Problems Day 5
PhD school: Vietri Sul Mare 2018

Problem 1: Kitaev chain as a quantum error correcting code

Consider a finite Kitaev chain at the special point $t = \Delta$ and $\mu = 0$. Remember that the Hamiltonian can be written in terms of Majorana operators as

$$H = -t \sum_{1}^{N-1} i\gamma_{Bj}\gamma_{Aj+1}. \tag{1}$$

- Show that this Hamiltonian can be viewed as a stabilizer code with $N-1$ stabilizers $S_j = i\gamma_{Bj}\gamma_{Aj+1}$. Make sure that you understand what it means to use this code for active error correction.
- Identify the code space and the logical operators.
- Show that local errors effected by fermion operators appear in pairs for fermion parity preserving perturbations. Argue why these errors can be detected and corrected. Identify the code distance.
- Show that the code has no protection for parity changing perturbations.

Problem 2: Dislocations in the toric code

Try to define a toric code on a lattice with a dislocation line (Bombin, PRL 2010).

- Show that a pair of e quasiparticles can be turned into e and m quasiparticles by moving one of the e's around the end of a dislocation line.
- Show by simply counting qubits and stabilizers that such a dislocation line comes with localized Majorana zero modes (more accurately: Ising anyons) at its ends.