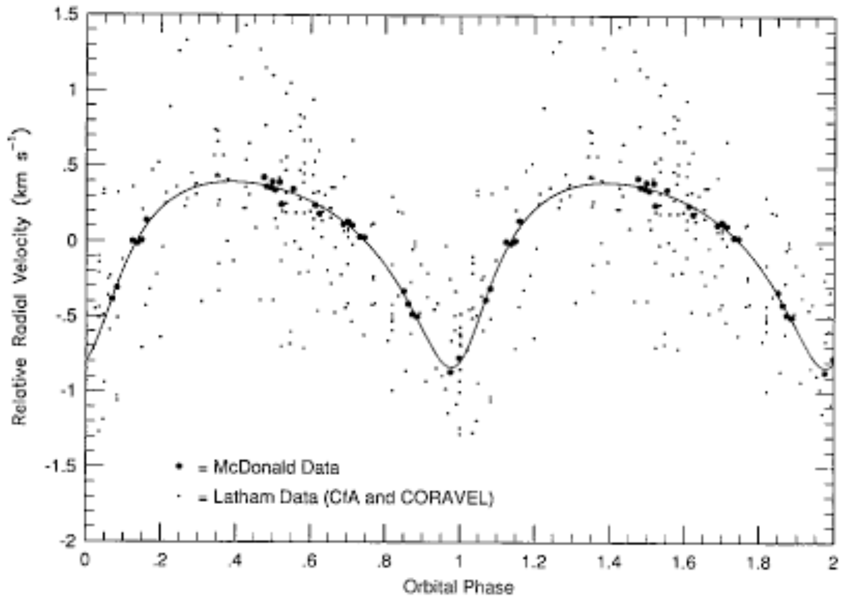


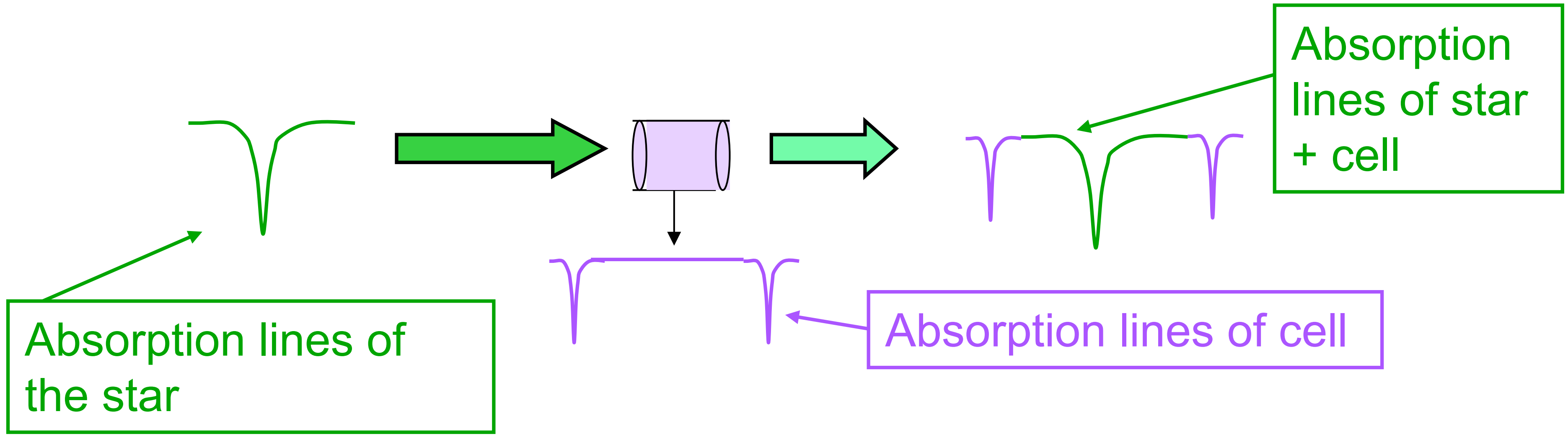
FIG. 1.—Radial velocity curve for HD 114762. The McDonald data presented in this paper are shown as large filled circles. The original discovery data of Latham et al. (1989) are shown as small dots. The solid curve is from the orbital solution derived from the McDonald data.

Filled circles are data taken at McDonald Observatory using the telluric lines at 6300 Ang.

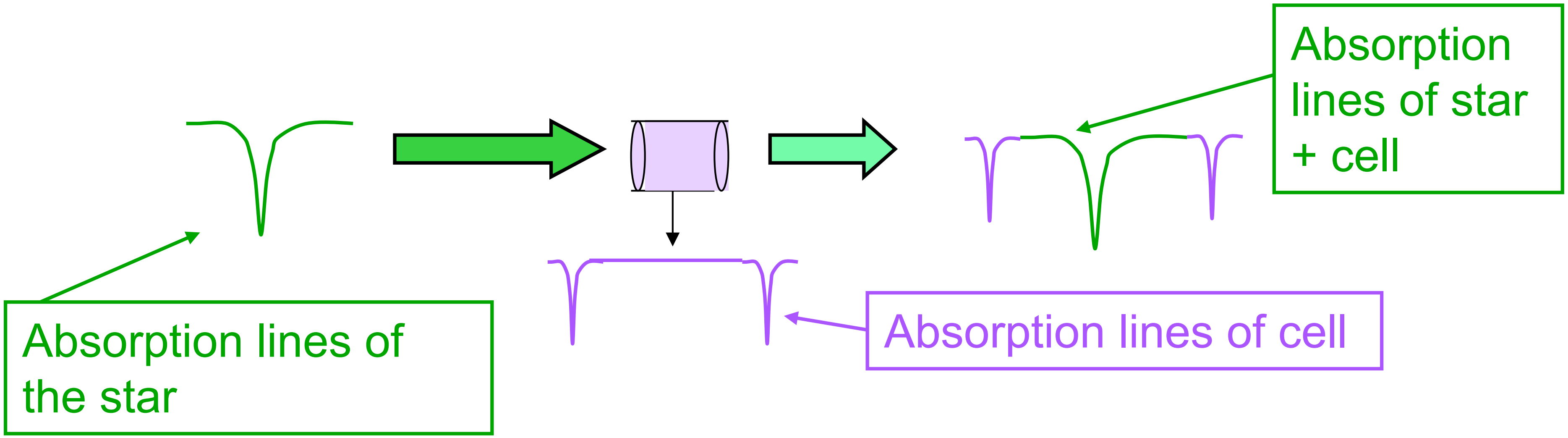
<u>Limitations of the telluric technique:</u>

- Limited wavelength range (≈ 10s Angstroms)
- Pressure, temperature variations in the Earth's atmosphere
- Winds
- Line depths of telluric lines vary with air mass
- Cannot observe a star without telluric lines which is needed in the reduction process.

b) Use a „controlled“ absorption cell



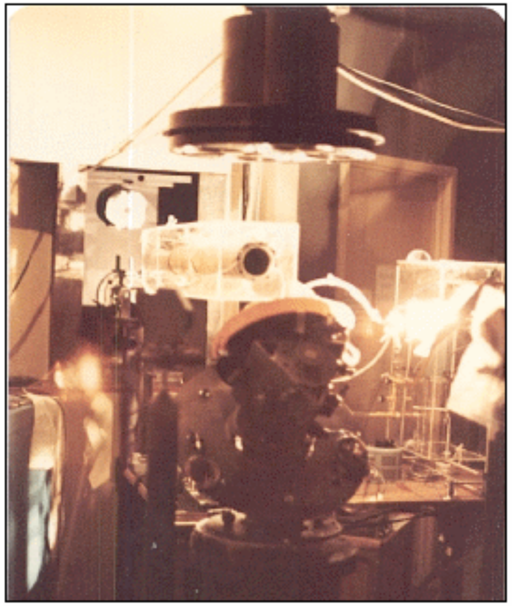
b) Use a „controlled“ absorption cell



The pioneers: Gordon Walker and Bruce Campbell

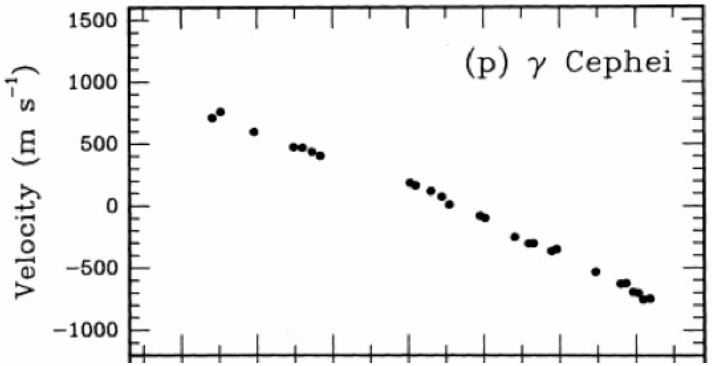
Planet search with the RV method started in 1979

Used an HF cell for calibration



Surveyed 26 stars and found evidence for the planet around γ Cep in 1988

"Probable third body variation of 25 m/s amplitude, 2.7 yr period.."



1988

"They invented the technique we all stole"

Geoff Marcy

Campbell & Walker: Hydrogen Fluoride cell:

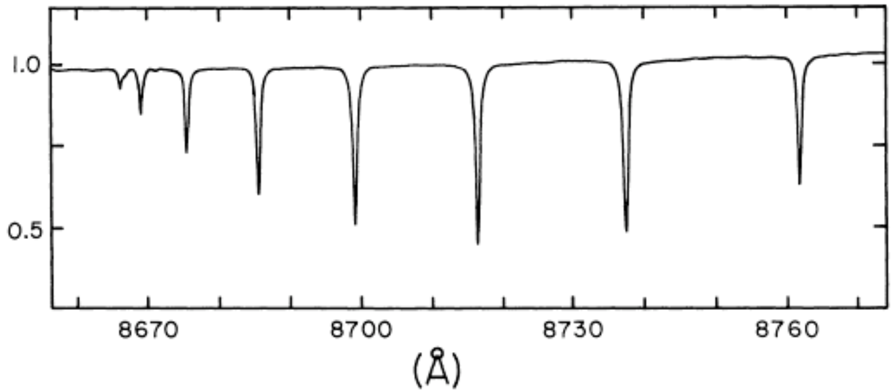


FIG. 1—Absorption spectrum of hydrogen fluoride 3–0 band R branch.

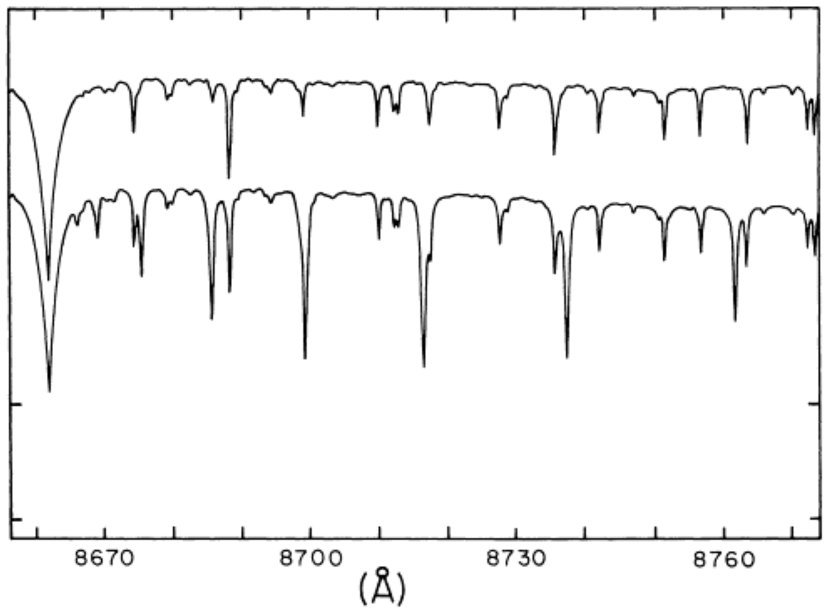
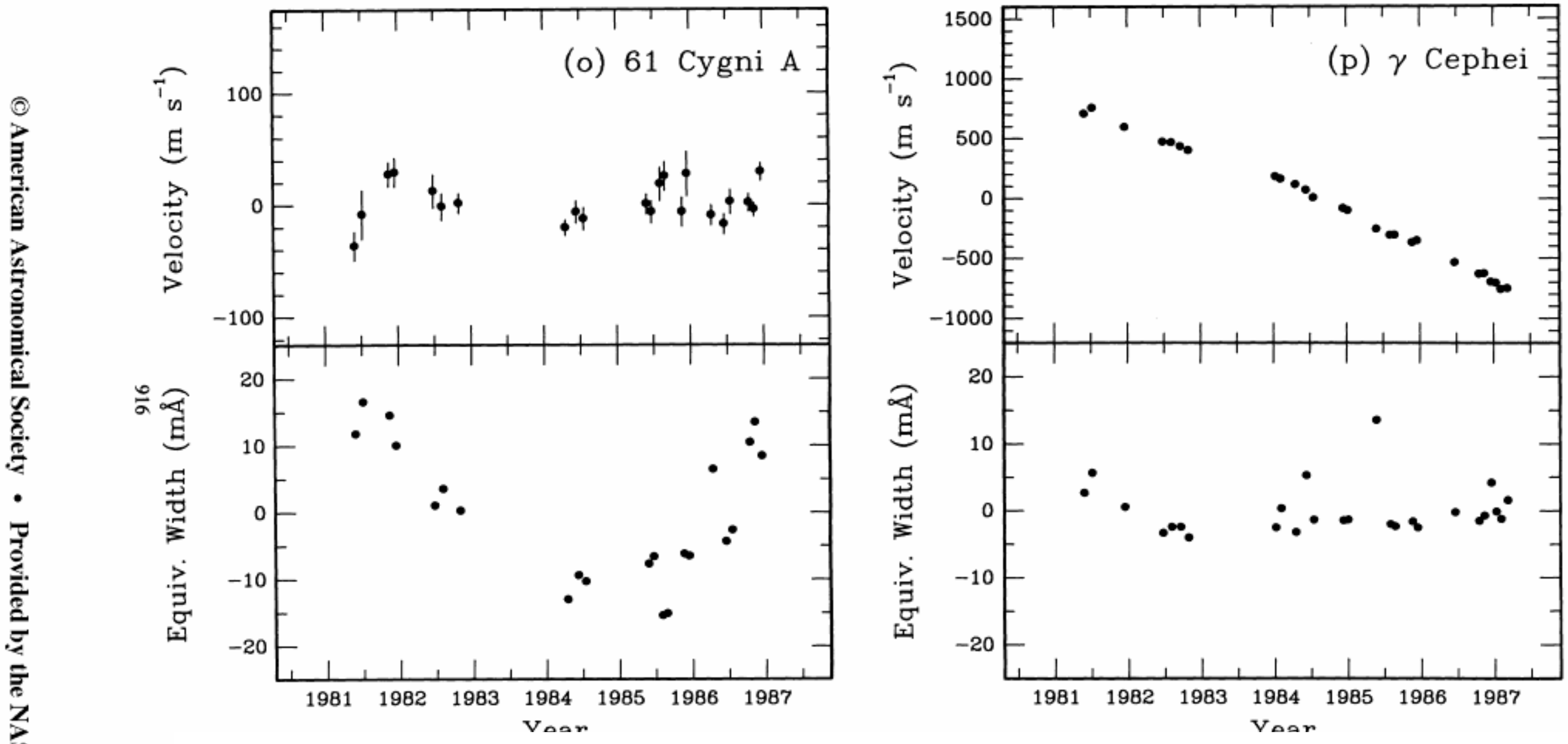


FIG. 3—Solar spectra with (*lower*) and without (*upper*) the hydrogen fluoride lines. The strong line at left is Ca II λ8662.

Demonstrated radial velocity precision of 13 m s^{-1} in 1980!



Drawbacks:

- Limited wavelength range (≈ 100 Ang.)
- Temperature stablized at 100 C
- Long path length (1m)
- Has to be refilled after every observing run
- **Dangerous**

A better idea: Iodine cell (first proposed by Beckers in 1979 for solar studies)

Spectrum of iodine

<u>Advantages over HF:</u>

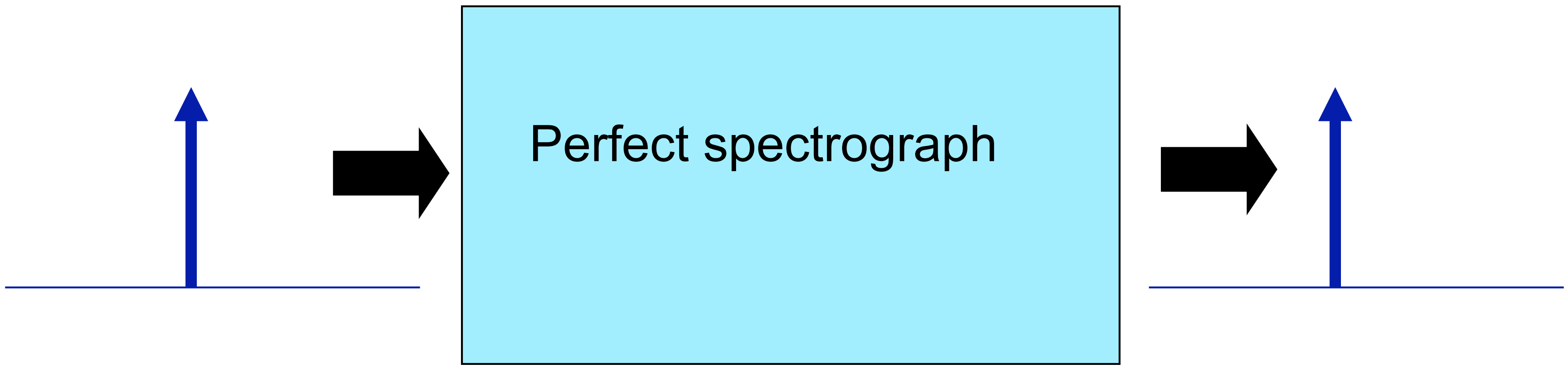
- 1000 Angstroms of coverage
- Stablized at 50–75 C
- Short path length (≈ 10 cm)
- Can model instrumental profile
- Cell is always sealed and used for >10 years
- If cell breaks you will not die!

Spectrum of star through Iodine cell:

Modelling the Instrumental Profile

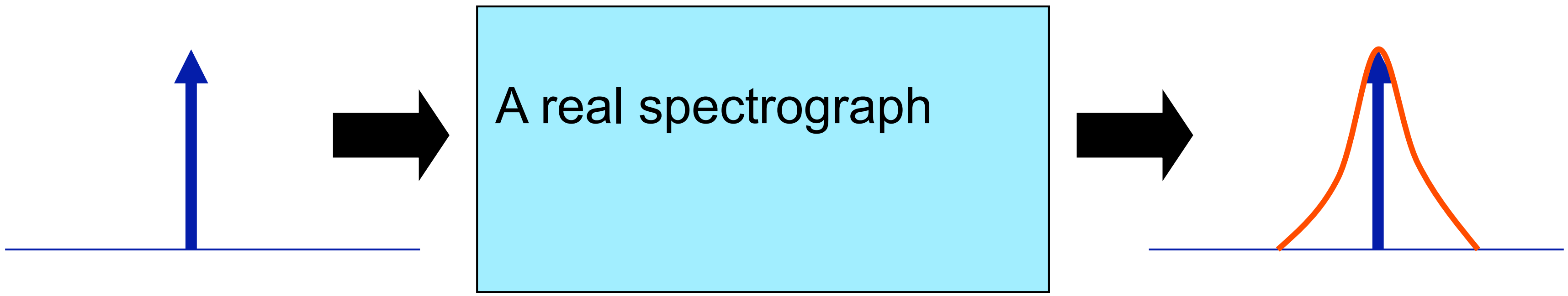
What is an instrumental profile (IP)?:

Consider a monochromatic beam of light (delta function)



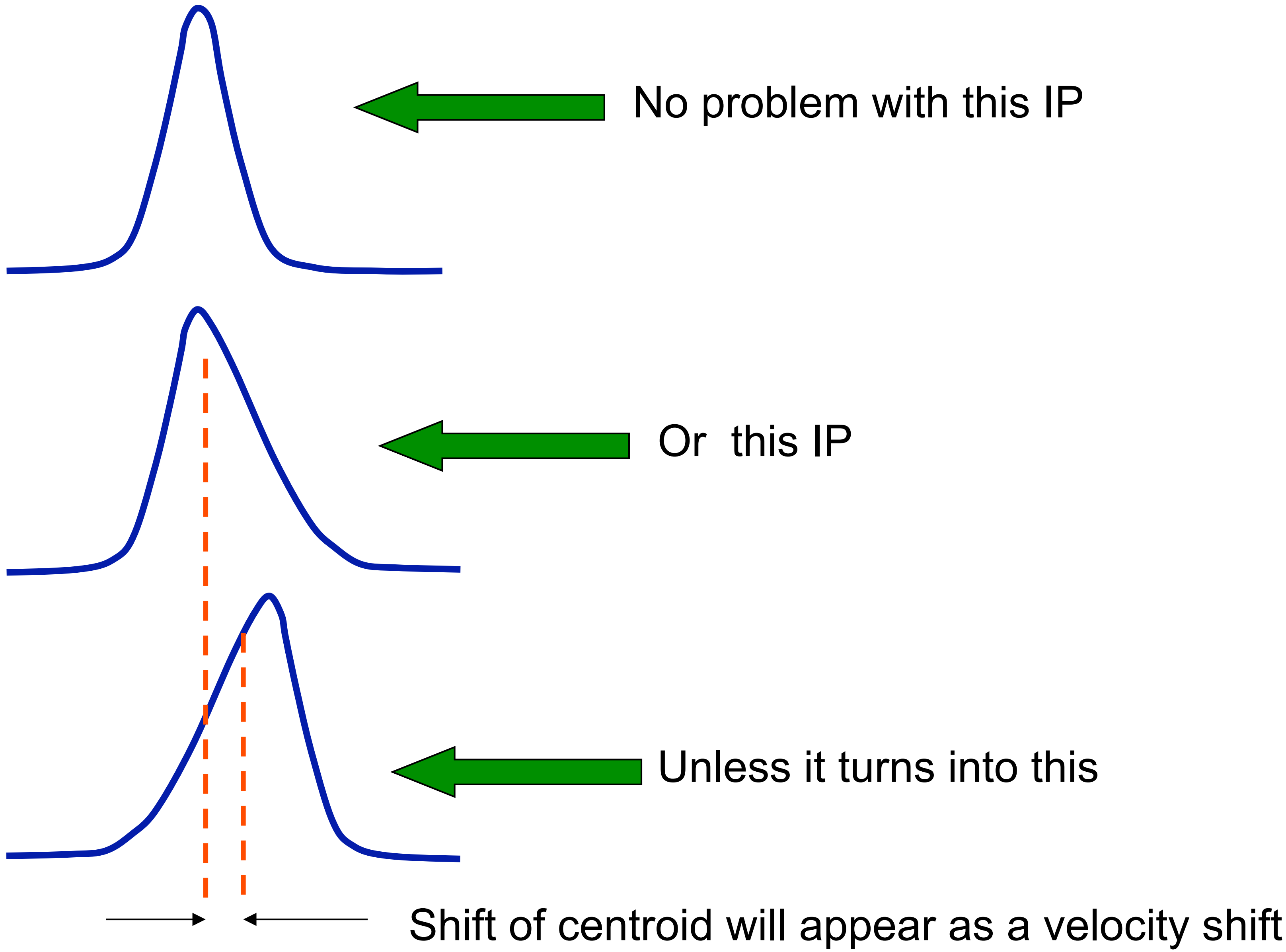
Modelling the Instrumental Profile

We do not live in a perfect world:



IP is usually a Gaussian that has a width of 2 detector pixels

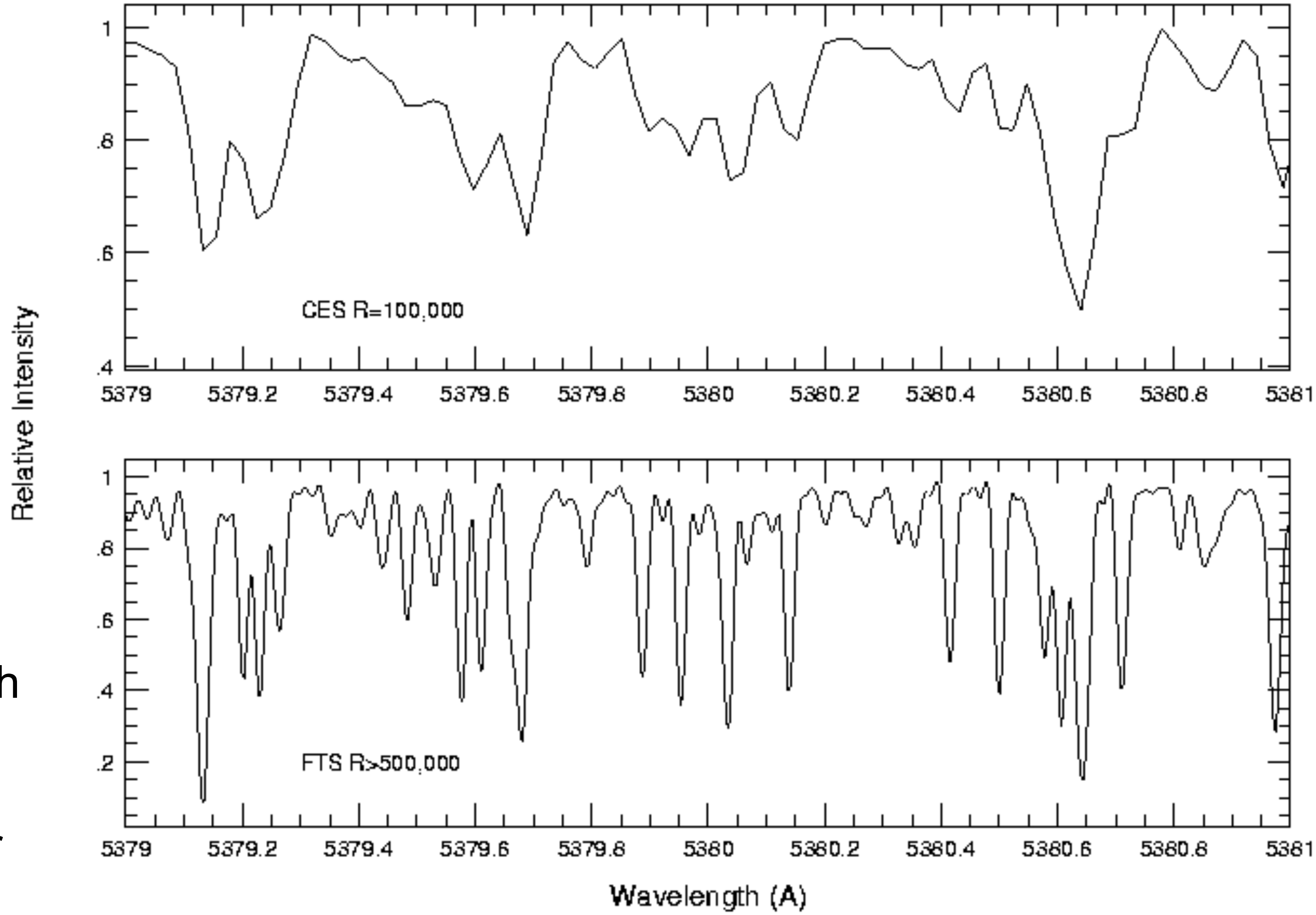
The IP is not so much the problem as *changes* in the IP

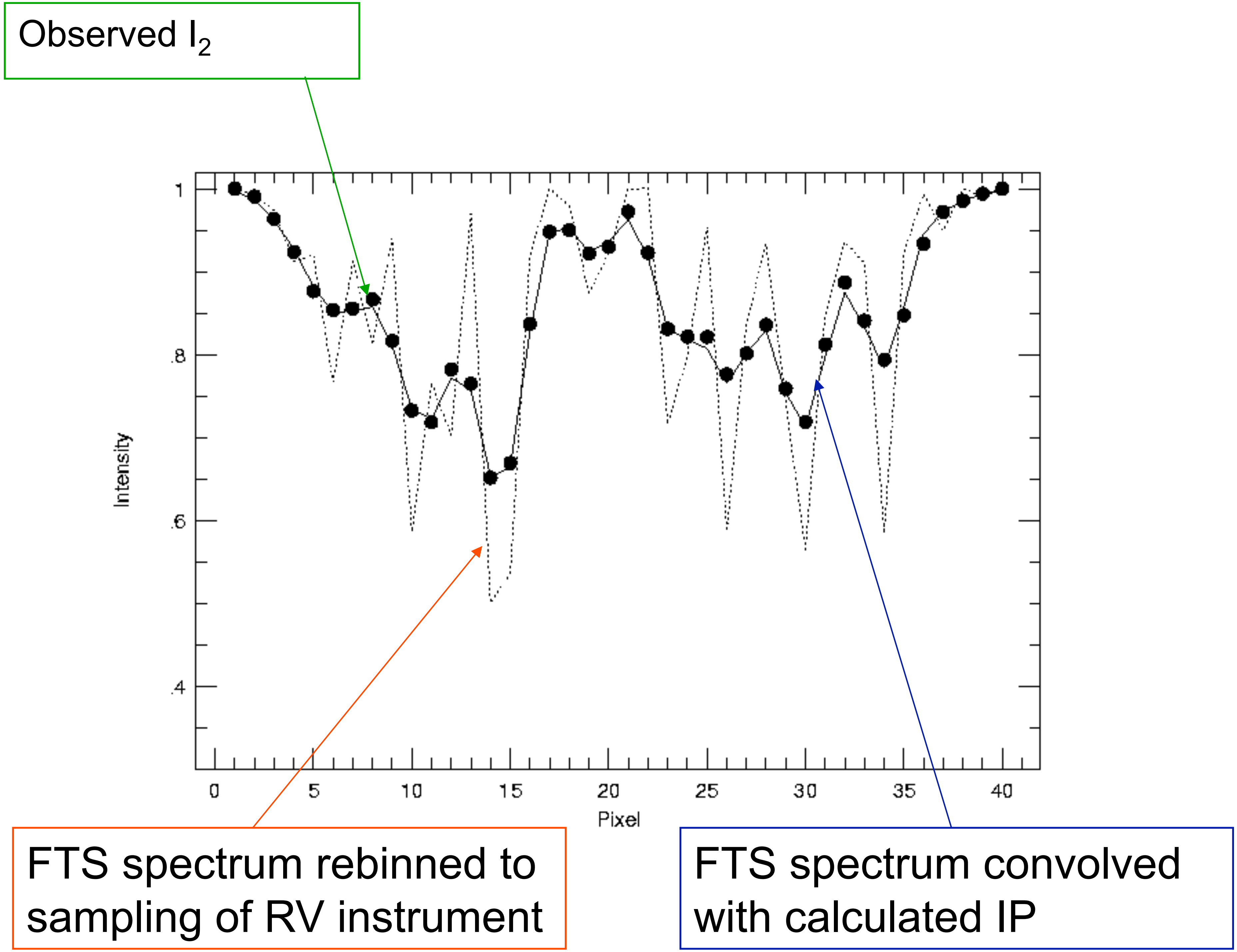


Use a high resolution spectrum of iodine to model IP

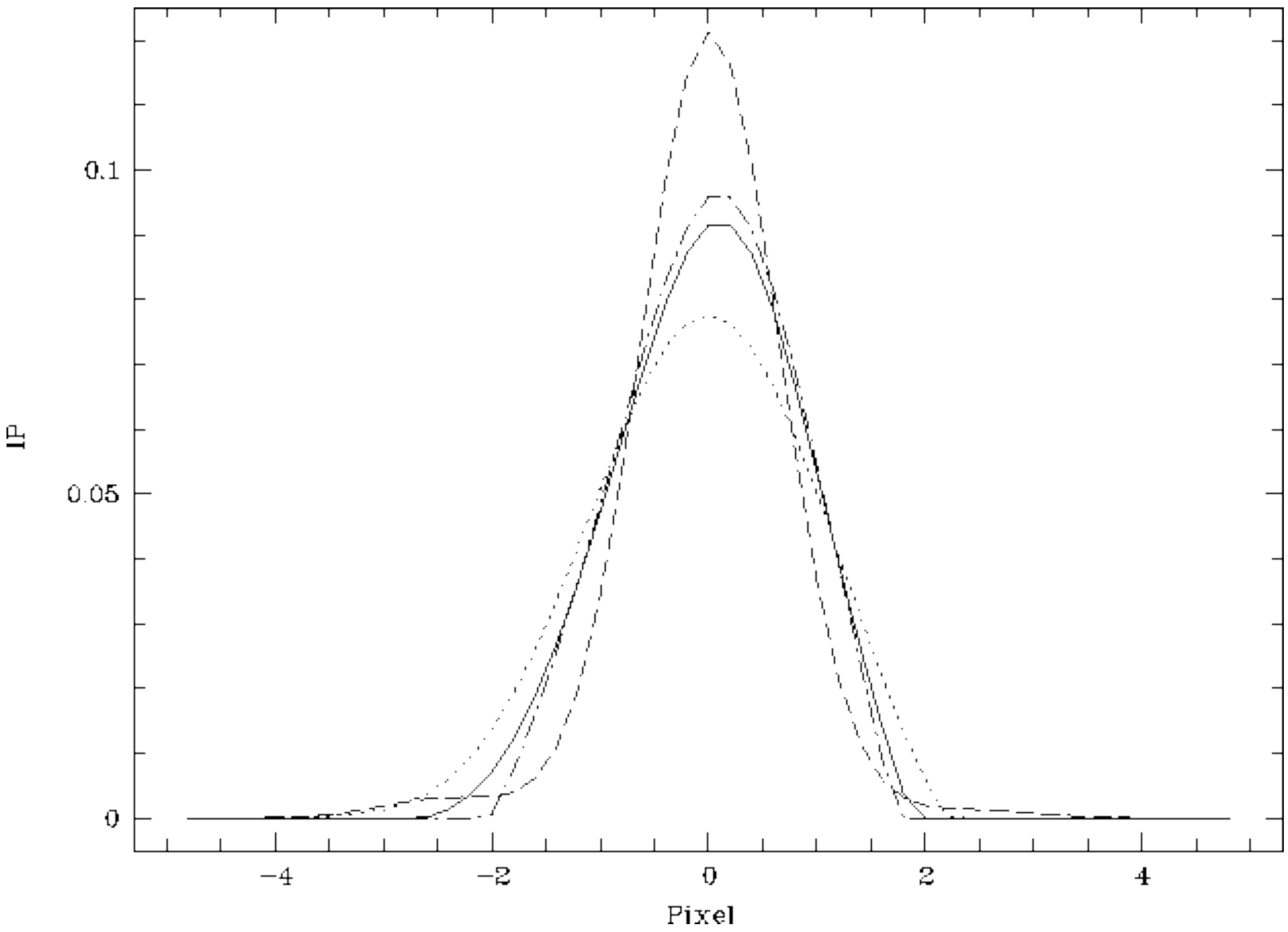
Iodine observed with RV instrument

Iodine Observed with a Fourier Transform Spectrometer




Observed I_2
Intensity
1
.8
.6
.4
0
5
10
15
20
25
30
35
40
Pixel
FTS spectrum rebinned to sampling of RV instrument
FTS spectrum convolved with calculated IP

Instrumental Profile Changes in ESO‘s CES spectrograph over 5 years:



IP is oversampled by a factor of 5

$$I_{\mathrm{obs}}(\lambda) = \mathrm{k}\,[T_{\mathrm{I2}}(\lambda) I_{\mathrm{s}}(\lambda + \Delta\lambda)]*\mathrm{PSF}$$

$I_{\mathrm{obs}}(\mathrm{l})$ = observed spectrum

k = normalization factor

T_{I2} = transmission function of iodine absorption cel

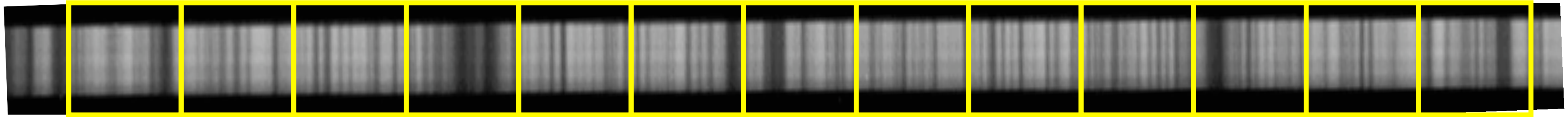
I_{s} = stellar spectrum

PSF = Point spread function of spectrograph

* = convolution

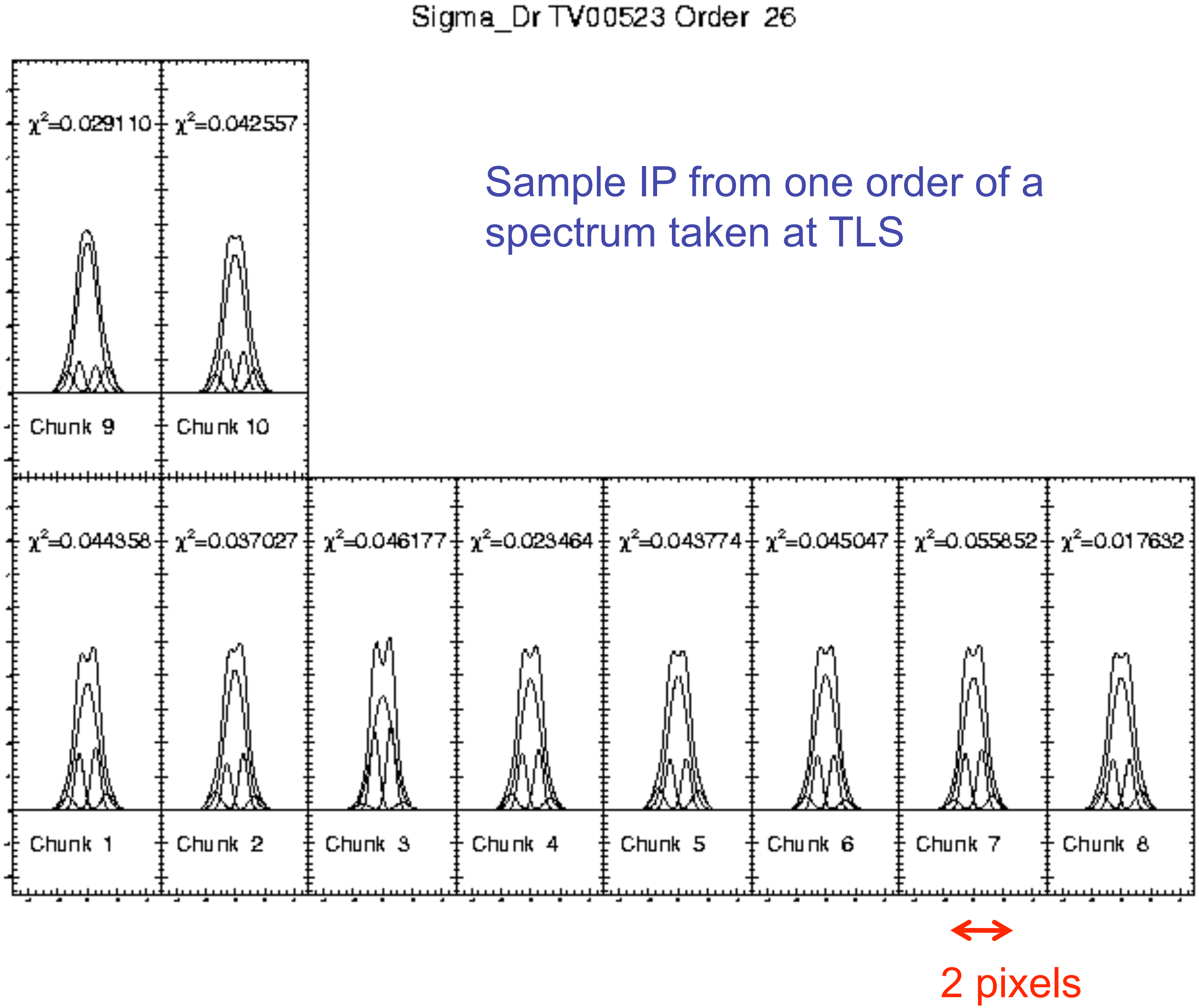
Butler et al. 1996

Modeling the Instrumental Profile



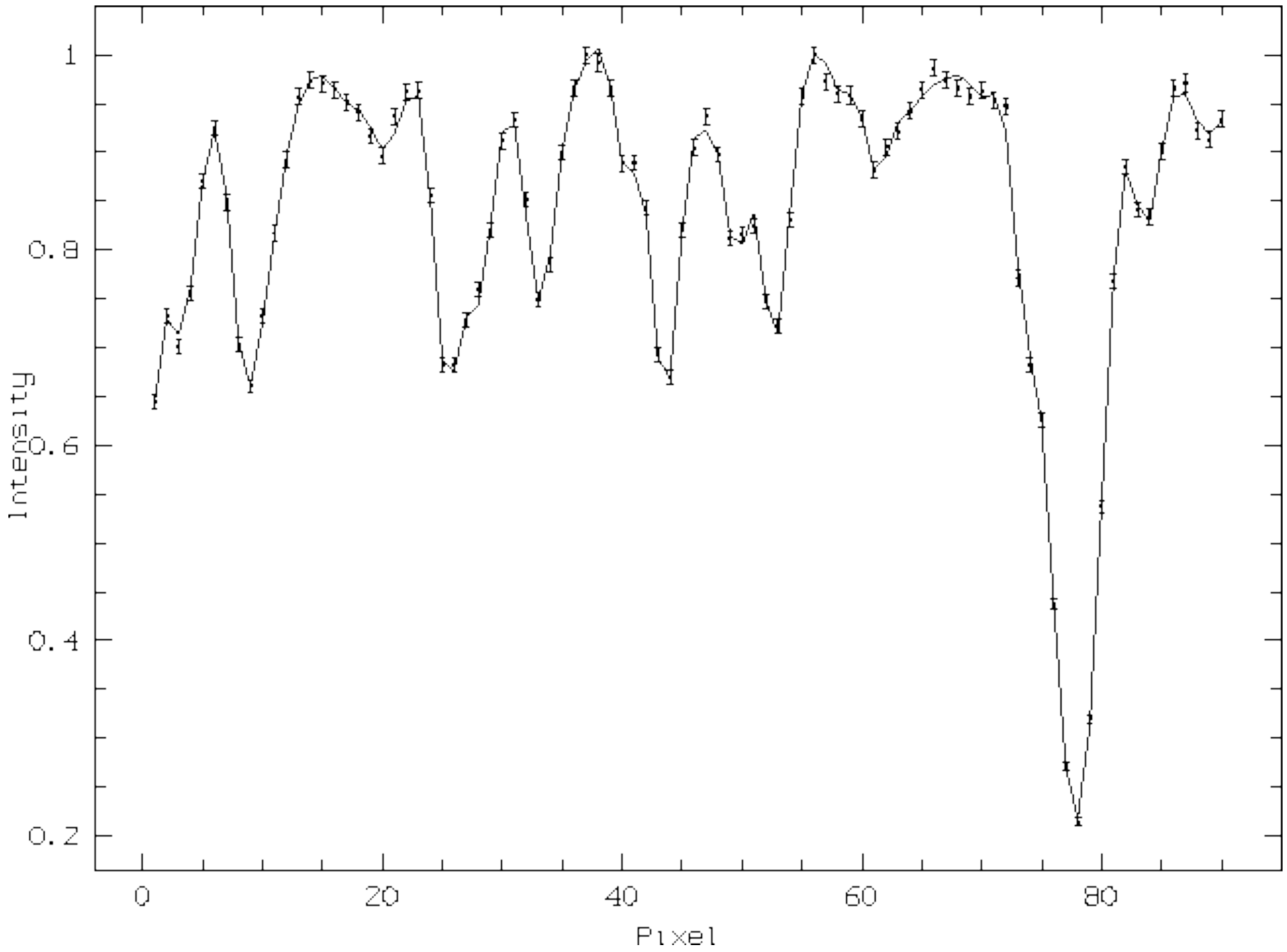
In each chunk:

- Remove continuum slope in data : 2 parameters
- Calculate dispersion (Å/pixel): 3 parameters (2nd order polynomial: a0, a1, a2)
- Calculate IP with 5 Gaussians: 9 parameters: 5 widths, 4 amplitudes (position and widths of satellite Gaussians fixed)
- Calculate Radial Velocity: 1 parameters
- Combine with high resolution iodine spectrum and stellar spectrum without iodine
- Iterate until model spectrum fits the observed spectrum

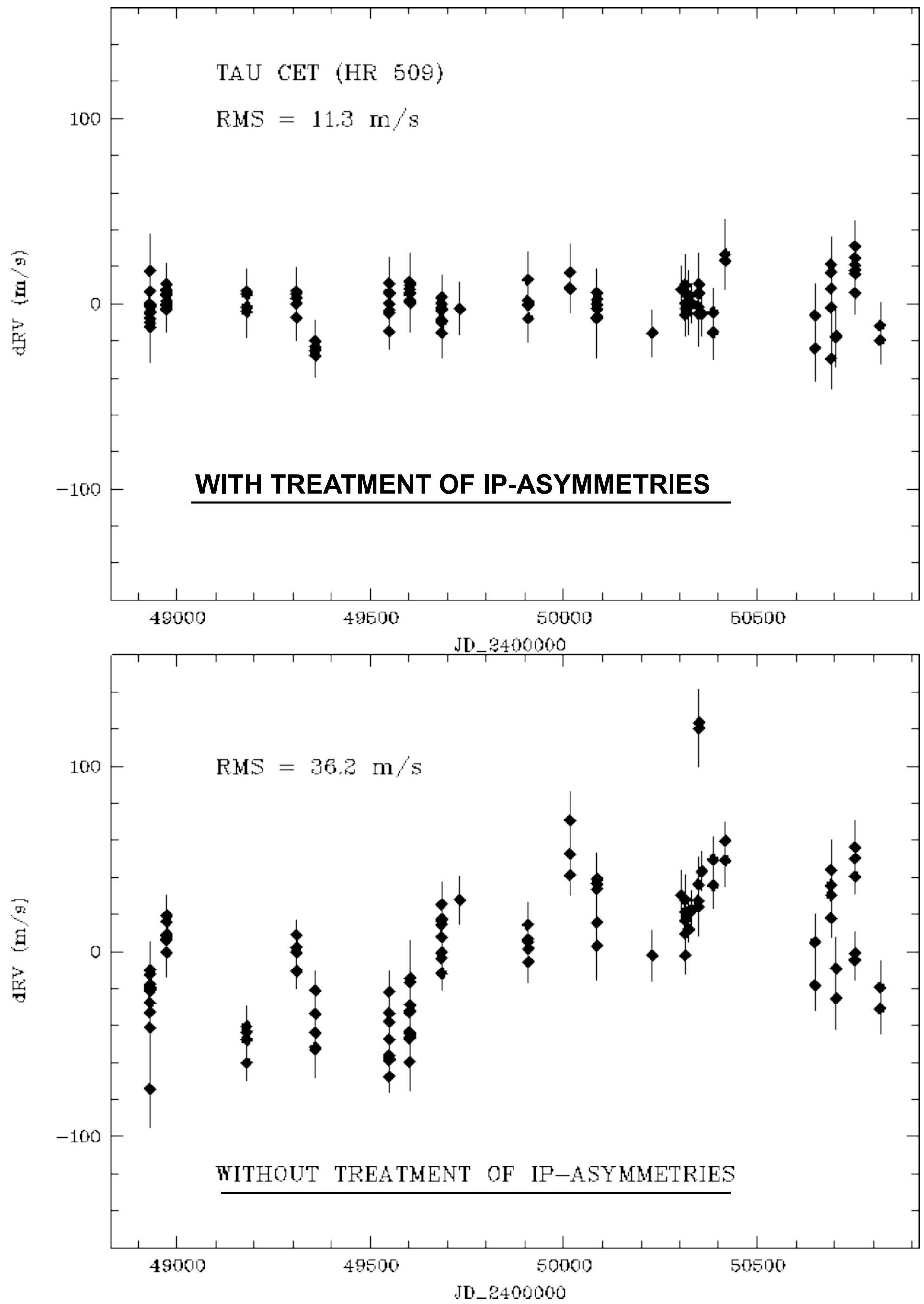


Sample IP from one order of a spectrum taken at TLS

Sample fit to an observed chunk of data

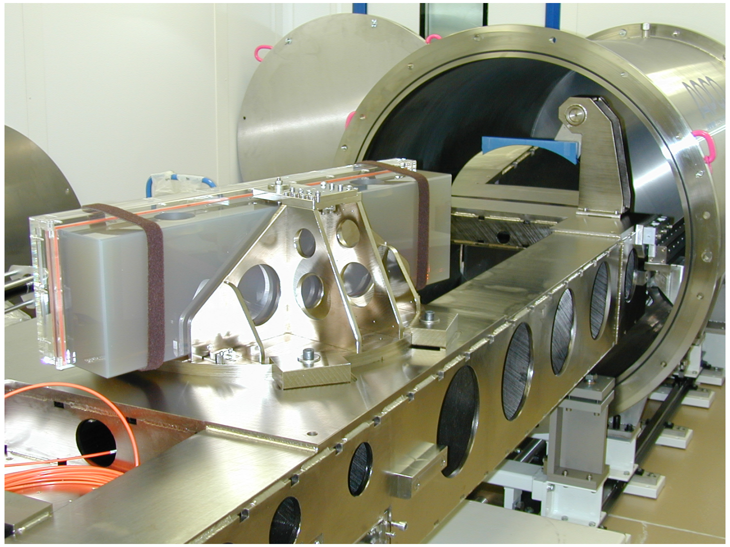
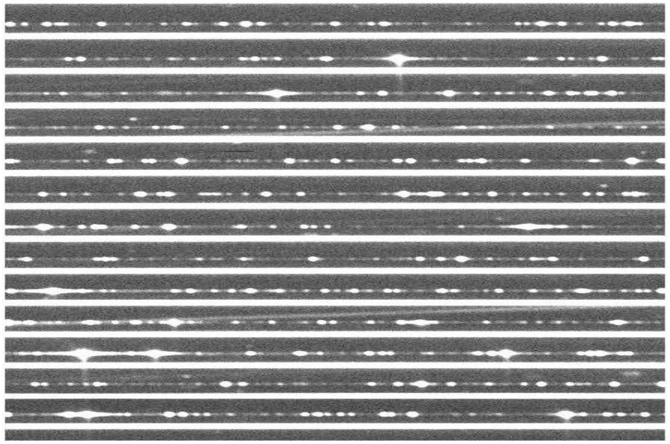


You need to produce an deconvolved and oversampled spectrum of your star!



TAU CET (HR 509)
RMS = 11.3 m/s
WITH TREATMENT OF IP-ASYMMETRIES
dRV (m/s)
JD_2400000
RMS = 36.2 m/s
WITHOUT TREATMENT OF IP–ASYMMETRIES

Techniques: Simultaneous Th-Ar versus Absorption Cell

Stability		
Wavelength Reference		
Best Precision	**0.5 – 1 m/s**	**1 – 2 m/s[1]** [1]On spectrographs not designed for stability

Telescope	Instrument	Wavelength Reference
1-m MJUO	Hercules	Th-Ar
1.2-m Euler Telescope	CORALIE	Th-Ar
1.8-m BOAO	BOES	Iodine Cell
1.88-m Okayama Obs,	HIDES	Iodine Cell
1.88-m OHP	SOPHIE	Th-Ar
2-m TLS	Coude Echelle	Iodine Cell
2.2m ESO/MPI La Silla	FEROS	Th-Ar
2.7m McDonald Obs.	Tull Spectrograph	Iodine Cell
3-m Lick Observatory	Hamilton Echelle	Iodine Cell
3.8-m TNG	SARG	Iodine Cell
3.9-m AAT	UCLES	Iodine Cell
3.6-m ESO La Silla	HARPS	Th-Ar
8.2-m Subaru Telescope	HDS	Iodine Cell
8.2-m VLT	UVES	Iodine Cell
9-m Hobby-Eberly	HRS	Iodine Cell
10-m Keck	HiRes	Iodine Cell