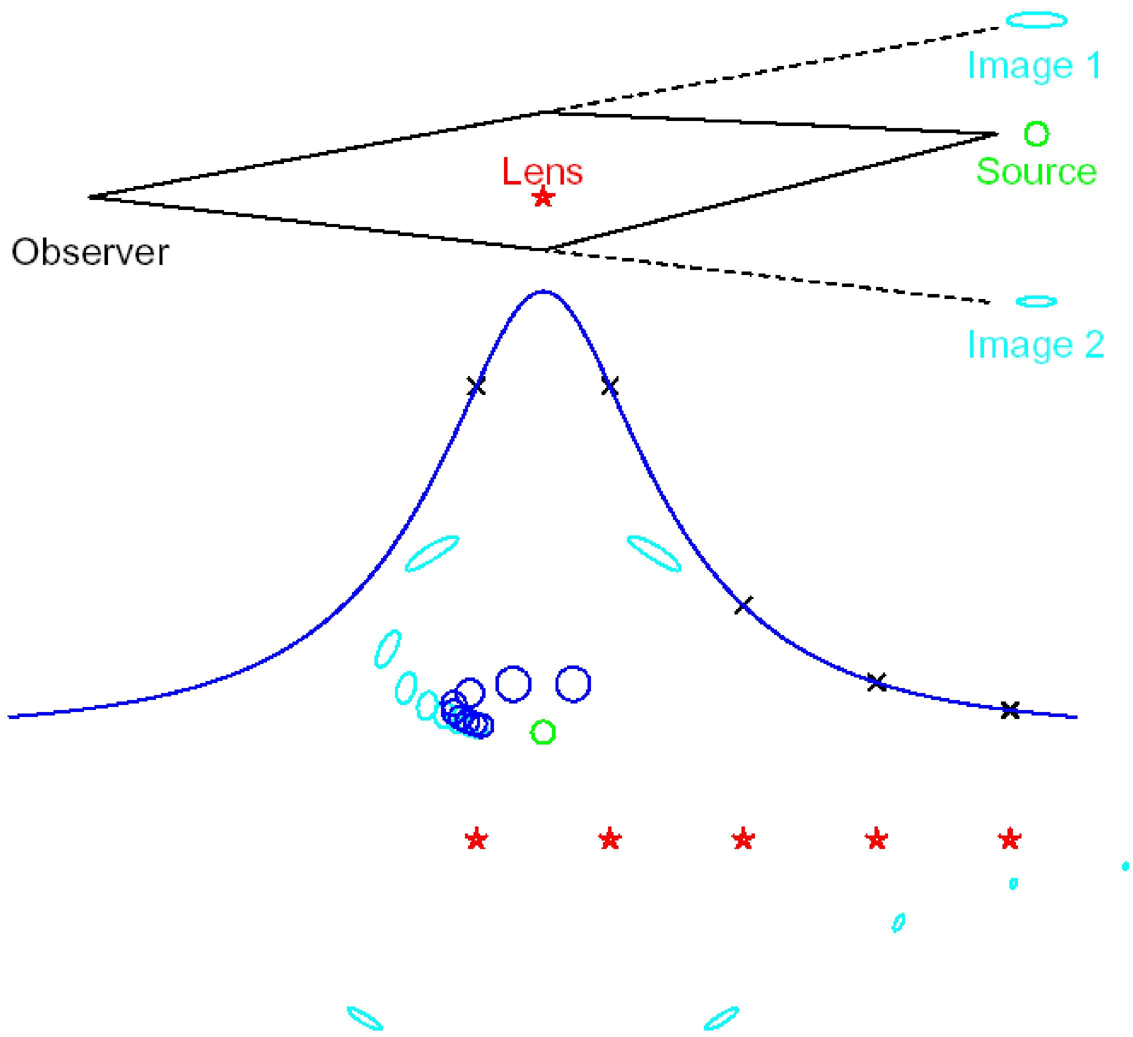
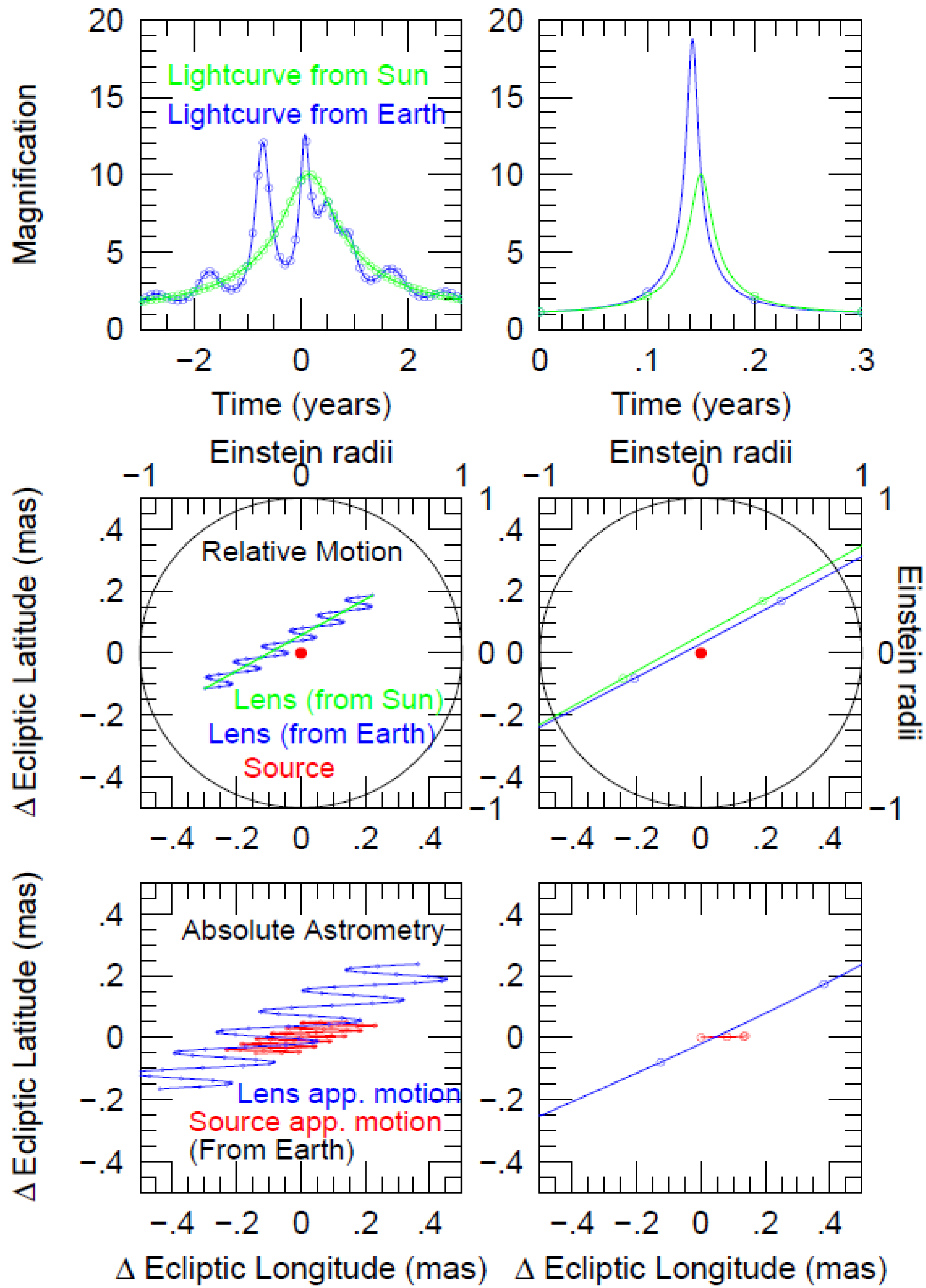
Microlensing III:

Higher-Order Effects and Degeneracies

Andy Gould (Ohio State)

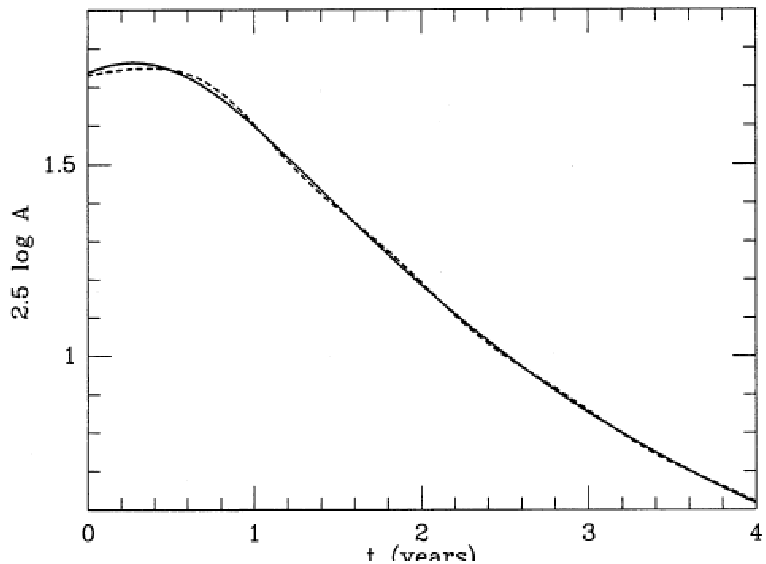


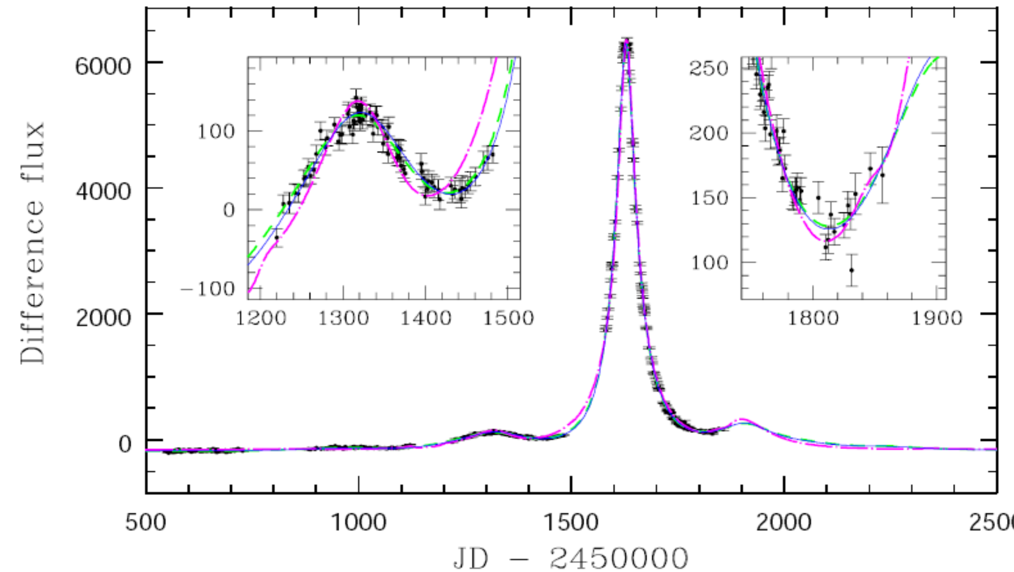


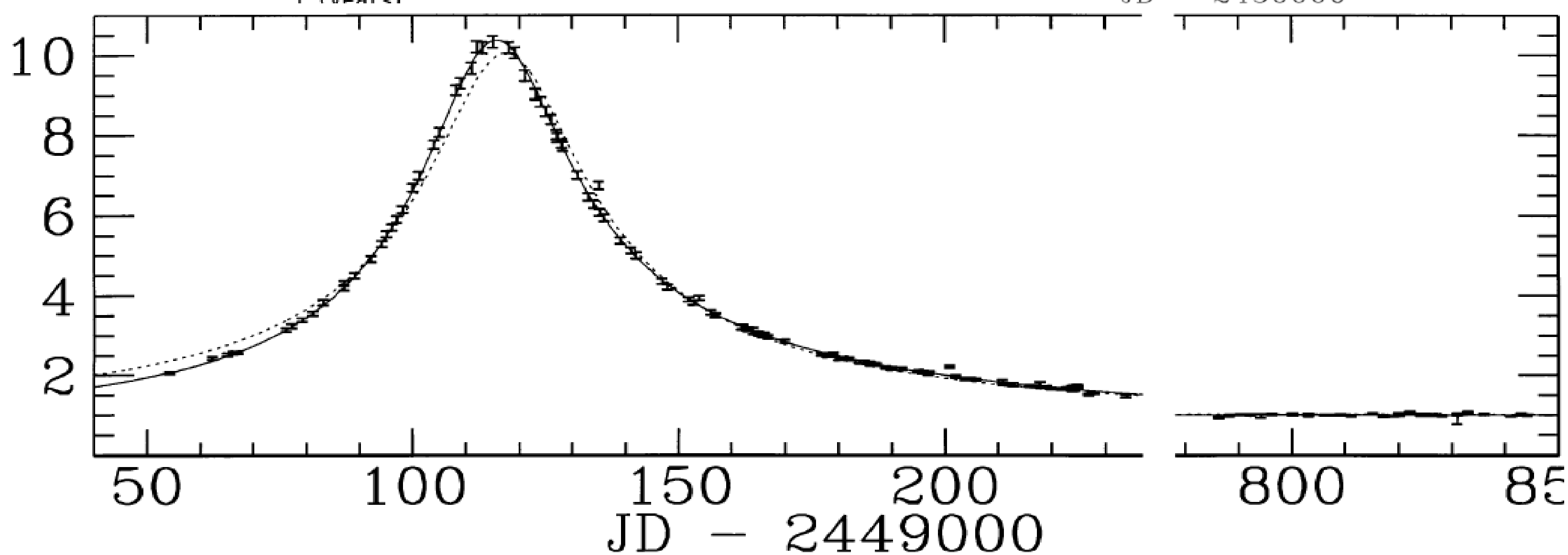
Magnification
Lightcurve from Sun
Lightcurve from Earth
Time (years)
Einstein radii
Relative Motion
Lens (from Sun)
Lens (from Earth)
Source
Δ Ecliptic Latitude (mas)
Absolute Astrometry
Lens app. motion
Source app. motion
(From Earth)
Δ Ecliptic Longitude (mas)

To measure parallax:

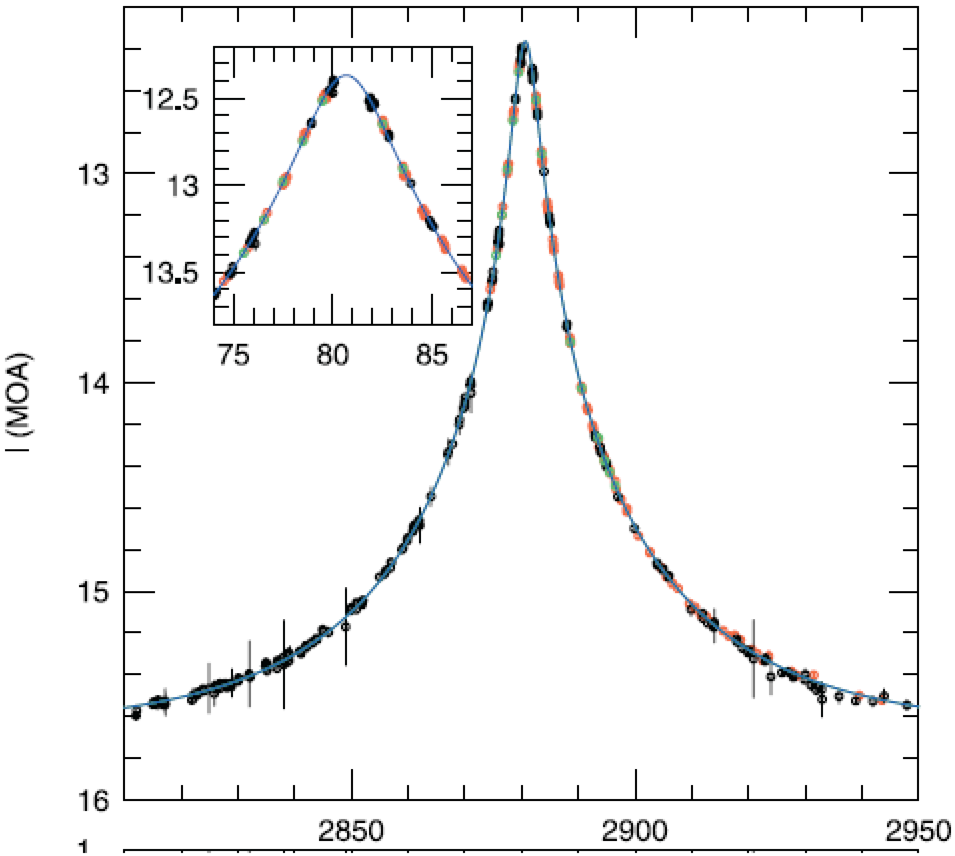
Standard Observer-Plane Rulers





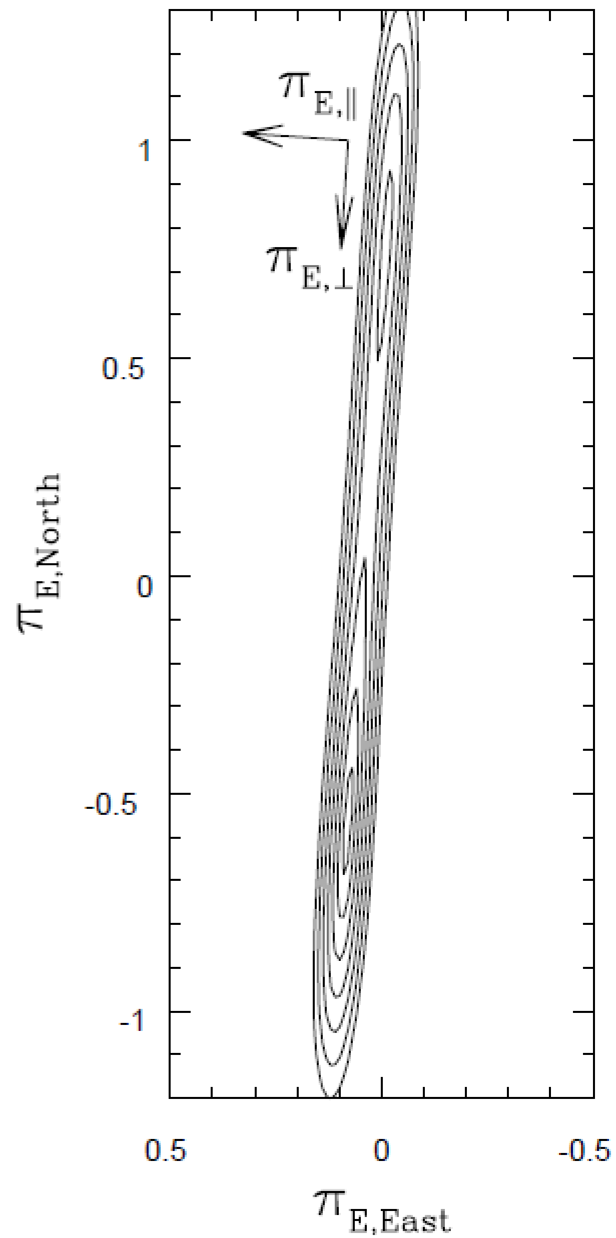


1-D Parallaxes Are "Common"



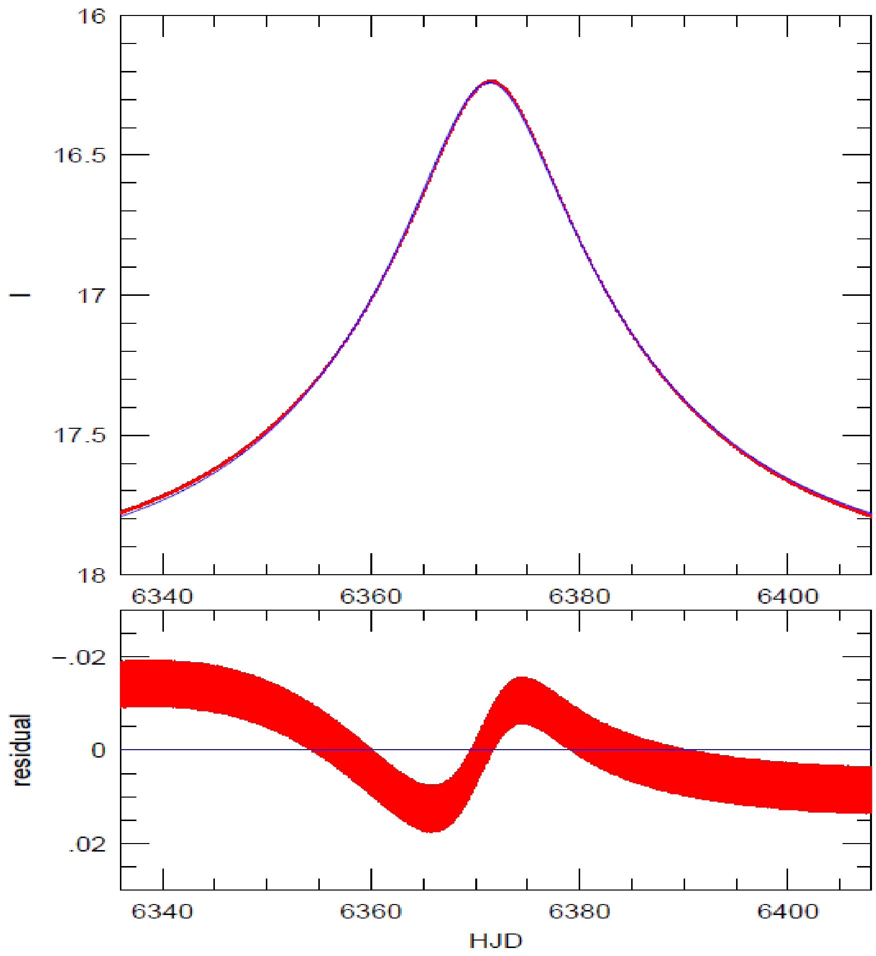
MOA-2003-BLG-37

Park et al. 2004, ApJ. 609, 166



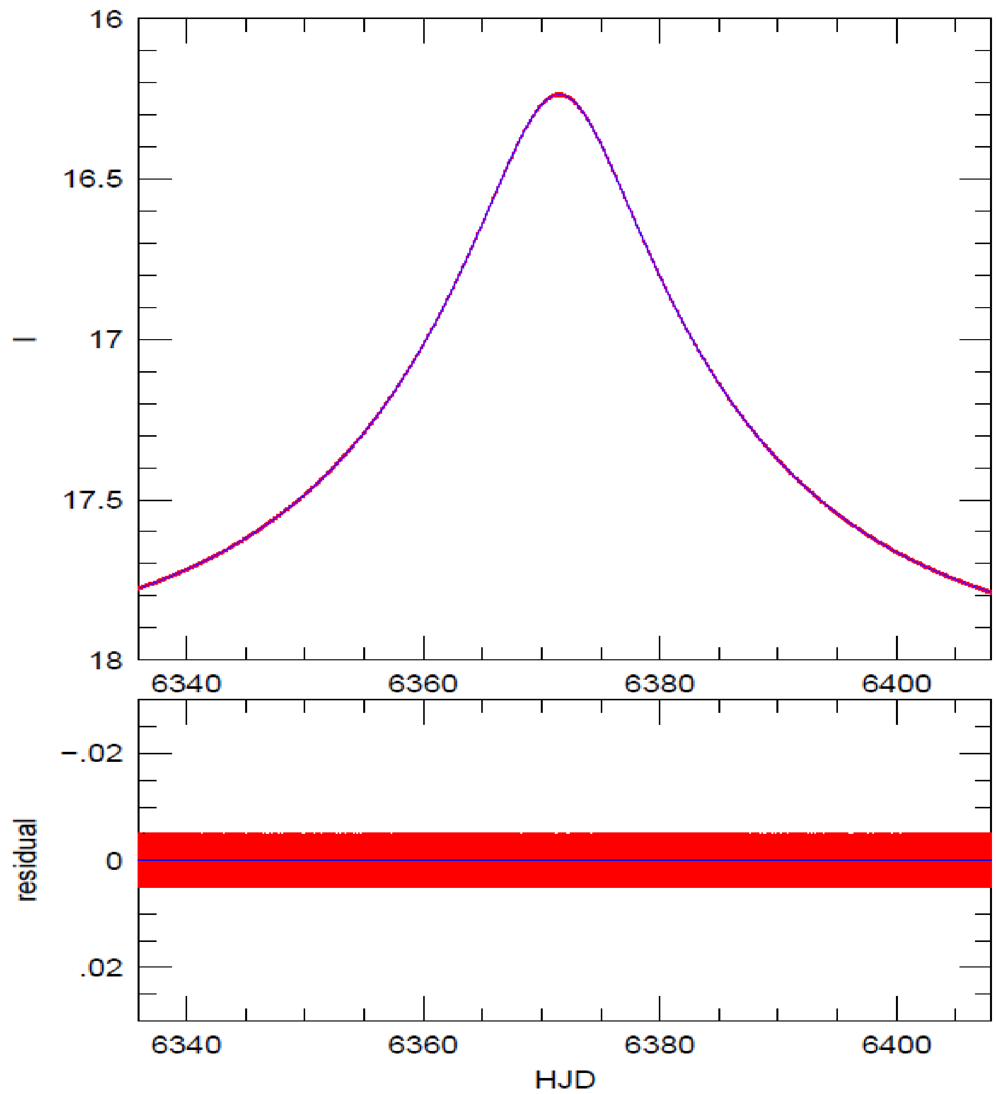
$\pi_{E,parallel}$ (square peg: round hole)

Component of π_E toward Sun

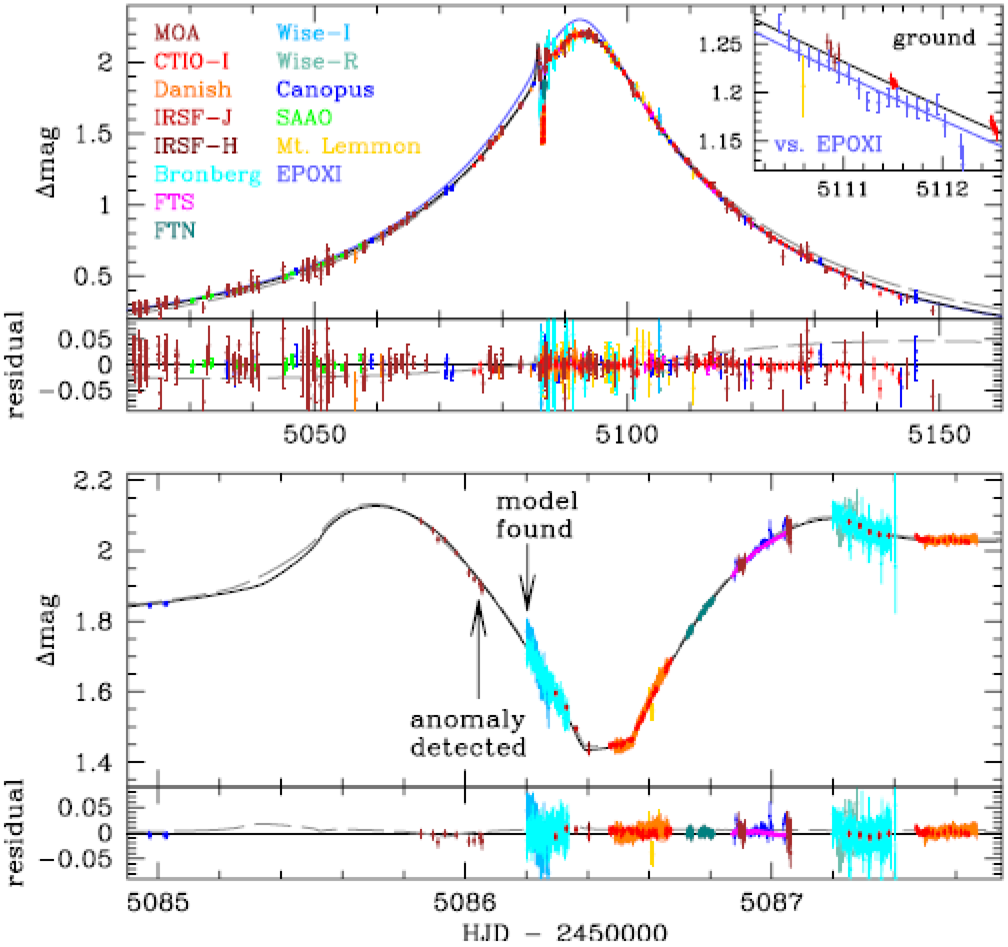


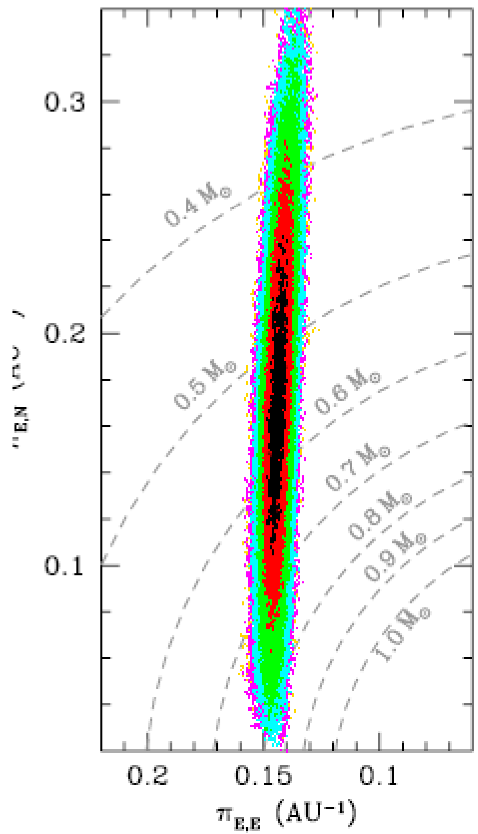
$\pi_{E,perp}$ (round peg: round hole)

Component of π_E perp to Sun



1-D Parallaxes Are "Common"

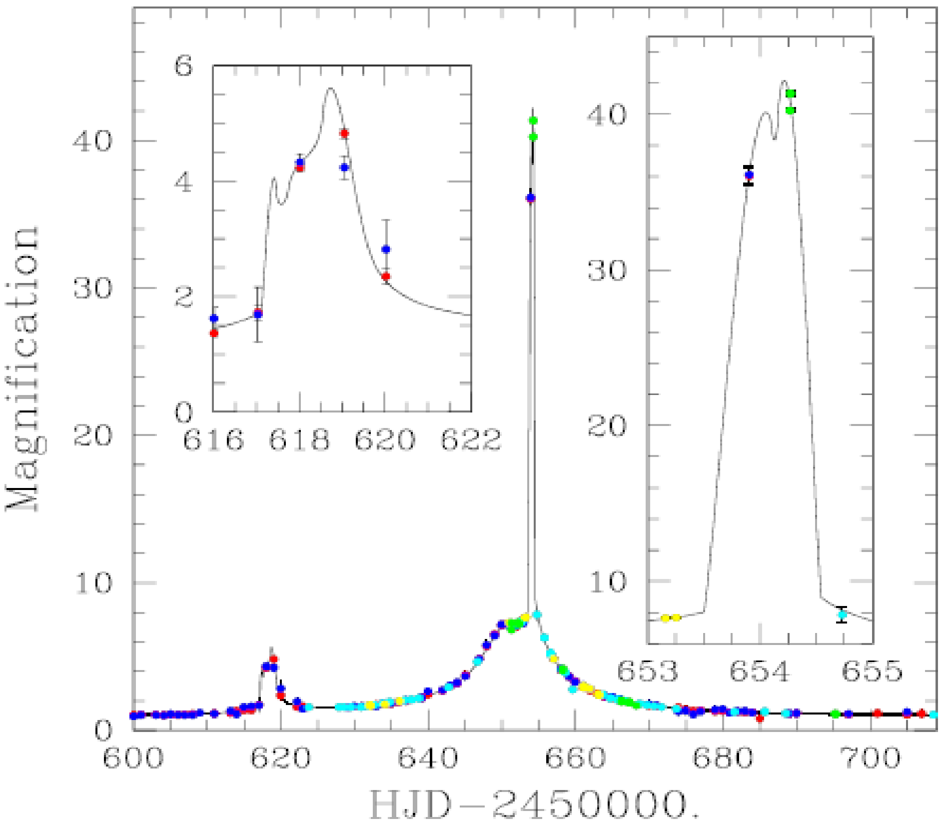


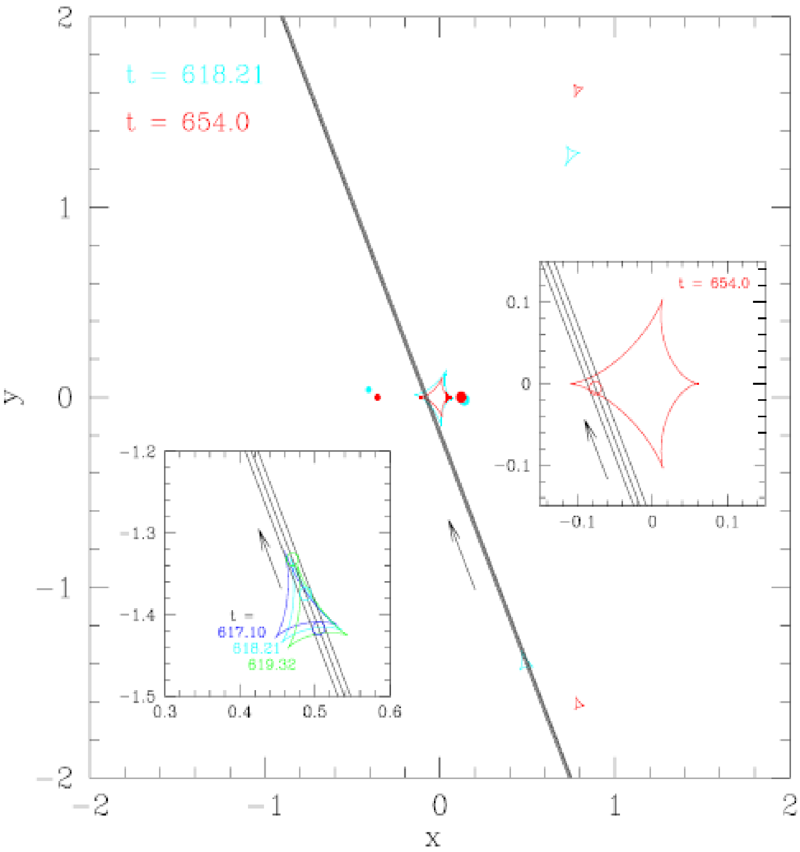


MOA-2009-BLG-266
Muraki et al. 2011, ApJ, 741, 22

Macho 97-41: Obvious Orbital Motion

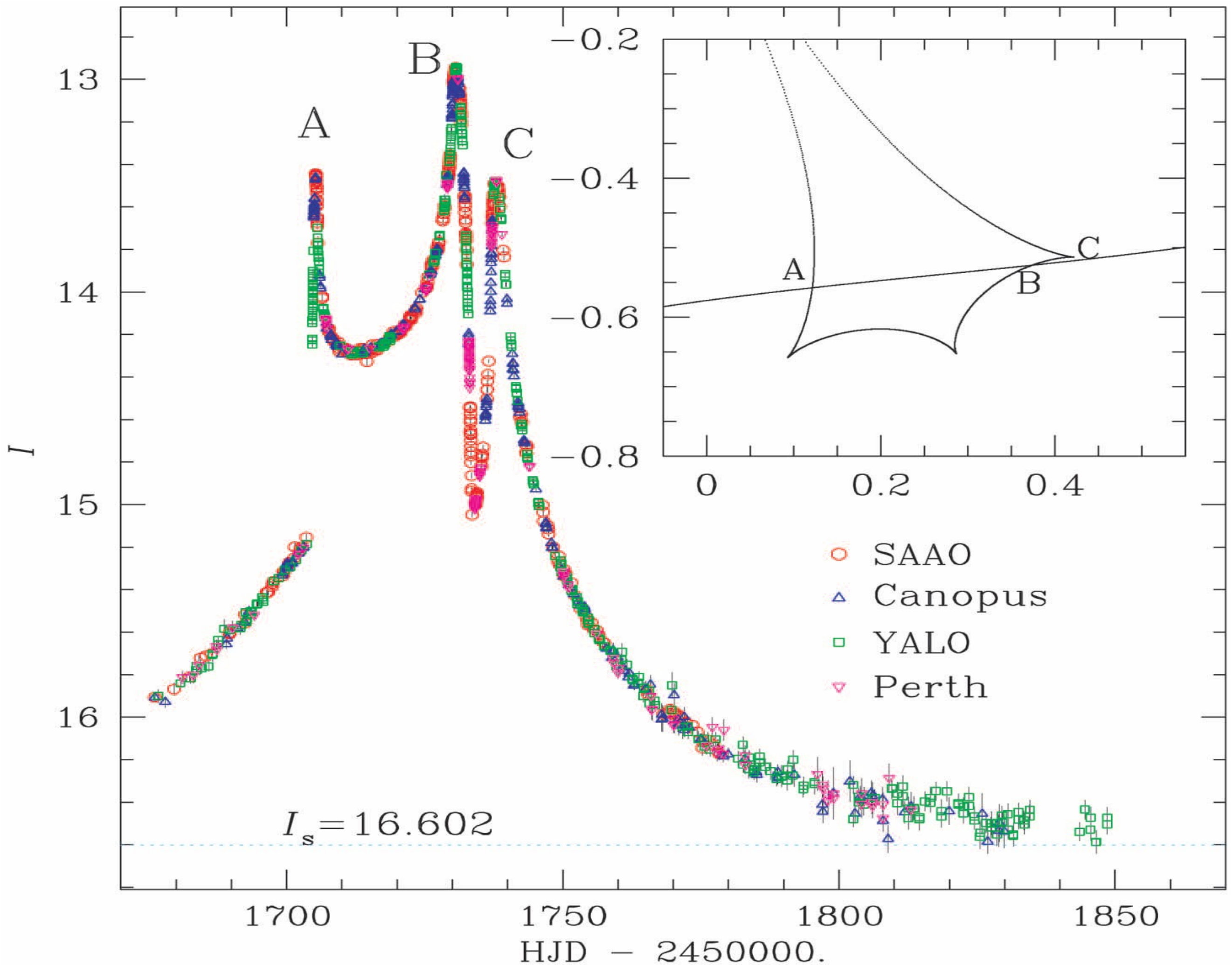
(But No Parallax)





EROS-BLG-2000-5

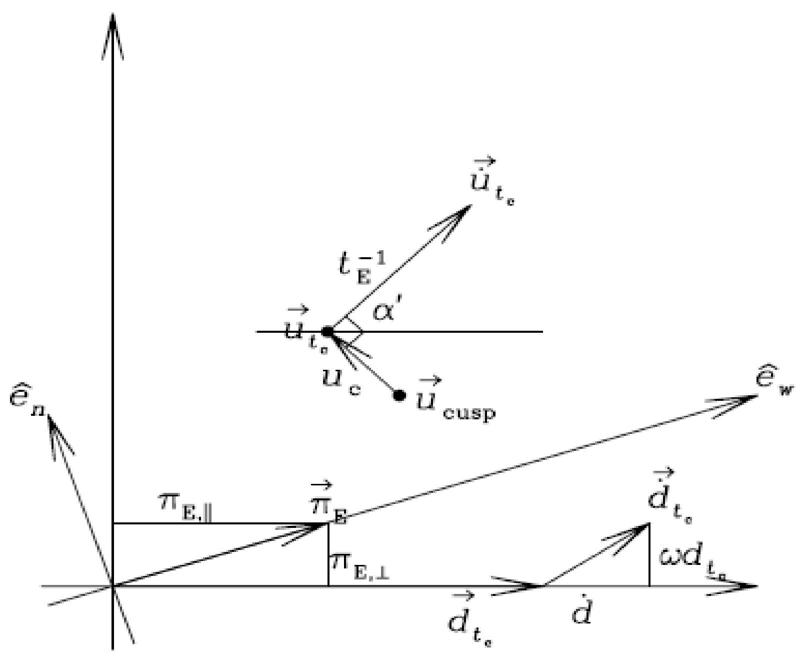
Parallax + Orbital Motion

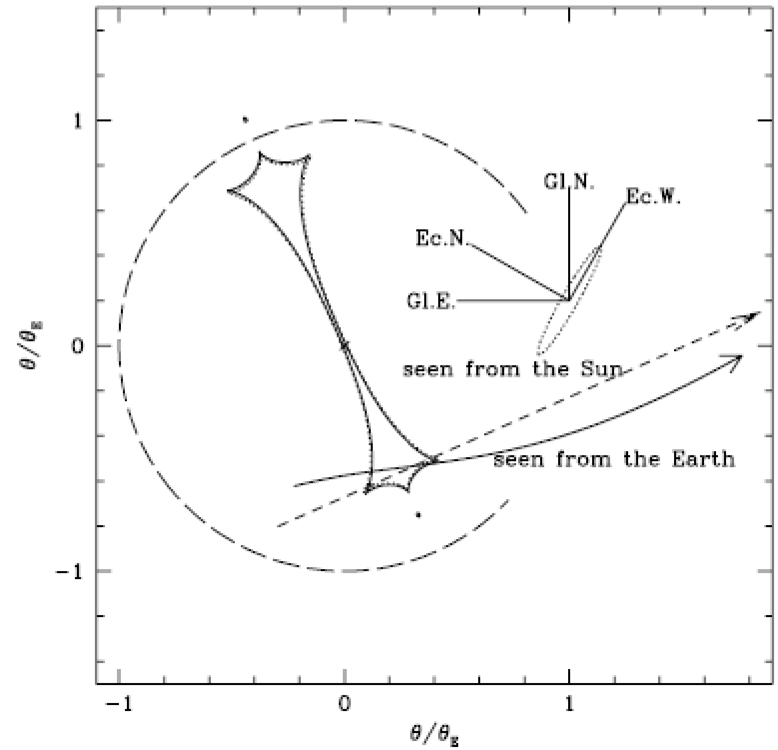


An et al. 2002, ApJ, 572, 521

EROS-BLG-2000-5

Parallax + Orbital Motion





An et al. 2002, ApJ, 572, 521

EROS-BLG-2000-5

Parallax from Triple-Peak Events

MICROLENS MASS MEASUREMENT USING TRIPLE-PEAK EVENTS

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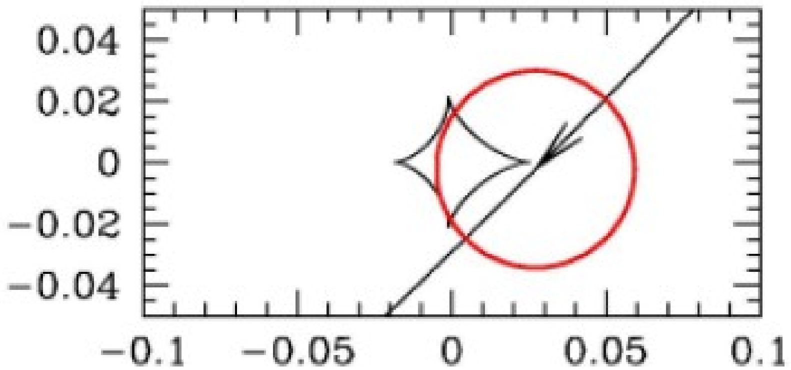
ABSTRACT

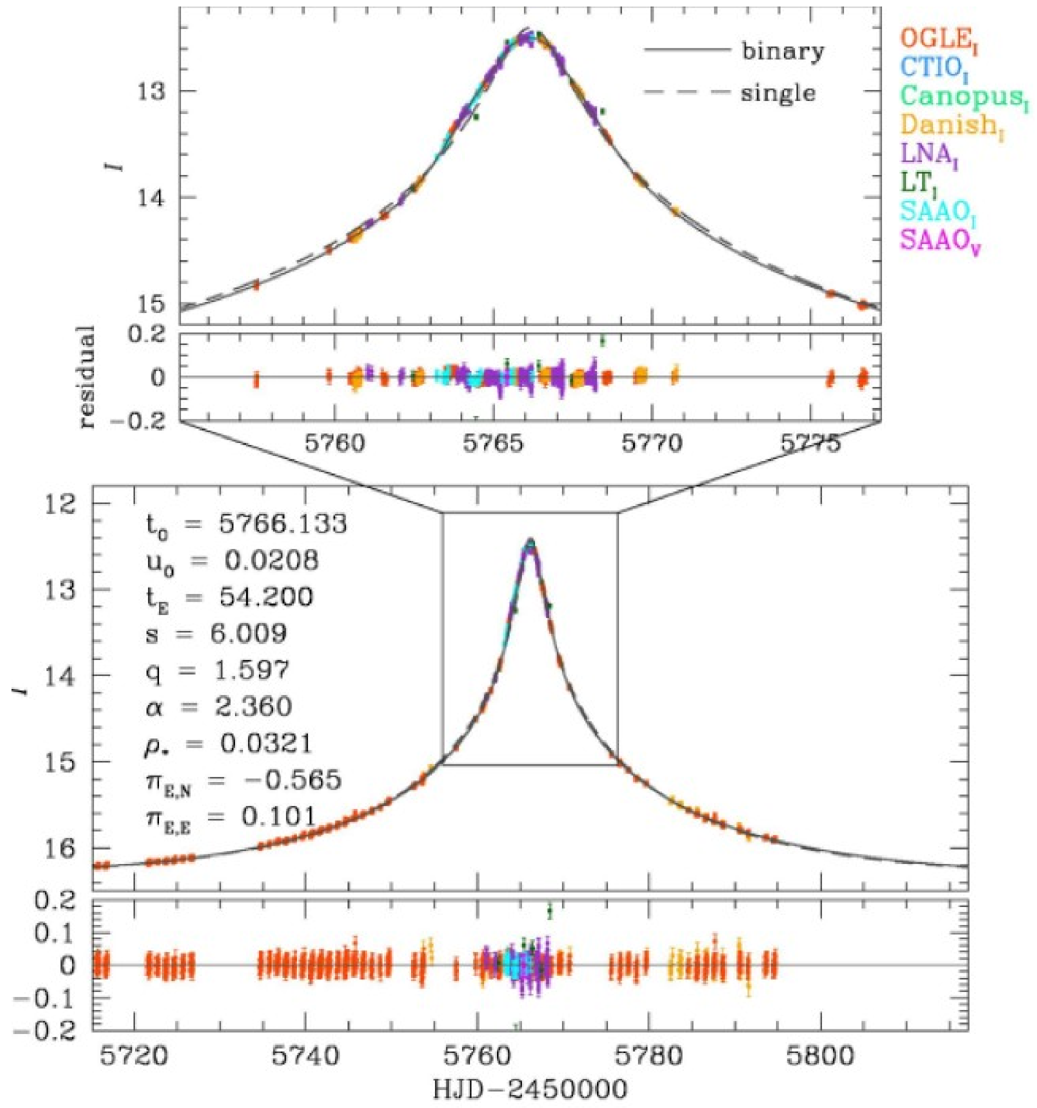
We show that one can measure the effects of microlens parallax for binary microlensing events with three well-measured peaks, two caustic crossings plus a cusp approach, and hence derive the projected Einstein radius $\tilde{r}_E$. Since the angular Einstein radius θ_E is measurable from finite-source effects for almost any well-observed caustic crossing, triple-peak events can yield the determination of the lens mass $M = (c^2/4G)\tilde{r}_E\theta_E$. We note that, to a certain extent, rotation of the binary can mimic the effects of parallax, but it should often be possible to disentangle parallax from rotation by making use of the late-time light curve.

An & Gould 2001, ApJ, 563, L111

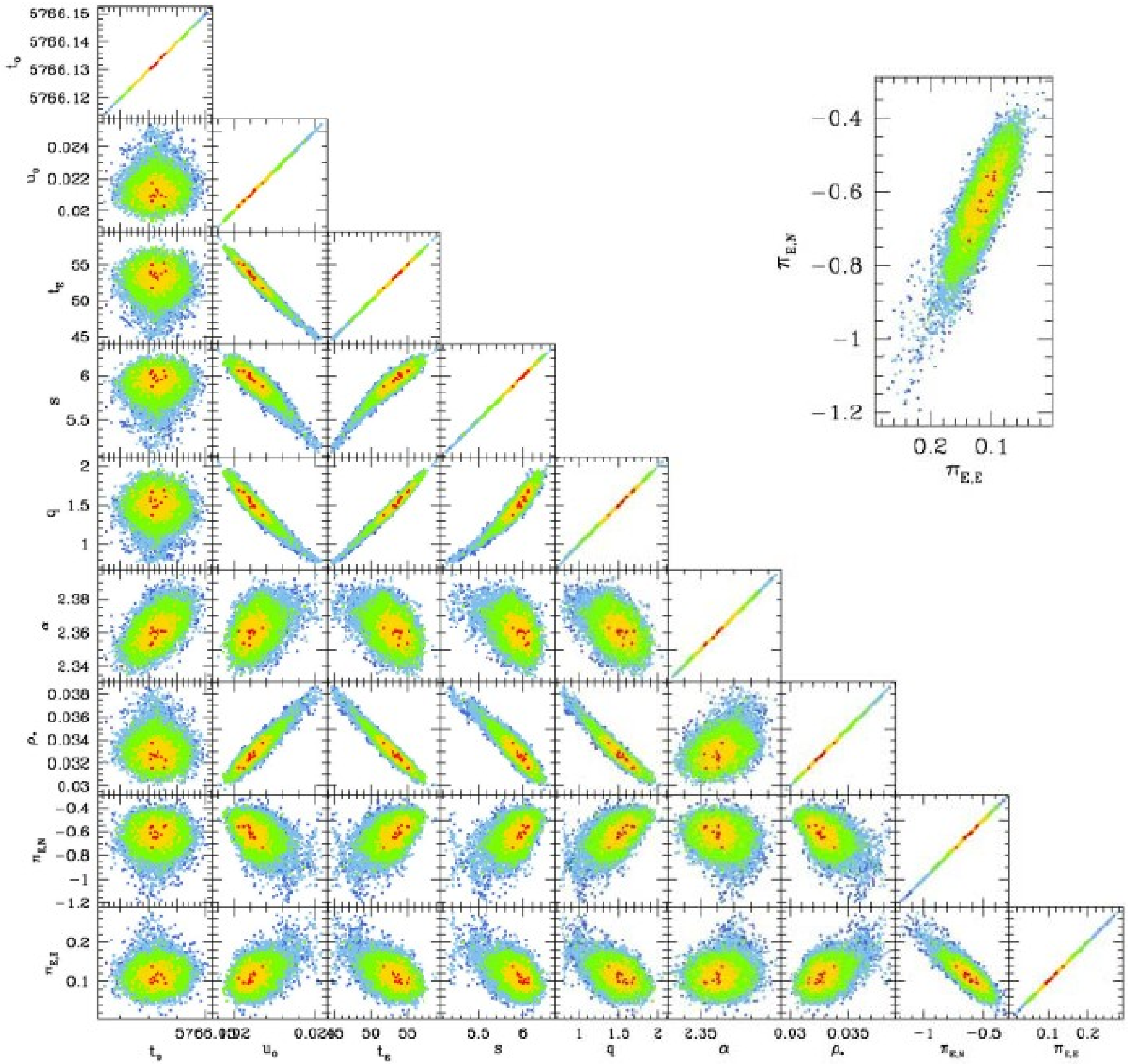
OGLE-2011-BLG-0420

Parallax + Orbital Motion





OGLE-2011-BLG-0420



OGLE-2011-BLG-0420

parameter	close	
	$u_0 > 0$	$u_0 < 0$
χ^2/dof	5427.4	5410.8
t_0 (HJD')	5766.110	5766.109
u_0	0.031	-0.030
t_E (days)	34.89	35.27
s	0.287	0.290
q	0.388	0.368
α	2.387	-2.383
$\rho_\star$	0.049	0.049
$\pi_{E,N}$	-1.03	-1.15
$\pi_{E,E}$	0.23	0.19
ds/dt (yr^{-1})	-2.44	-2.48
$d\alpha/dt$ (yr^{-1})	-8.09	7.08
KE/PE	0.36	0.32

quantity	close ($u_0 < 0$)
M_1	0.024± 0.001 $M_\odot$
M_2	0.0088 ±0.0005 $M_\odot$ (9.3 ±0.5 M_J)
D_L (kpc)	2.1 ±0.1
projected separation (AU)	0.19 ±0.01

Planet Lenses: + Projected Motion

11 Features & 11 Parameters

3 Point-Lens	t_0, u_0, t_E
3 Binary-Lens	α_0, s_0, q
Width of Caustic Cr.	$t_* = \rho * t_E$
Symmetric Distortion	$\pi_{E,perp}$
Anti-symmetric Dist.	$\pi_{E,parallel}$
Rotational Motion	$\gamma_{perp} = d\alpha/dt$
Radial Motion	$\gamma_{parallel} = (ds/dt)/s_0$

(KE/PE)_perp: Ratio of Transverse Kinetic to Potential Energy

$$\mathrm{KE} = \frac{M_1 M_2}{M_1+M_2}\frac{v_{\rm rel}^2}{2}; \quad \mathrm{PE} = \frac{GM_1M_2}{r}$$

$$(\mathrm{KE})_\perp \equiv \frac{M_1 M_2}{M_1+M_2}\frac{v_\perp^2}{2}; \quad (\mathrm{PE})_\perp \equiv \frac{GM_1M_2}{r_\perp}$$

$$\left(\frac{\mathrm{KE}}{\mathrm{PE}}\right)_\perp = \left(\frac{\mathrm{KE}}{\mathrm{PE}}\right)\left(\frac{v_{\rm rel}}{v_\perp}\right)^2\frac{r_\perp}{r} \le \left(\frac{\mathrm{KE}}{\mathrm{PE}}\right)$$

$$\left(\frac{\mathrm{KE}}{\mathrm{PE}}\right)_\perp = \frac{r_\perp v_{\rm rel}^2}{2GM} = \frac{r_\perp^3\gamma^2}{2GM}$$

$$r_\perp = D_{\rm L}\theta_{\rm E}s = \frac{\mathrm{AU}\,\theta_{\rm E}s}{\pi_{\rm E}\theta_{\rm E}+\pi_s} = \frac{\mathrm{AU}\,s}{\pi_{\rm E}+\pi_s/\theta_{\rm E}}$$

$$\frac{\mathrm{AU}^3}{GM_\odot} = \left(\frac{\mathrm{yr}}{2\pi}\right)^2; \quad \frac{M}{M_\odot} = \frac{\theta_{\rm E}}{\kappa M_\odot \pi_{\rm E}} = \frac{\theta_{\rm E}/8.14\,\mathrm{mas}}{\pi_{\rm E}}$$

$$\left(\frac{\mathrm{KE}}{\mathrm{PE}}\right)_\perp = \frac{8.14}{8\pi^2}\frac{\pi_{\rm E}s^3(\gamma\,\mathrm{yr})^2}{(\theta_{\rm E}/\mathrm{mas})(\pi_E+\pi_s/\theta_E)^3}$$

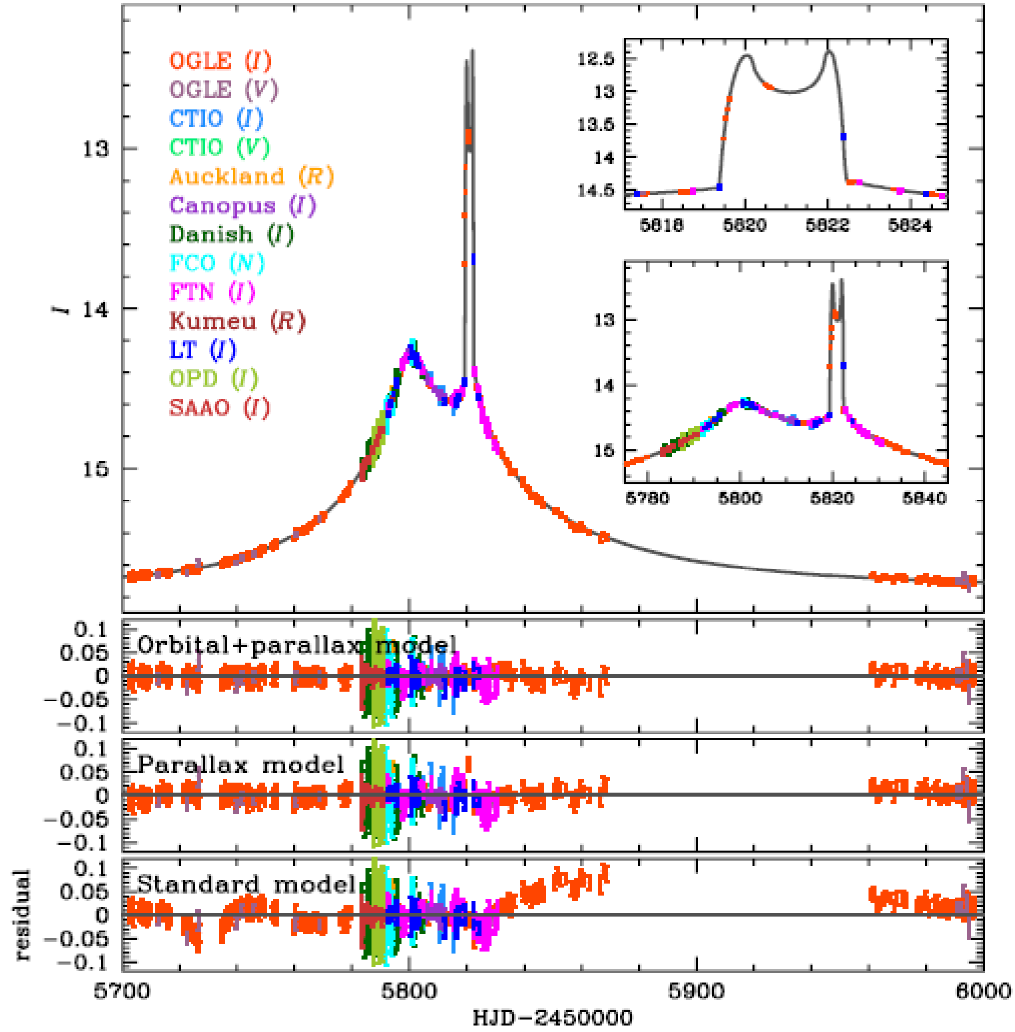
Complete Orbital Motion

13 "Features" & 13 Parameters

3 Point-Lens	t_0, u_0, t_E
3 Binary-Lens	α_0, s_0, q
Width of Caustic Cr.	$t_* = \rho * t_E$
2 Parallax	$\pi_{E,perp}$, $\pi_{E,parallel}$
2 Transverse Motion	γ_{perp}, $\gamma_{parallel}$
Out-of-plane Position	$s_{parallel}$
Out-of-plane Motion	$ds_{parallel}/dt$

OGLE-2011-BLG-0417

Complete Orbital Solution



Shin et al. 2012, ApJ, 755, 91

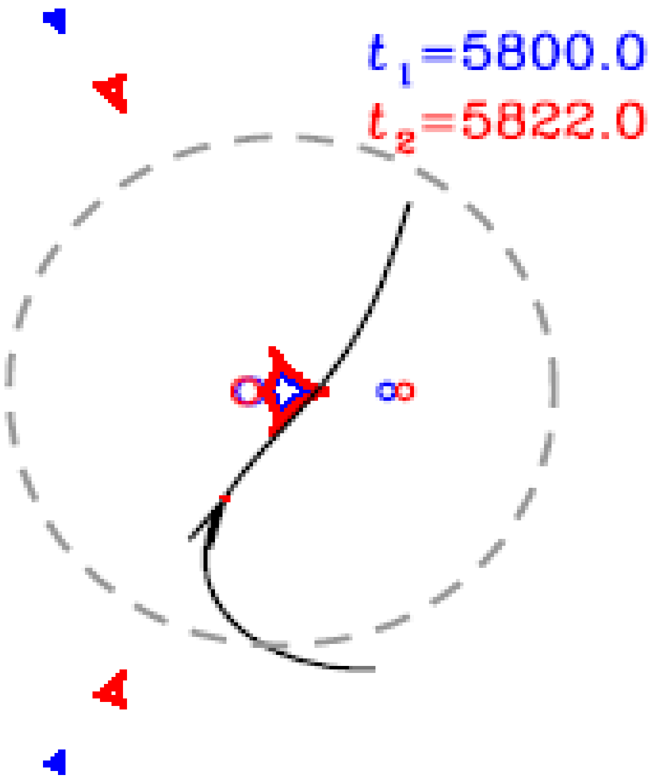
OGLE-2011-BLG-0417

Complete Orbital Solution

Parameters	Standard	Model Parallax	Orbital+Parallax
χ^2/dof	4415/2627	2391/2625	1735/2621
t_0 (HJD′)	5817.302 ± 0.018	5815.867 ± 0.030	5813.306 ± 0.059
u_0	0.1125 ± 0.0001	-0.0971 ± 0.0003	-0.0992 ± 0.0005
t_E (days)	60.74 ± 0.08	79.59 ± 0.36	92.26 ± 0.37
$s_\perp$	0.601 ± 0.001	0.574 ± 0.001	0.577 ± 0.001
q	0.402 ± 0.002	0.287 ± 0.002	0.292 ± 0.002
α (rad)	1.030 ± 0.002	-0.951 ± 0.002	-0.850 ± 0.004
ρ_* (10^{-3})	3.17 ± 0.01	2.38 ± 0.02	2.29 ± 0.02
$\pi_{E,N}$	...	0.125 ± 0.004	0.375 ± 0.015
$\pi_{E,E}$	...	-0.111 ± 0.005	-0.133 ± 0.003
$ds_\perp/dt$ (yr^{-1})	...	...	1.314 ± 0.023
$d\alpha/dt$ (yr^{-1})	...	...	1.168 ± 0.076
$s_\parallel$	...	...	0.467 ± 0.020
$ds_\parallel/dt$ (yr^{-1})	...	...	-0.192 ± 0.036

OGLE-2011-BLG-0417

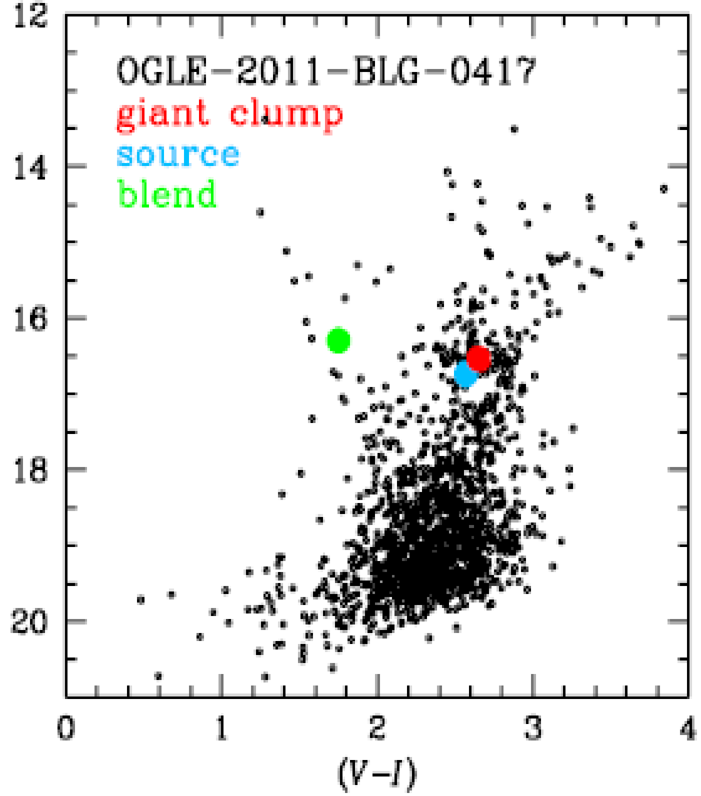
Complete Orbital Solution



Parameter	OGLE-2011-BLG-0417
M_{tot} ($M_\odot$)	0.74 ± 0.03
M_1 ($M_\odot$)	0.57 ± 0.02
M_2 ($M_\odot$)	0.17 ± 0.01
θ_E (mas)	2.44 ± 0.02
μ (mas yr^{-1})	9.66 ± 0.07
D_L (kpc)	0.89 ± 0.03
a (AU)	1.15 ± 0.04
P (yr)	1.44 ± 0.06
e	0.68 ± 0.02
i (deg)	116.95 ± 1.04

OGLE-2011-BLG-0417

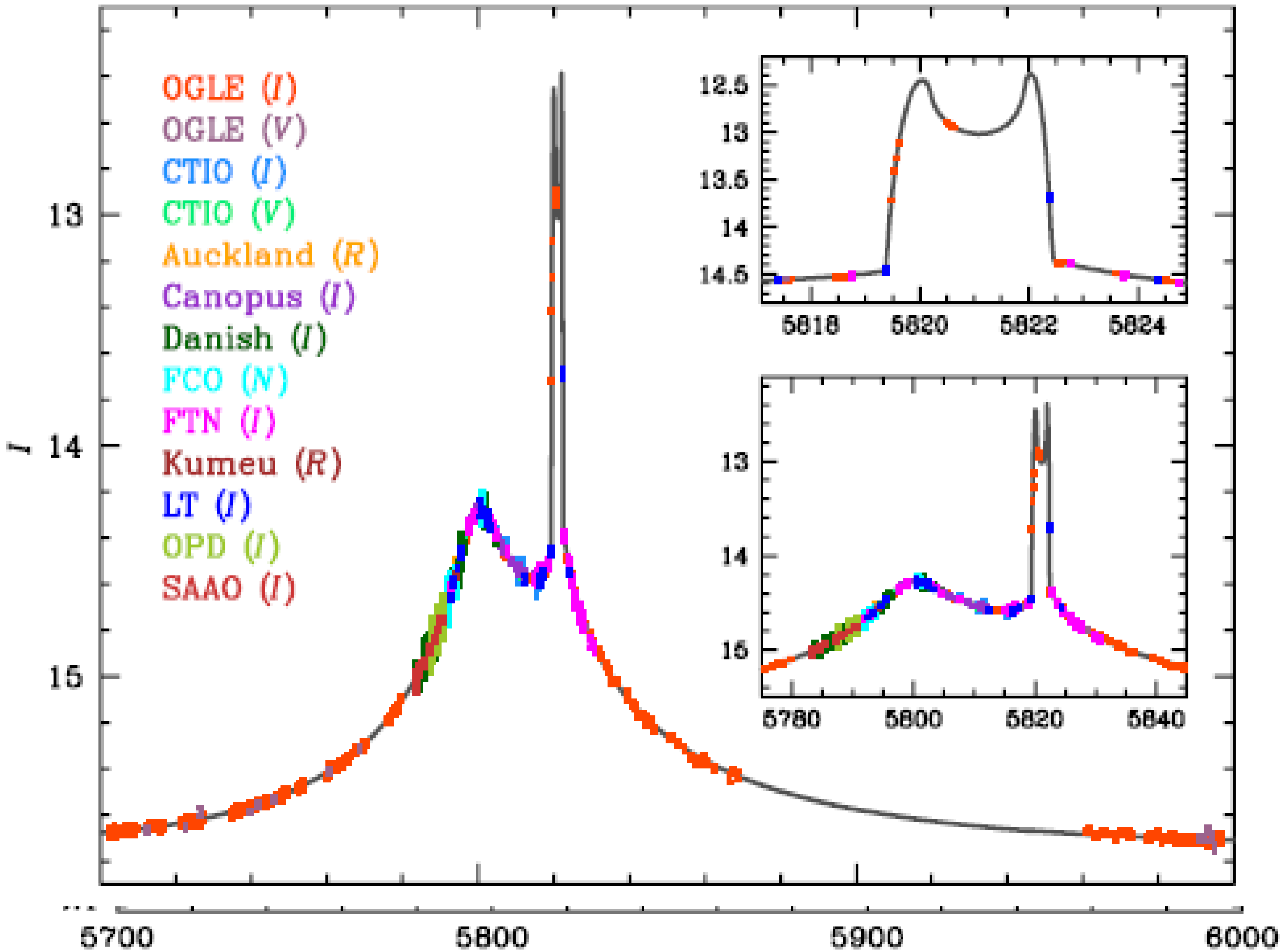
Complete Orbital Solution



ants MOA 2011 BLG 090 (left panel) and OGLE 2011 F

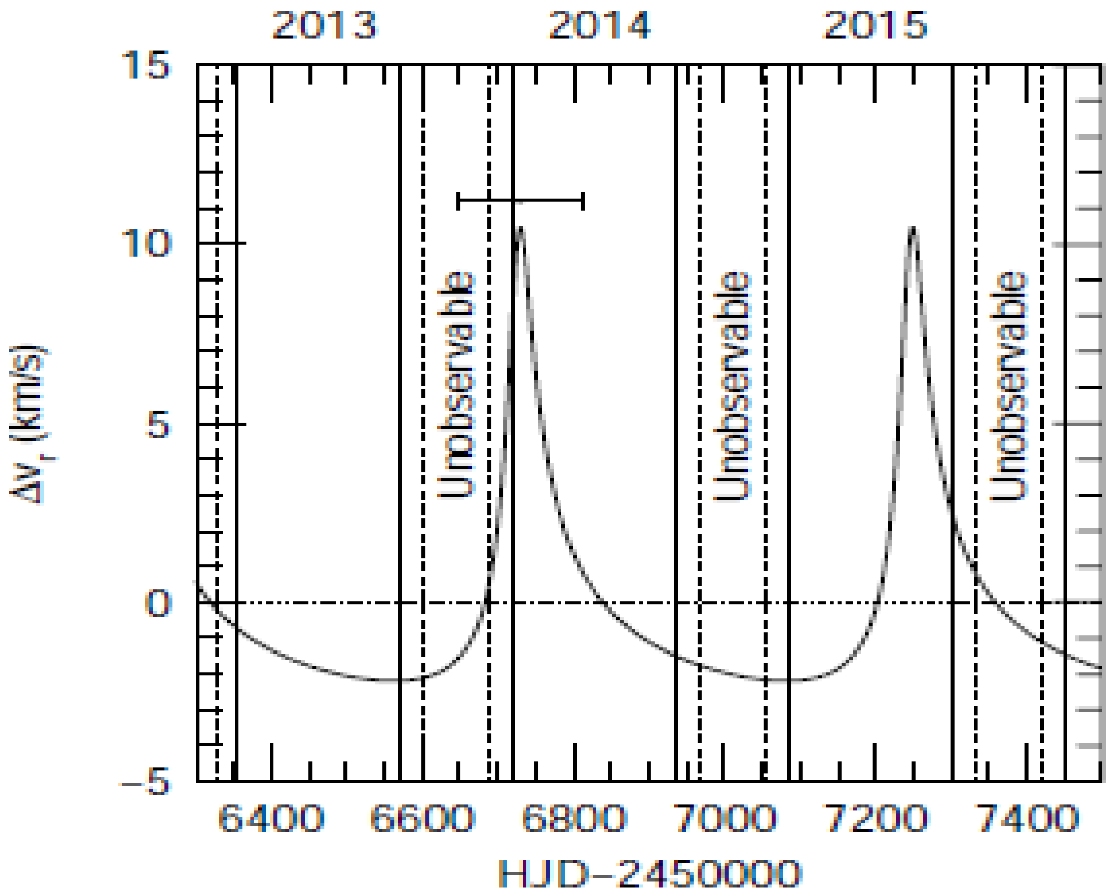
Parameter	OGLE-2011-BLG-0417
M_{tot} ($M_\odot$)	0.74 ± 0.03
M_1 ($M_\odot$)	0.57 ± 0.02
M_2 ($M_\odot$)	0.17 ± 0.01
θ_E (mas)	2.44 ± 0.02
μ (mas yr^{-1})	9.66 ± 0.07
D_L (kpc)	0.89 ± 0.03
a (AU)	1.15 ± 0.04
P (yr)	1.44 ± 0.06
e	0.68 ± 0.02
i (deg)	116.95 ± 1.04

OGLE-2011-BLG-0417



OGLE-2011-BLG-0417

Predictions for RV



Gould et al. 2013, ApJ, 786, 126

OGLE-2011-BLG-0417

Predictions for RV

Table 1: Microlens Measurements

	M_{tot}	M_2/M_1	P	e	i	ω	Ω	t_{peri}	D_L
	($M_\odot$)		(yr)		(deg)	(deg)	(deg)	(HJD)	(kpc)
Value	0.677	0.292	1.423	0.688	60.963	341.824	125.374	5686.344	0.951
Error	0.047	0.003	0.113	0.027	1.554	2.655	1.649	6.960	0.058
C.C.	1.000	−0.204	0.101	0.511	−0.024	−0.008	0.511	0.133	−0.065
C.C.	−0.204	1.000	−0.055	−0.150	0.161	−0.065	−0.302	−0.015	−0.136
C.C.	0.101	−0.055	1.000	−0.118	0.523	0.484	0.595	−0.756	0.791
C.C.	0.511	−0.150	−0.118	1.000	−0.217	−0.247	0.151	0.485	0.211
C.C.	−0.024	0.161	0.523	−0.217	1.000	−0.257	0.141	−0.781	0.093
C.C.	−0.008	−0.065	0.484	−0.247	−0.257	1.000	0.667	−0.013	0.518
C.C.	0.511	−0.302	0.595	0.151	0.141	0.667	1.000	−0.141	0.469
C.C.	0.133	−0.015	−0.756	0.485	−0.781	−0.013	−0.141	1.000	−0.367
C.C.	−0.065	−0.136	0.791	0.211	0.093	0.518	0.469	−0.367	1.000

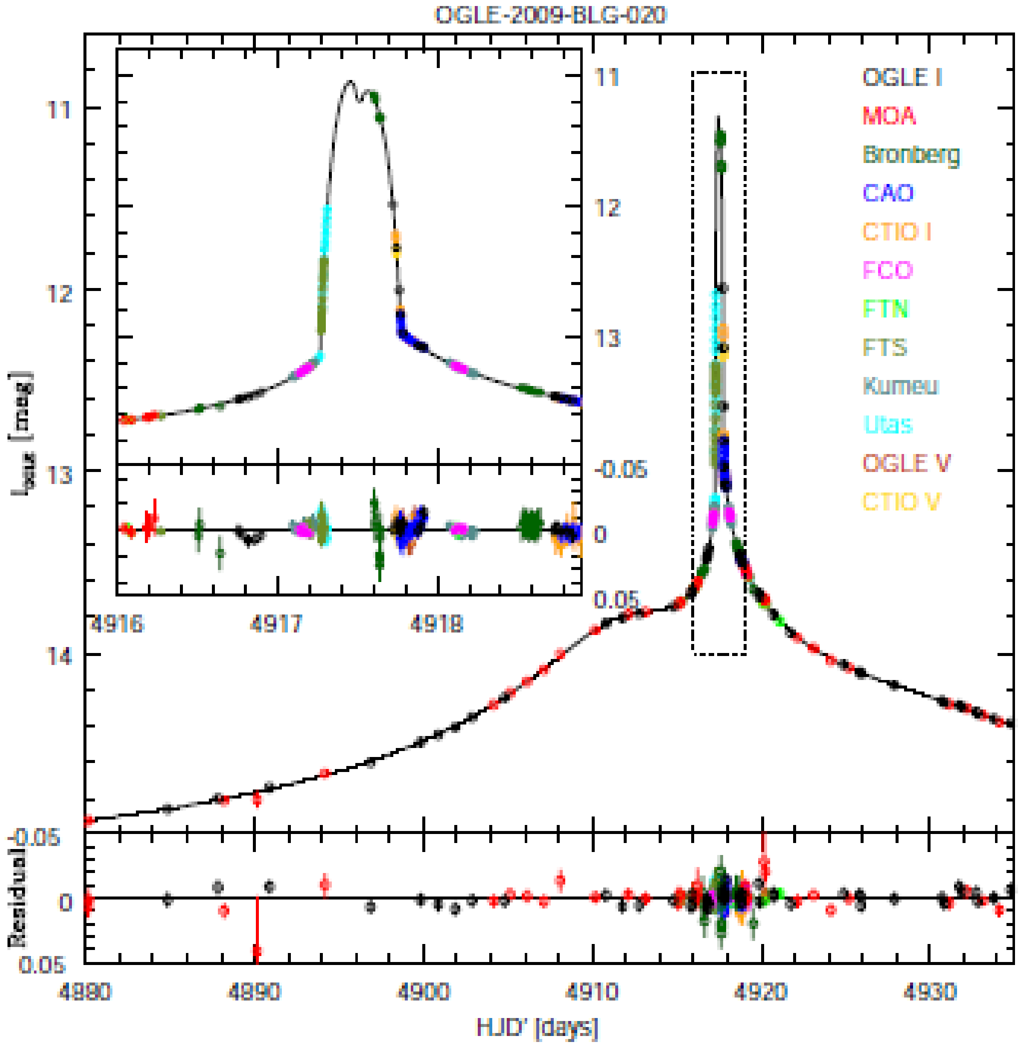
Table 2: Predictions for RV Measurements

	K	P	e	ω	t_{peri}	M_1	D_L
	(km s^{-1})	(yr)		(deg)	(HJD)	($M_\odot$)	(kpc)
Value	6.352	1.423	0.688	341.824	5686.344	0.524	0.951
Error	0.340	0.113	0.027	2.655	6.960	0.036	0.058
C.C.	1.000	−0.365	0.838	−0.473	0.503	0.693	−0.267
C.C.	−0.365	1.000	−0.118	0.484	−0.756	0.101	0.791
C.C.	0.838	−0.118	1.000	−0.247	0.485	0.511	0.211
C.C.	−0.473	0.484	−0.247	1.000	−0.013	−0.008	0.518
C.C.	0.503	−0.756	0.485	−0.013	1.000	0.133	−0.367
C.C.	0.693	0.101	0.511	−0.008	0.133	1.000	−0.065
C.C.	−0.267	0.791	0.211	0.518	−0.367	−0.065	1.000

Gould et al. 2013, ApJ, 786, 126

OGLE-2009-BLG-020

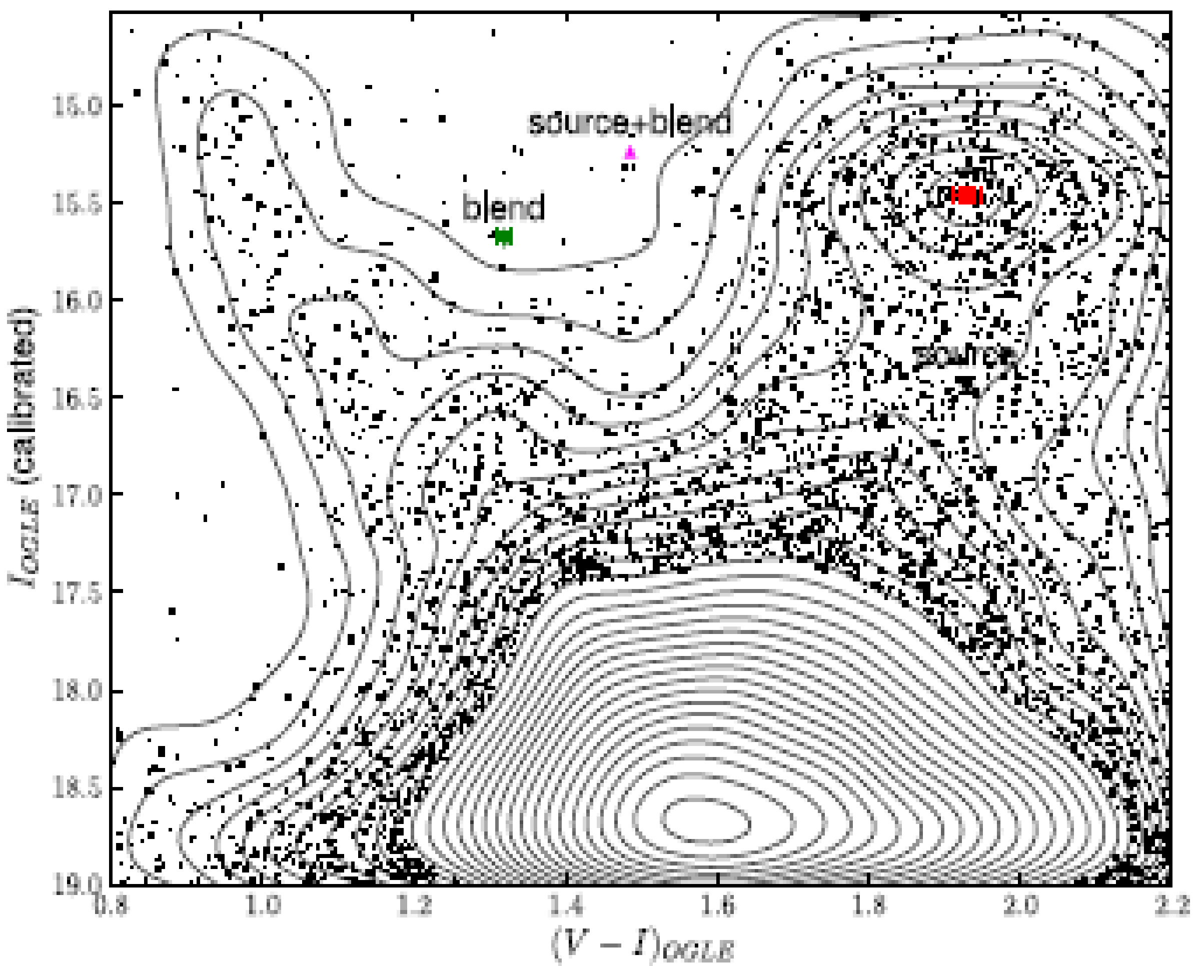
Predictions for RV



Skowron et al. 2011, ApJ, 738, 87

OGLE-2009-BLG-020

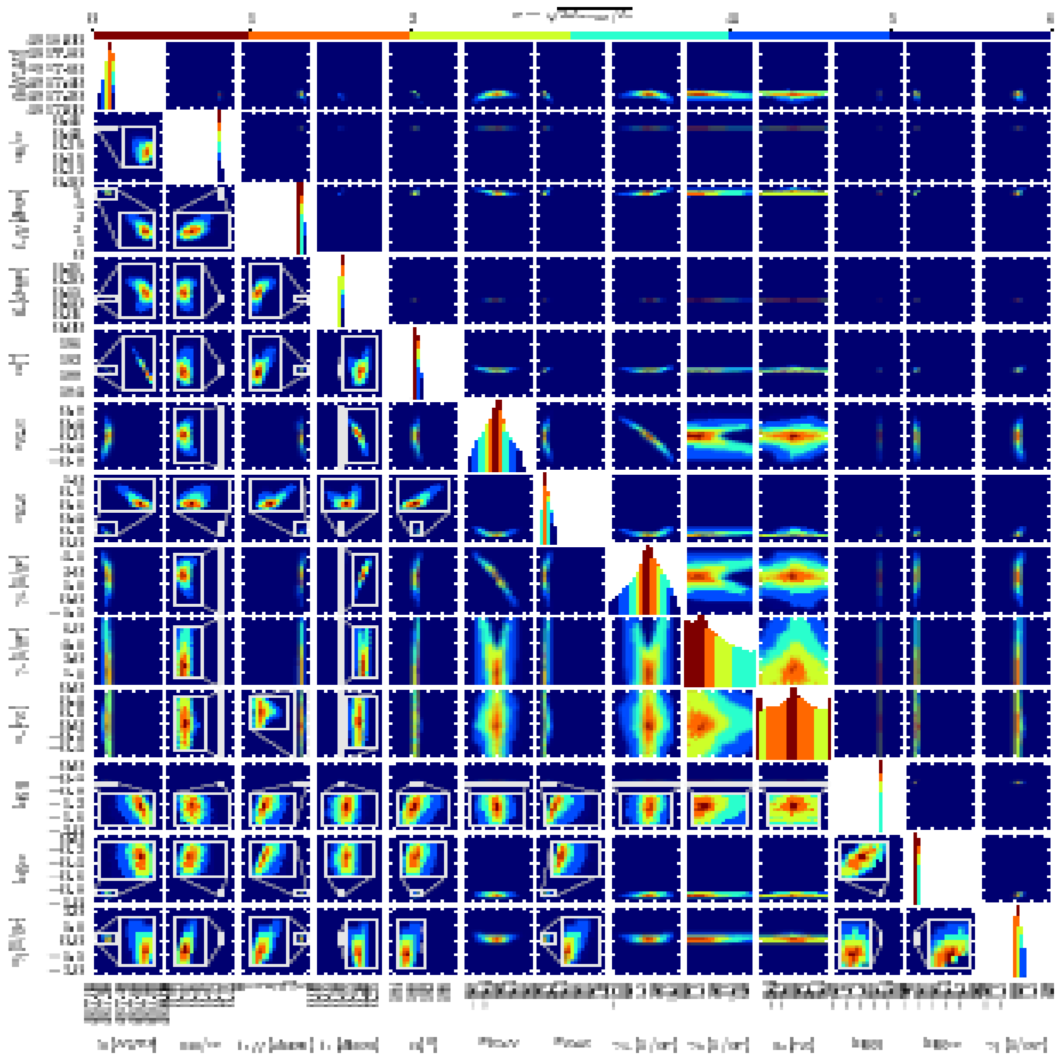
Predictions for RV



Skowron et al. 2011, ApJ, 738, 87

OGLE-2009-BLG-020

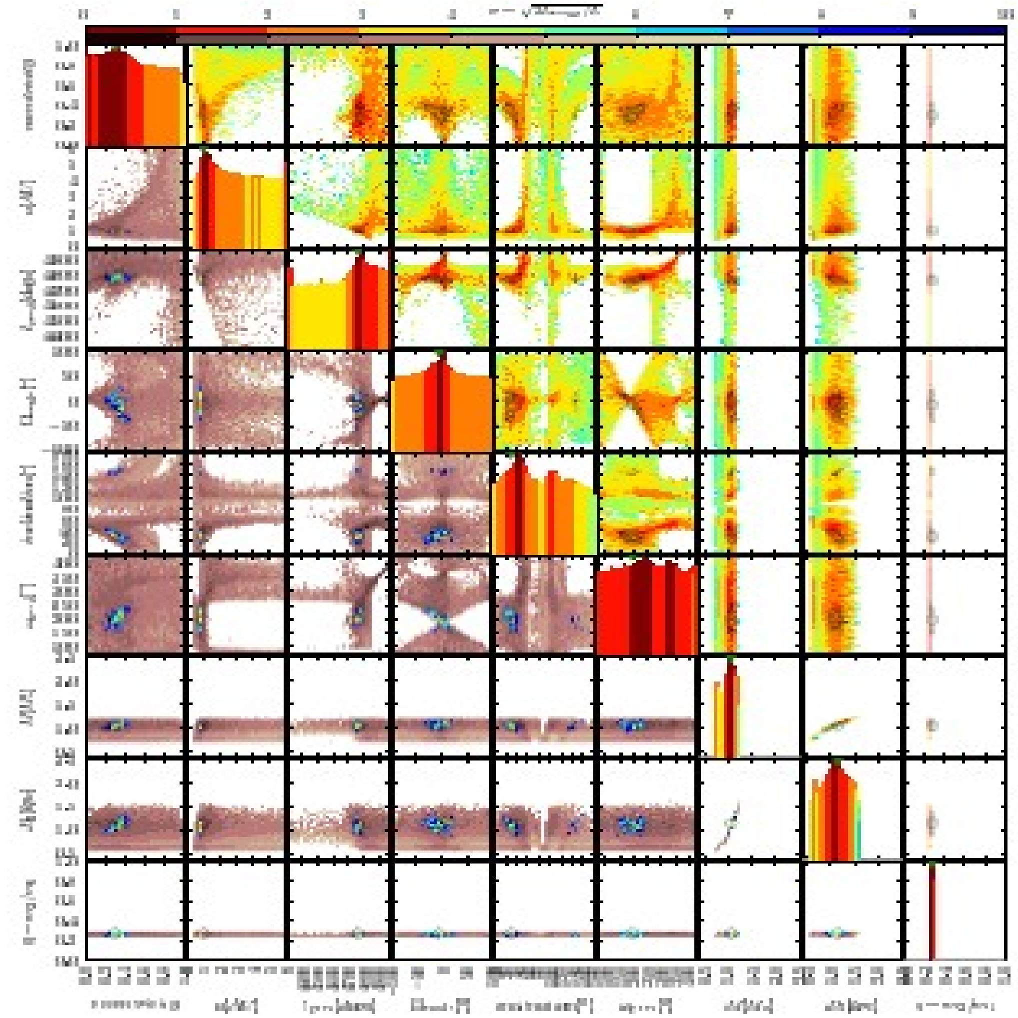
Predictions for RV



Skowron et al. 2011, ApJ, 738, 87

OGLE-2009-BLG-020

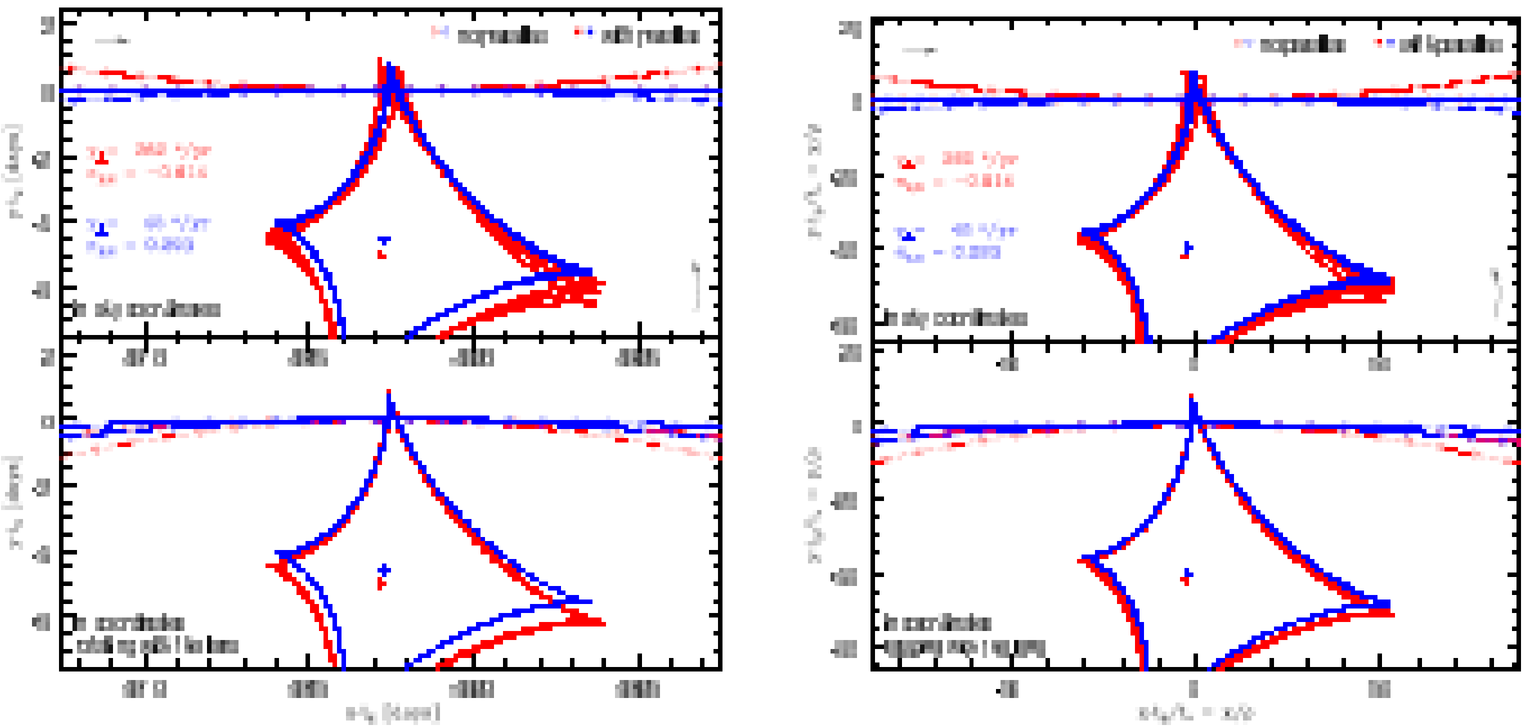
Predictions for RV



Skowron et al. 2011, ApJ, 738, 87

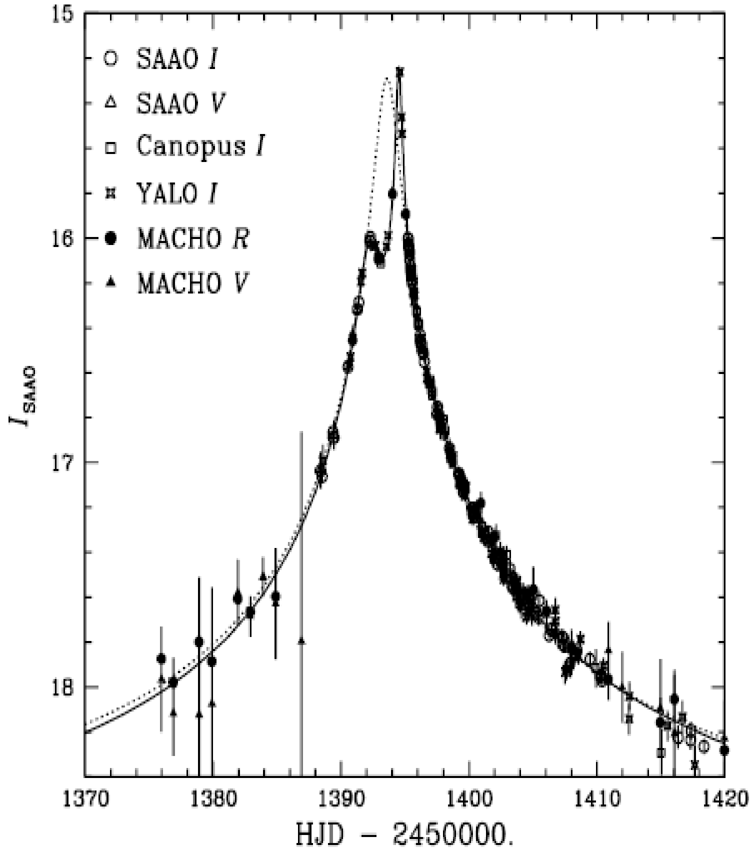
OGLE-2009-BLG-020

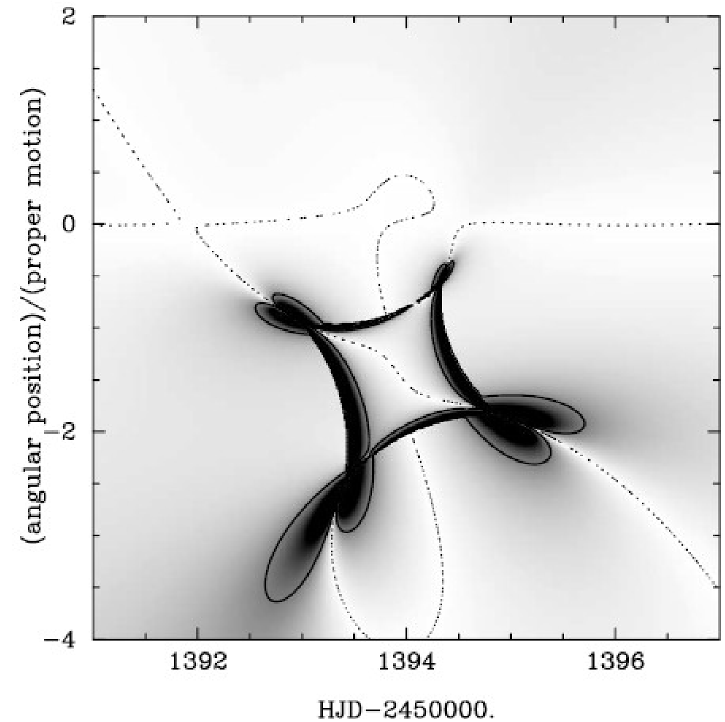
Predictions for RV

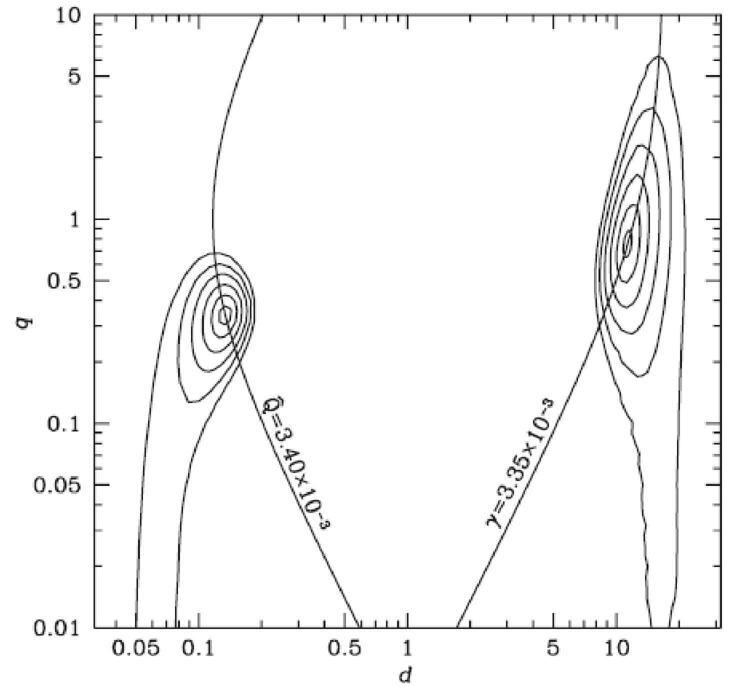


Skowron et al. 2011, ApJ, 738, 87

Macho-99-BLG-47







Jin An: Close/Wide Degeneracy (At Lowest Order) [d & q]

$$\zeta = z - \frac{\epsilon_1}{\bar z - d_c\epsilon_2} - \frac{\epsilon_2}{\bar z + d_c\epsilon_1}$$
$$\approx z - \frac{1}{\bar z} - \frac{d_c^2\epsilon_1\epsilon_2}{\bar z^3} + \frac{d_c^3\epsilon_1\epsilon_2(\epsilon_1-\epsilon_2)}{\bar z^4} + \cdots$$

$$\delta z_c \approx \hat Q\left(1 - \frac{1}{|z_0|^4}\right)^{-1}\left[\left(\frac{1}{\bar z_0^3} - \frac{1}{z_0^3\bar z_0^2}\right) + \left(\frac{1}{z_0^4\bar z_0^2} - \frac{1}{\bar z_0^4}\right)\frac{1-q_c}{1+q_c}d_c + \cdots\right]$$

$$\frac{\partial\zeta}{\partial\bar z} \approx \frac{1}{\bar z^2} + \frac{3\hat Q}{\bar z^4}\left[1 - \frac{4(1-q_c)}{3(1+q_c)}\frac{d_c}{\bar z} + \cdots\right]$$

$$\left|\frac{\partial\zeta}{\partial\bar z}\right|^2 \approx \frac{1}{|z_0|^4} + \hat Q\left[\frac{3|z_0|^4 - 2|z_0|^2 - 1}{|z_0|^8 - |z_0|^4}\left(\frac{1}{z_0^2} + \frac{1}{\bar z_0^2}\right) - \frac{4|z_0|^4 - 2|z_0|^2 - 2}{|z_0|^8 - |z_0|^4}\left(\frac{1}{z_0^3} + \frac{1}{\bar z_0^3}\right)\frac{1-q_c}{1+q_c}d_c + \cdots\right]$$

$$A^{-1} \approx \left|4\Delta - 2\hat Q\left(\frac{1}{z_0^2} + \frac{1}{\bar z_0^2}\right) + 3\hat Q\left(\frac{1}{z_0^3} + \frac{1}{\bar z_0^3}\right)\frac{1-q_c}{1+q_c}d_c\right|$$
$$= 4\left|(|z_0| - 1) - \hat Q\Re(z_0^{-2}) + \frac{3(1-q_c)}{2(1+q_c)}d_c\hat Q\Re(z_0^{-3})\right|$$

$$\zeta = z - \frac{1}{\bar z} - \frac{q_w}{\bar z + d_1}\,,$$
$$\approx z - \frac{1}{\bar z} - \frac{q_w}{d_1} + \frac{q_w}{d_1^2}\bar z - \frac{q_w}{d_1^3}\bar z^2 + \cdots \qquad (d_w \gg |z|)$$

$$\delta z_w \approx \gamma\left(1 - \frac{1}{|z_0|^4}\right)^{-1}\left[\left(\frac{z_0}{\bar z_0^2} - \bar z_0\right) + \left(\bar z_0^2 - \frac{z_0^2}{\bar z_0^2}\right)\frac{1}{(1+q_w)^{1/2}d_w} + \cdots\right]$$

$$\frac{\partial\zeta}{\partial\bar z} \approx \frac{1}{\bar z^2} + \gamma\left[1 - \frac{2}{(1+q_w)^{1/2}}\frac{\bar z}{d_w} + \cdots\right]$$

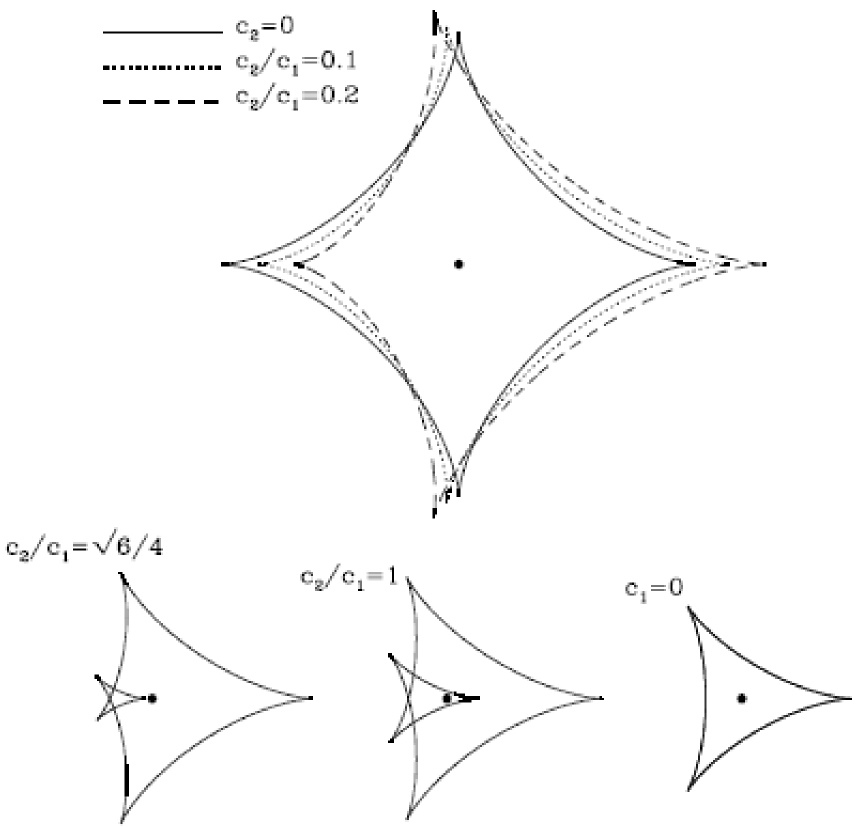
$$\left|\frac{\partial\zeta}{\partial\bar z}\right|^2 \approx \frac{1}{|z_0|^4} + \gamma\left[\frac{|z_0|^4 + 2|z_0|^2 - 3}{|z_0|^4 - 1}\left(\frac{1}{z_0^2} + \frac{1}{\bar z_0^2}\right) - \frac{2|z_0|^6 + 2|z_0|^4 - 4|z_0|^2}{|z_0|^4 - 1}\left(\frac{1}{z_0^3} + \frac{1}{\bar z_0^3}\right)\frac{1}{(1+q_w)^{1/2}d_w} + \cdots\right]$$

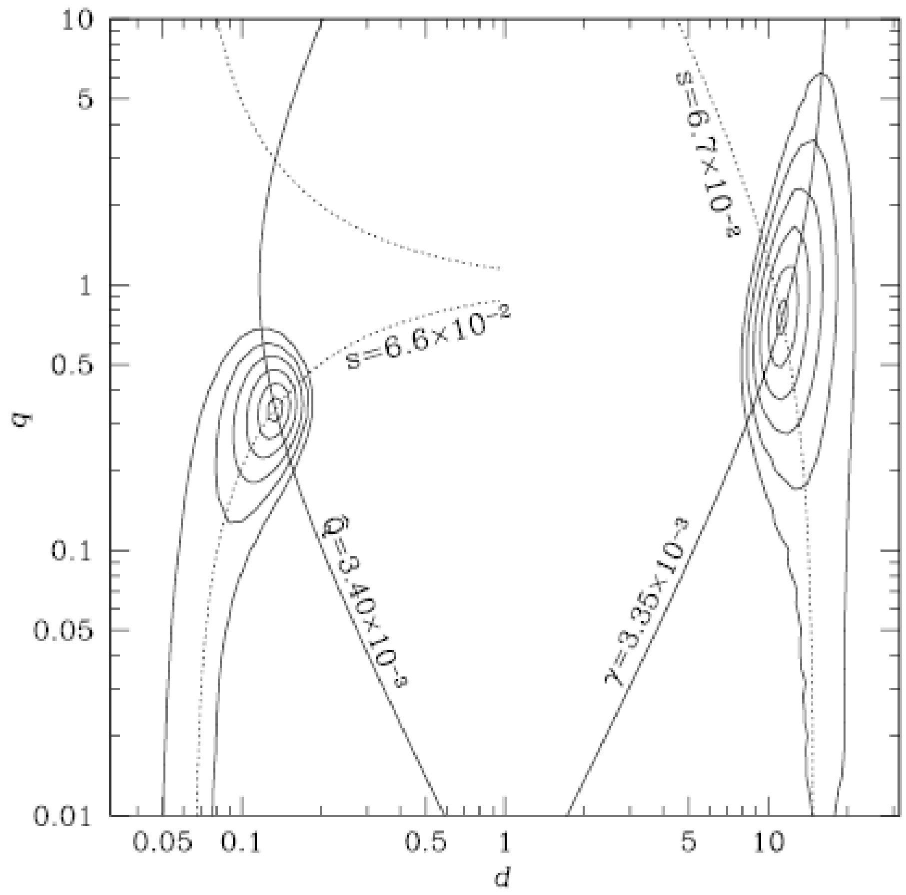
$$d_c^2 d_w^2(1+q_w) = \frac{q_w}{q_c}(1+q_c)^2\,,$$

$$A^{-1} \approx \left|4\Delta - 2\gamma\left(\frac{1}{z_0^2} + \frac{1}{\bar z_0^2}\right) + 3\gamma\left(\frac{1}{z_0^3} + \frac{1}{\bar z_0^3}\right)\frac{1}{(1+q_w)^{1/2}d_w}\right|$$
$$= 4\left|(|z_0| - 1) - \gamma\Re(z_0^{-2}) + \frac{3}{2(1+q_w)^{1/2}}\frac{\gamma}{d_w}\Re(z_0^{-3})\right|$$

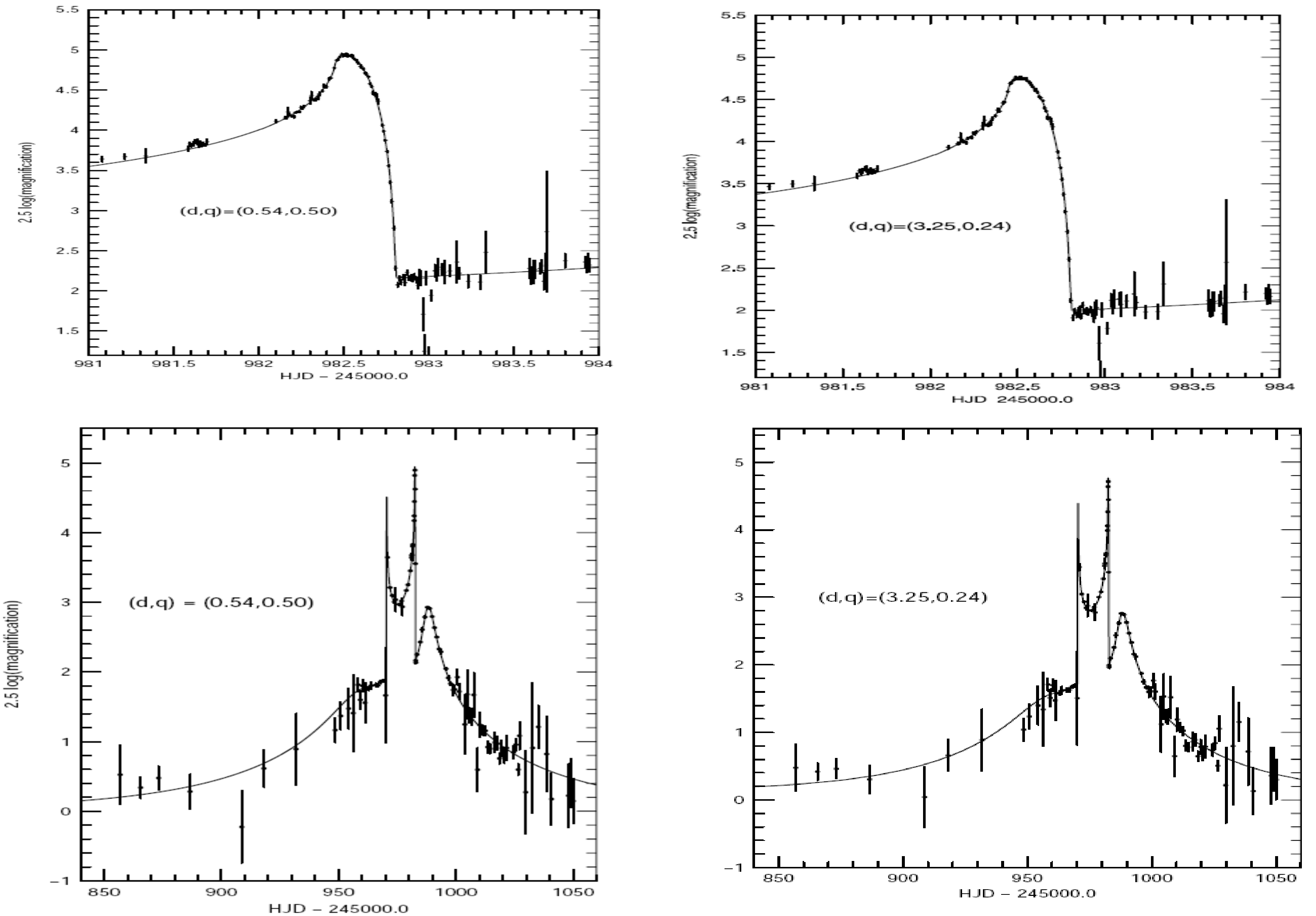
$$d_c d_w(1+q_w)^{1/2} = \frac{1+q_c}{1-q_c}\,,$$

Jin An: Wide/Close Degeneracy
(At Second Order)
[Shape Parameter: s = c_2/c_1]





Macho-98-SMC-1
Close/Wide Binary Degeneracy

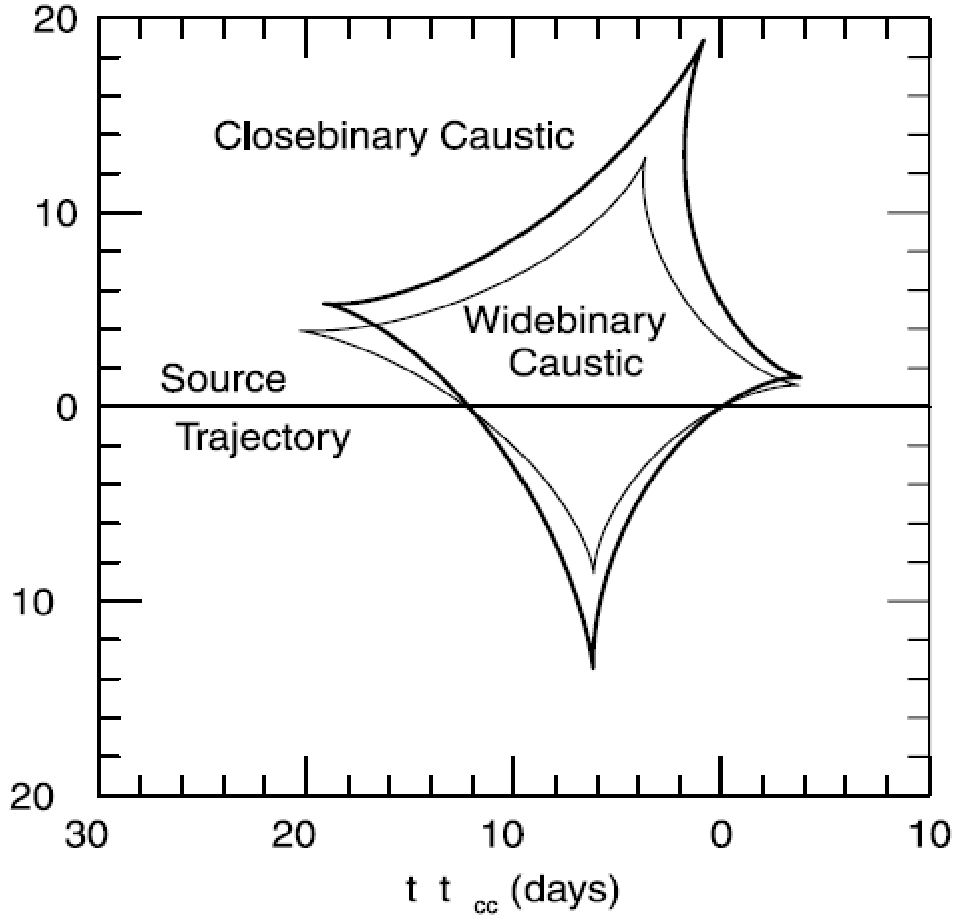


Different caustics -> Same lightcurve

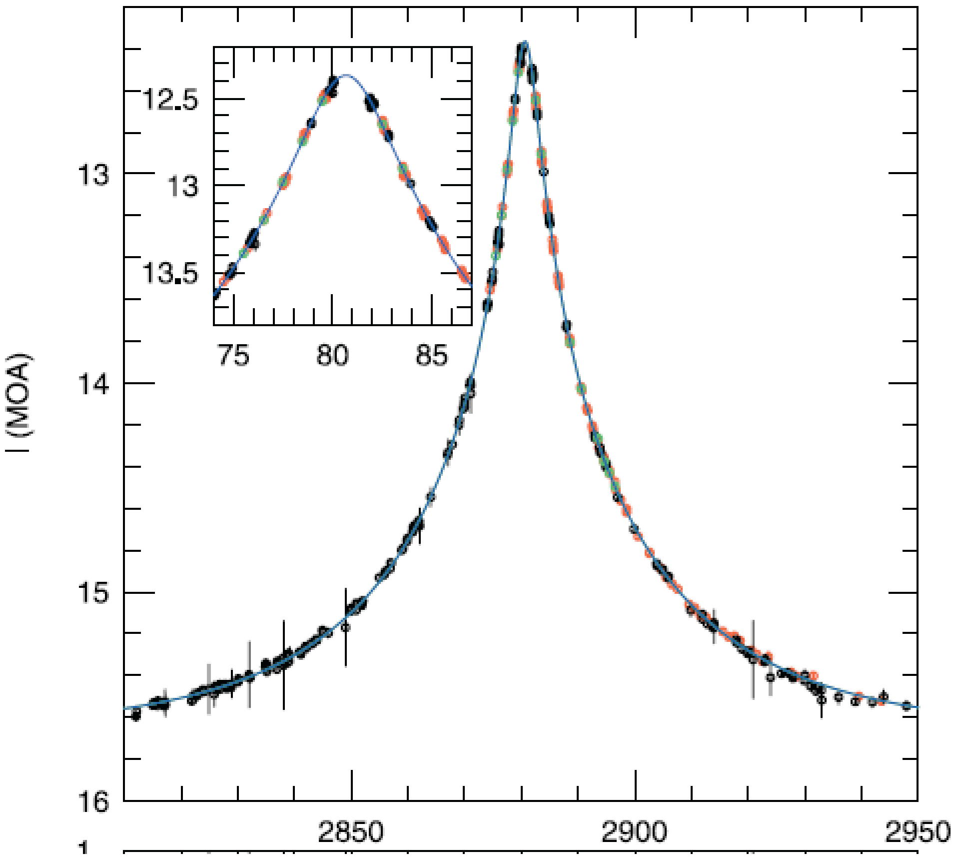
An 2005, MNRAS, 356, 1409

$$q_c \to 1 - \frac{\sqrt{1+4q_w} - 1}{2q_w}$$

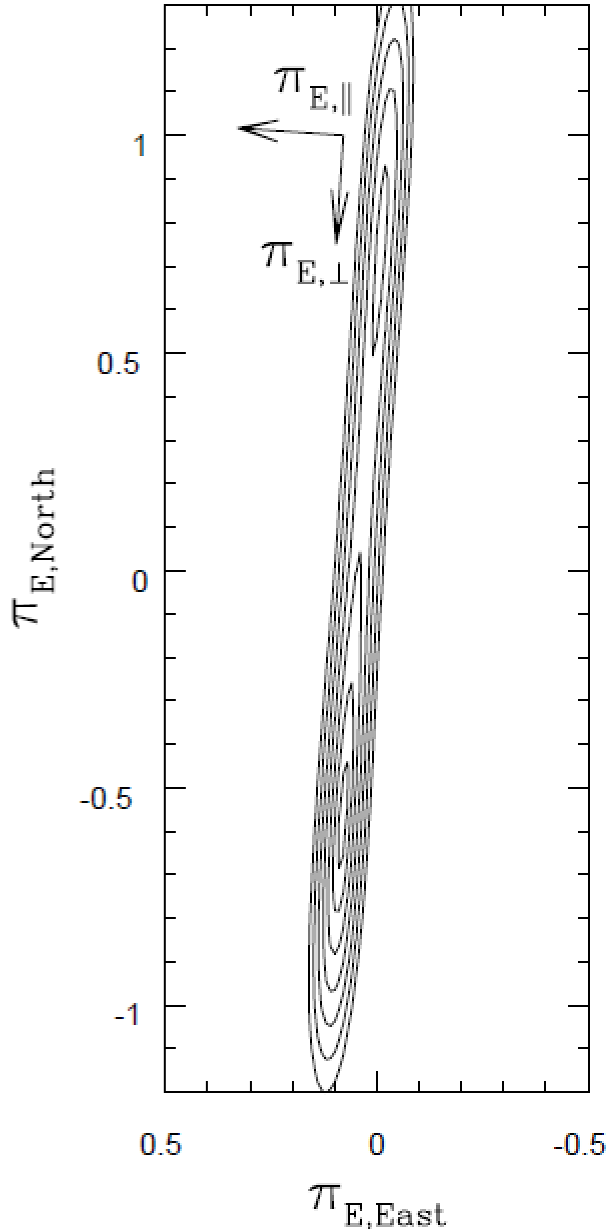
$$b_c \to b_w^{-1} \sqrt{\frac{1+4q_w}{1+q_w}}$$



1-D Parallaxes Are “Common”



MOA-2003-BLG-37
Park et al. 2004, ApJ. 609, 166

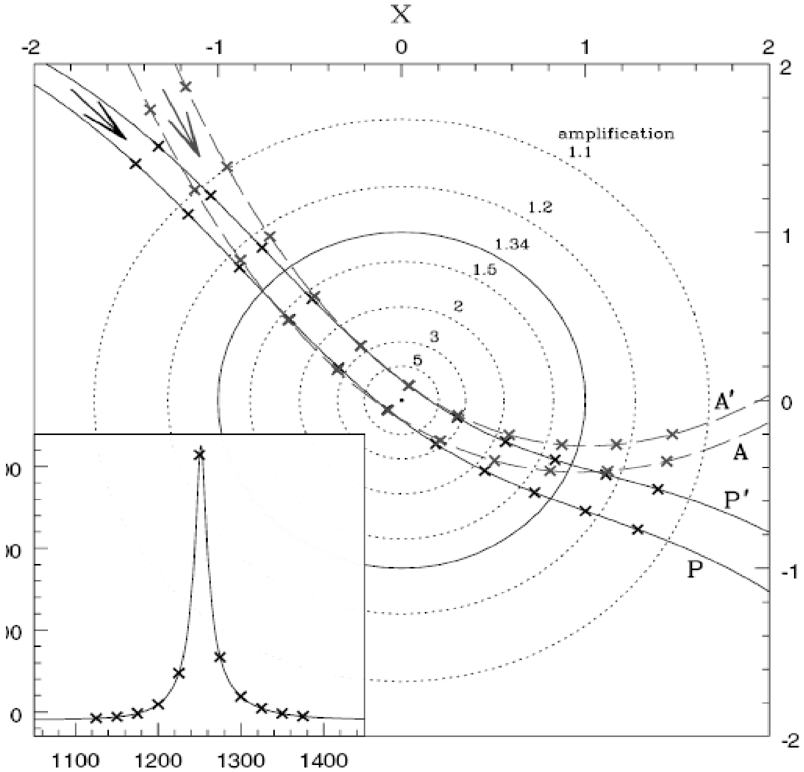


Ecliptic Degeneracy

Begins in 'constant acceleration' model

u_0 --> -u_0

Smith, Mao, & Paczynski (2003)



Ecliptic Degeneracy

Embedded in 'jerk parallax' formalism

u_0 --> -u_0

SMP (2003)

|u_0|<<1 ==> jerk-par

Gould (2004)

$$\pi'_{\mathrm{E},\parallel} = \pi_{\mathrm{E},\parallel}, \qquad \pi'_{\mathrm{E},\perp} = -(\pi_{\mathrm{E},\perp} + \pi_{j,\perp}),$$

$$\pi_{j,\perp} = -\frac{4}{3}\frac{\mathrm{yr}}{2\pi t_{\mathrm{E}}}\frac{\sin\beta_{\mathrm{ec}}}{\left(\cos^2\psi\sin^2\beta_{\mathrm{ec}} + \sin^2\psi\right)^{3/2}}$$

Ecliptic Degeneracy

Jiang et al.: Exact Degenercy (β_{ec}=0)

u_0 --> -u_0

|u_0|<<1 ==> jerk-par

$(u_0, \pi_{E,perp})$->-$(u_0, \pi_{E,perp})$

SMP (2003)

Gould (2004)

Jiang et al. (2004)

$$\pi_{j,\perp} = -\frac{4}{3}\frac{\mathrm{yr}}{2\pi t_{\mathrm{E}}}\frac{\sin\beta_{\mathrm{ec}}}{\left(\cos^2\psi\sin^2\beta_{\mathrm{ec}} + \sin^2\psi\right)^{3/2}}$$

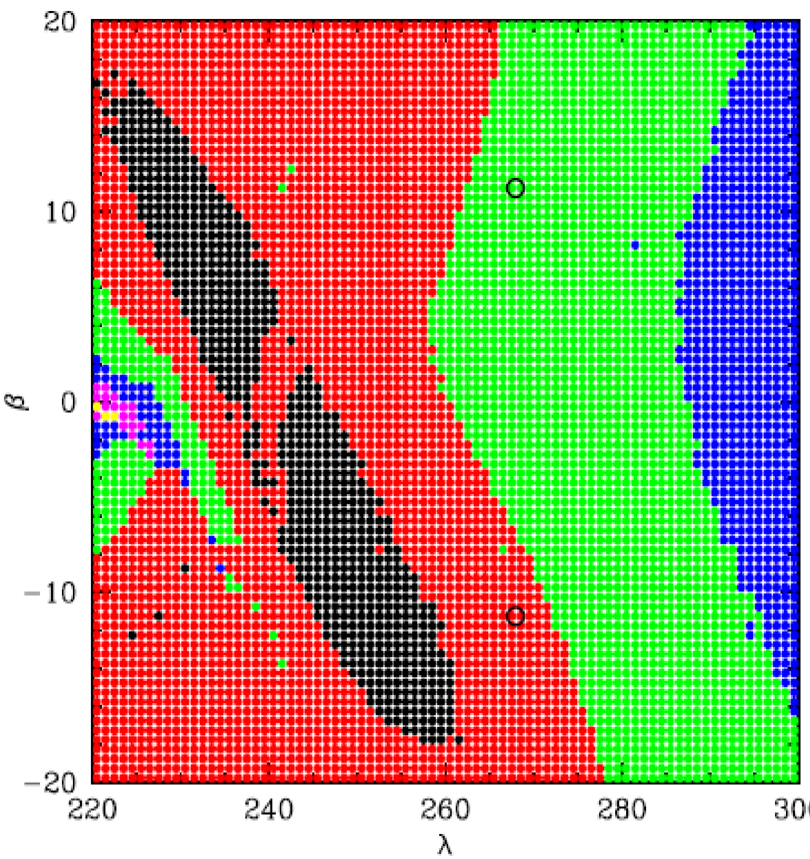
Ecliptic Degeneracy

Skowron et al. 2011, ApJ, 738,87

generalize to binaries

u_0 --> -u_0	SMP (2003)
|u_0|<<1 ==> jerk-par	Gould (2004)
$(u_0,\pi_{E,perp})$-> $-(u_0,\pi_{E,perp})$	Single
$(u_0,\pi_{E,perp},\alpha)$ -> $-(u_0,\pi_{E,perp},\alpha)$	Static Binary
$(u_0,\pi_{E,perp},\alpha_0,d\alpha/dt)$ -> $-(u_0,\pi_{E,perp},\alpha_0,d\alpha/dt)$	Rotating Binary

Xallarap vs. Parallax



Xallarap vs. Parallax